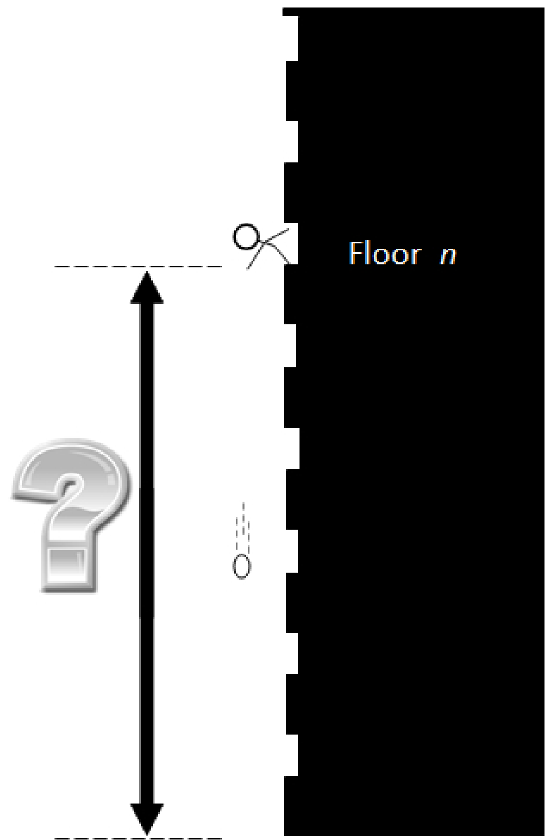
CHAMP Activity — October 13, 2014

The Egg Drop Problem: Explorations in Search Strategies

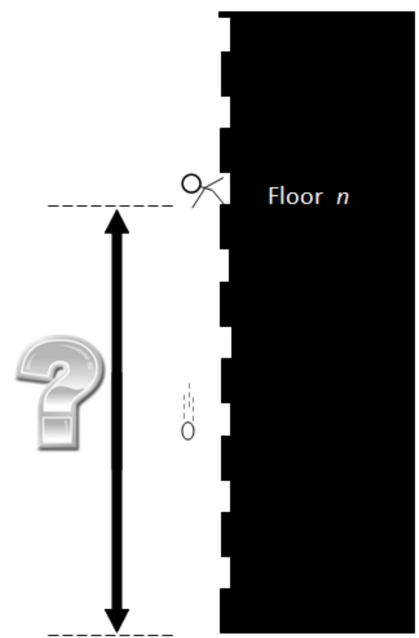
1. **Warm-up Question:** Consider the following situation. You have an egg and a 100 floor building. You would like to determine the highest floor you can drop the egg from without it breaking. Once the egg breaks, you are done - you can't collect any more information. What strategy should you use to determine the highest floor you can drop the egg on, without it breaking? What is the maximum number of times you will have to drop the egg?



Start at the first floor. Drop the egg. If it does not break, pick it up and go to the 2nd floor and drop. If it does not break, pick it up and go to the 3rd floor and drop. Repeat until the egg drops on the nth floor. Then, you know the $(n-1)$th floor is the highest floor you can drop from. Since there are 100 floors, we could have up to 100 drops.

2. **Two eggs:** Here is a problem typically used as a Google interview questions. You have two eggs, identical to one another. You would like to know the highest floor of a 100 floor building you can drop them from without them breaking. How can you do this in the fewest drops possible?

   - Hint #1: Try something different than the strategy you used with 1 egg.
   - Hint #2: Think about how you can move up the building more quickly than one floor at a time.
   - Hint #3: The best strategy makes use of the formula $n + (n-1) + (n-2) + \cdots + 2 + 1 = (n+1)n/2 = 100$.



Our "one egg" strategy will work, but it will be very slow. Basically, it may still take up to 100 drops.

Consider instead skipping floors. What if we try dropping from every other floor (2, 4, 6, 8, 10, ...) with the egg #1? Then, once egg #1 breaks, drop from the floor in between the previous and current floor to determine the precise max floor to drop from. This will only take us 50+1=51 drops at most.

We can do even better though if we skip every 3 floors. Then go back and go up one by one once egg 1 breaks. That would only take us up to 33+2=35 drops (you can work this out).

The best number, using this skipping strategy, is to skip every 10 floors (10, 20, 30, 40, ...), then it will take us up to 10+9=19 drops.

Amazingly, we can do even better by varying how many floors we skip as we go up the building. It turns out the optimal solution is 14,27,39,50,60,69,77,84,90,95,99,100. Then stop and go one at a time once the first egg breaks. To find this sequence, note that we can cap the number of drops at n**, if we skip** n, $n-1$, $n-2$, **....,** 2, 1 **floors and then return to the previous plus 1 once egg 1 breaks. To find this number** n**, we just want to know the minimum integer such that** $n+(n-1)+(n-2)+\cdots+2+1 \geq 100$**. Just solve the quadratic equation** $n(n+1)/2 = 100$ **or** $n^2+n-200=0$**, so** $(-1+\sqrt{801})/2 < 14$**.**

3. **As many eggs as we like:** What is the best strategy if we have as many eggs as we like?

Here, we would like to bite off as much as possible with each drop. To do so, drop on the 50th floor. If it breaks, drop on the 25th, if not drop on the 75th. Then, drop on the floor in between where it broke and where it did not. For instance, if it breaks on 75th, then drop on the 63rd. If it breaks there, drop on 57th...

4. **Varying the number of floors:** What is the best strategy if we have two eggs and 25 floors?

First try: Skip 5 floors with egg 1 (5,10,15,20,25). Then go up one by one, once egg 1 breaks. This yields a maximum of 4+5=9 drops.

Best strategy: Solve $(n+1)n/2 = 25$, $n^2 + n - 50 = 0$, $(-1 \pm \sqrt{1+200})/2 < 7$. **Then skip to the 7th, 13th, 18th, 22nd, 23rd, 24th, 25th. If the egg breaks on any of these floors, return to the previous floor plus 1, counting up until it breaks. The previous floor is then the maximal floor. This yields a maximum of 7 drops. Note, we cannot do fewer, since 6, 11, 15, 18, 20, 21 terminates before 25.**

5. **Varying the number of eggs:** What is the best strategy if we have three eggs and 100 floors?