CHAMP Activity — September 29, 2014

Escape!

Today we explore escape strategies that use many different techniques in mathematics. Such problems are commonly encountered in nature, games, sports, or disaster situations.

1. **Pigeons coops:** There are 100 carrier pigeons in 100 separate **locked** rooftop cages. During the night each of 100 pigeon keepers visits the cages. The first keeper visits every cage. The second keeper visits cages 2,4,6,... etc (every 2nd cage), the third keeper visits cages 3,6,9,..etc (every third cage), the fourth keeper visits every fourth cage, and so on until the 100th keeper visits the 100th cage. On a visit each keeper unlocks the cage if it is locked or locks the cage if it is unlocked. If a cage remains unlocked after all keepers have completed their rounds, the pigeon inside can escape. In the morning, how many pigeons have escaped and why?

   Ten pigeons will have escaped, specifically the ones in the 1st, 4th, 9th, 16th, 25th, 36th, 49th, 64th, 81st, and 100th cages. Essentially we are looking for cages visited an odd number of times. In other words, what numbers 1-100 have an ODD number of factors, including 1 and themselves? For any number that is not square, each divisor will have a distinct partner, e.g., $6 = 2 \cdot 3$ **or** $18 = 6 \cdot 3$**. Square numbers have divisors that are unpaired, e.g.,** $16 = 4 \cdot 4$ **counts only one factor.**

2. **Escape from an active volcano:** Four people are on a volcanic island, and the volcano will explode and cover part of the island with lava in 15 minutes. There is a narrow bridge to safety, but it can only hold two people at a time. They have one torch and, because it's night, the torch has to be used when crossing the bridge. Ella can cross the bridge in 1 minute, Miles in 2 minutes, Nina in 5 minutes, and Riley in 8 minutes. When two people cross the bridge together, they must move at the slower person's pace. How can they all cross in 20 minutes? Is it possible for them all to cross in 15 minutes?

An unavoidable time sink here is carrying the torch back and forth across the bridge. Thus, we might think to just always have Ella be the torch carrier, since she's the fastest. In this case, we have:

- **EM cross: 2 minutes**
- **E runs back: 1 minute**
- **EN cross: 5 minutes**
- **E runs back: 1 minutes**
- **ER cross: 8 minutes**

This totals 17 minutes, under 20 minutes. Notice we can cut time further if we get our most careful movers, Nina and Riley, to cross together. To do this, have:

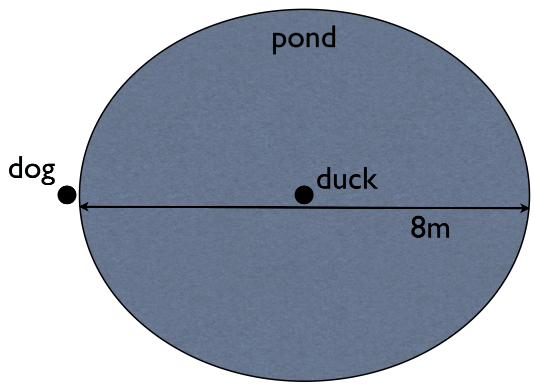
- **EM cross: 2 minutes**
- **E runs back: 1 minute**
- **NR cross: 8 minutes**
- **M runs back: 2 minutes**
- **EM cross: 2 minutes**

for a total of 15 minutes

3. **Duck in a pond:** A duck is in the center of a **circular** pond 8 meters across but he cannot take flight from the water, only on land. On the perimeter of the lake there is a hunting dog that always runs to be as close to the duck as he can. The duck must make it to the land before taking off and must do so before the dog makes it to him. The dog is 3.5 times faster than the swan and always runs to the point around the lake closest to the swan.

If the duck swims straight for the opposite edge of the pond, will it be caught by the dog? Why or why not?

How can the duck get out of the lake and take flight before the dog gets him?



If the duck swims straight for the opposite edge, it will get there in 4 time units (time unit=time it takes duck to swim 1 meter). The dog will already be there, since it will take the dog $4\pi/3.5 < 4$ **time units to get there.**

Here's one strategy to escape the dog, the duck should swim 1/4 of the way to the shore and then start swimming in circles. Since the duck is swimming a circle 1/4 the radius of the dog, it will always be able to do it faster, since (1/4 length)/(1 m/time)) < (1 length)/(3 m / time). Thus, the duck can eventually get to the furthest place on the circle from the dog and make a break in the opposite direction of the dog. This will take the duck 3 time units, while it will take the dog $4\pi/3.5 > 12/3.5 > 3$ **time units.**

4. **Hurricane evacuation:** Now we focus on the problem of many individuals escaping at one time. We will take a specific example, evacuating Houston during a hurricane. People want to go inland, away from the hurricane. They should thus go one of four ways: west on I-10, north on I-45, northwest on 290, or north on 59. The time it takes for people to get to safety along these routes is directly proportional to how many thousands of cars (x) are on them, in the following way:

$$\begin{aligned} &\mathrm{I}-10: \quad x/10 \text{ hours},\\ &\mathrm{I}-45: \quad x/5 \text{ hours},\\ &290: \quad x/2 \text{ hours},\\ &59: \quad x/5 \text{ hours}. \end{aligned}$$

Rick Flanagan, director of emergency management, wants to get 44,000 cars full of people to safety as soon as possible. How should he split them up on these four routes?

This problem actually has similarities to the bridge and torch problem - the total evacuation time will always be as slow as the slowest route. Therefore, there is no use having one route be way faster than the others. The optimal solution is 20,000 cars on I-10; 10,000 on I-45; 4,000 on 290; 10,000 on 59. Each route will take two hours each. One way to figure this out is trial and error. Another is to use algebra. You'll just need to generate 3 equations for all routes taking the same time:

- $x/10 = y/5$
- $y/5 = z/2$
- $(44 - x - y - z)/5 = y/5$

This has a unique solution given by $x = 20$**,** $y = 10$**,** $z = 4$**.**