

A composite image of the Orion nebula, showing vibrant, multi-colored gas clouds in shades of blue, green, yellow, orange, and red, with numerous bright stars scattered throughout. The nebula's structure is complex, with various filaments and regions of higher density.

The Cosmic Distance Ladder

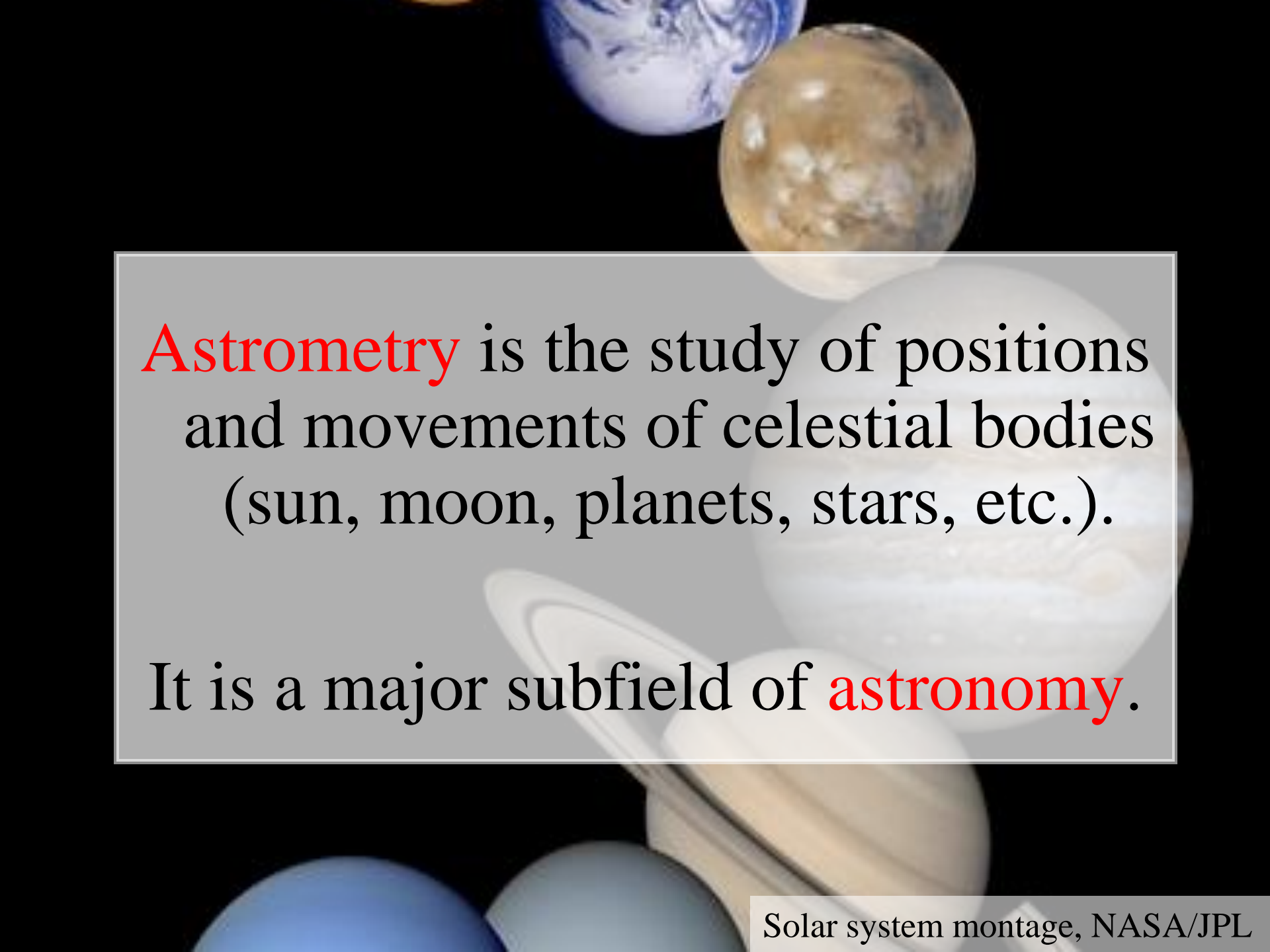
Terence Tao (UCLA)

Orion nebula, Hubble & Spitzer telescopes, composite image, NASA/JPL



Astrometry

Solar system montage, NASA/JPL



Astrometry is the study of positions and movements of celestial bodies (sun, moon, planets, stars, etc.).

It is a major subfield of **astronomy**.

A collage of celestial bodies including Earth, the Moon, Saturn, and other planets. The Earth is at the top left, the Moon is at the top right, Saturn is in the middle right, and other planets are at the bottom.

Typical questions in astrometry are:

- How far is it from the Earth to the Moon?
- From the Earth to the Sun?
- From the Sun to other planets?
- From the Sun to nearby stars?
- From the Sun to distant stars?

These distances are far too vast to be measured **directly**.

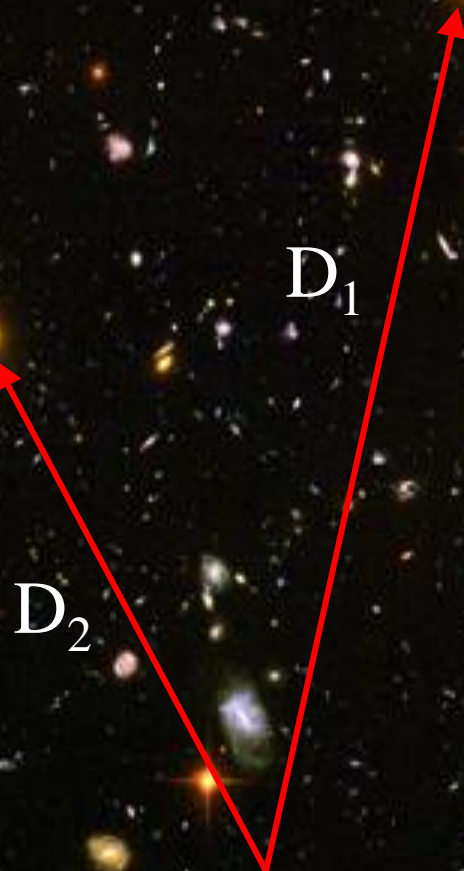
D_2

D_1

$D_1 = ???$

$D_2 = ???$

Nevertheless, there are several ways to measure these distances **indirectly**.



$$D_1 / D_2 = 3.4 \pm 0.1$$

The methods often rely more on **mathematics** than on technology.

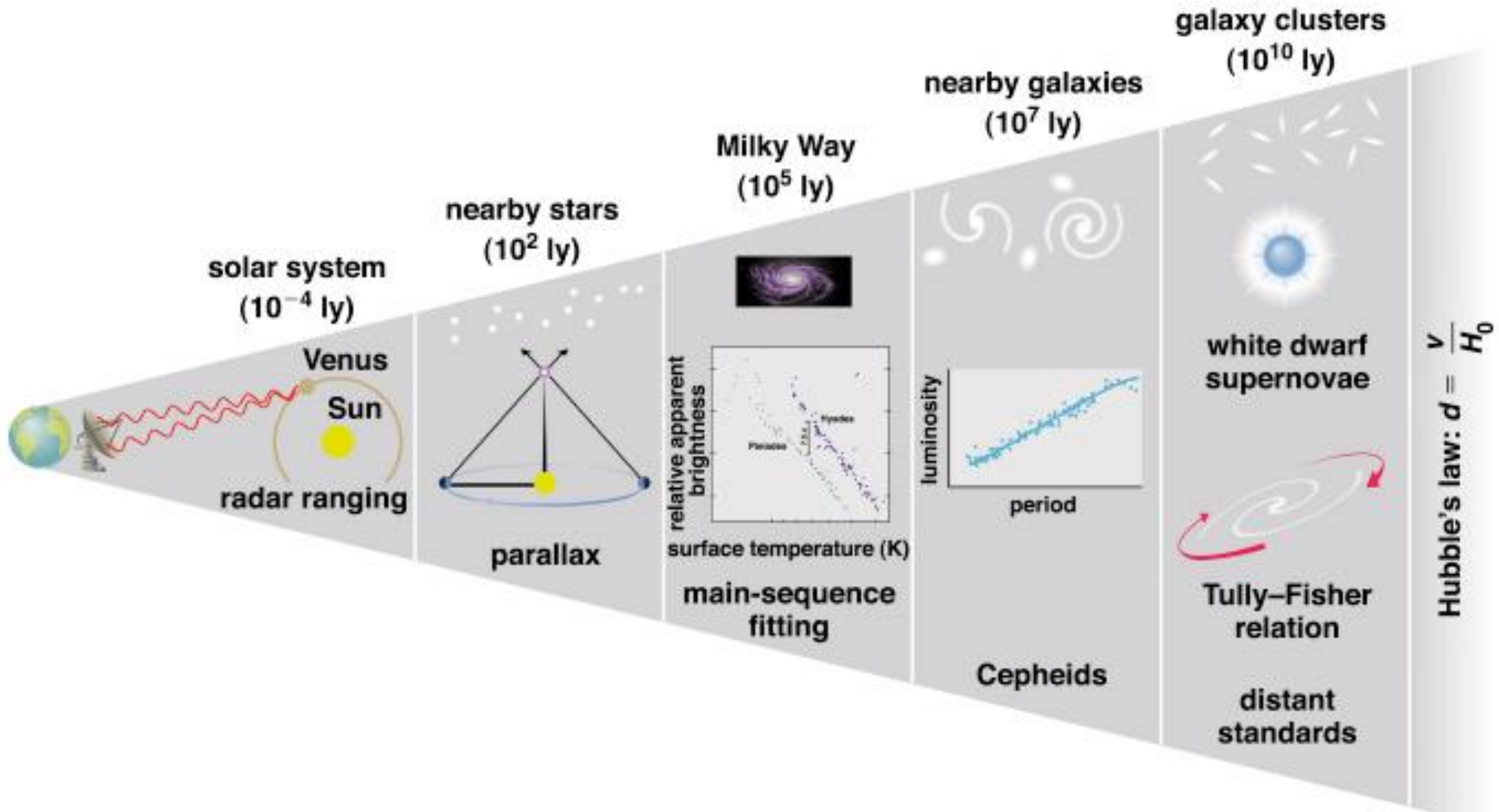
D_2

D_1

$$\begin{aligned}v_1 &= H D_1 \\v_2 &= H D_2 \\v_1 / v_2 &= 3.4 \pm 0.1\end{aligned}$$

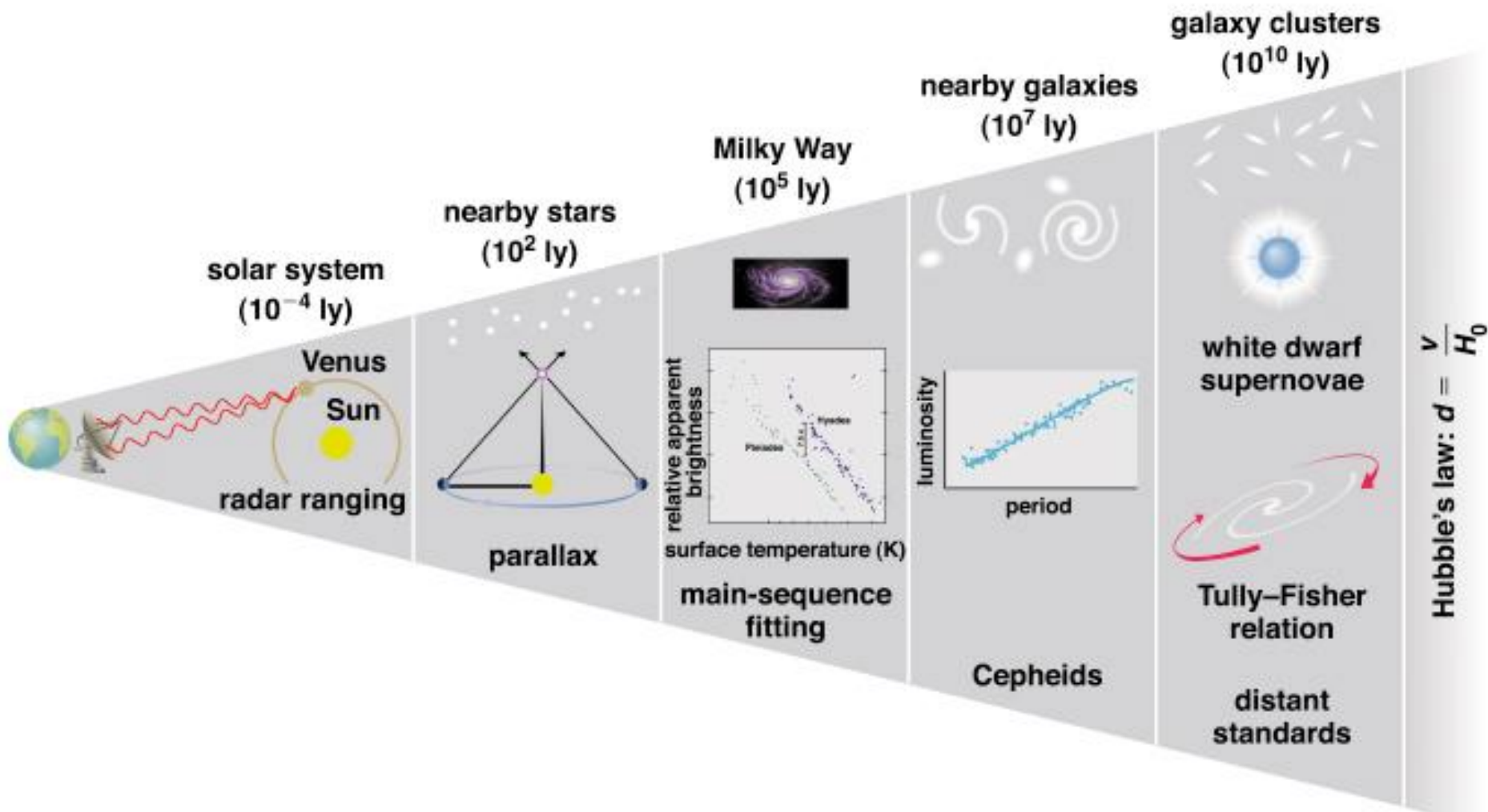
$$D_1 / D_2 = 3.4 \pm 0.1$$

The indirect methods control large distances in terms of smaller distances.



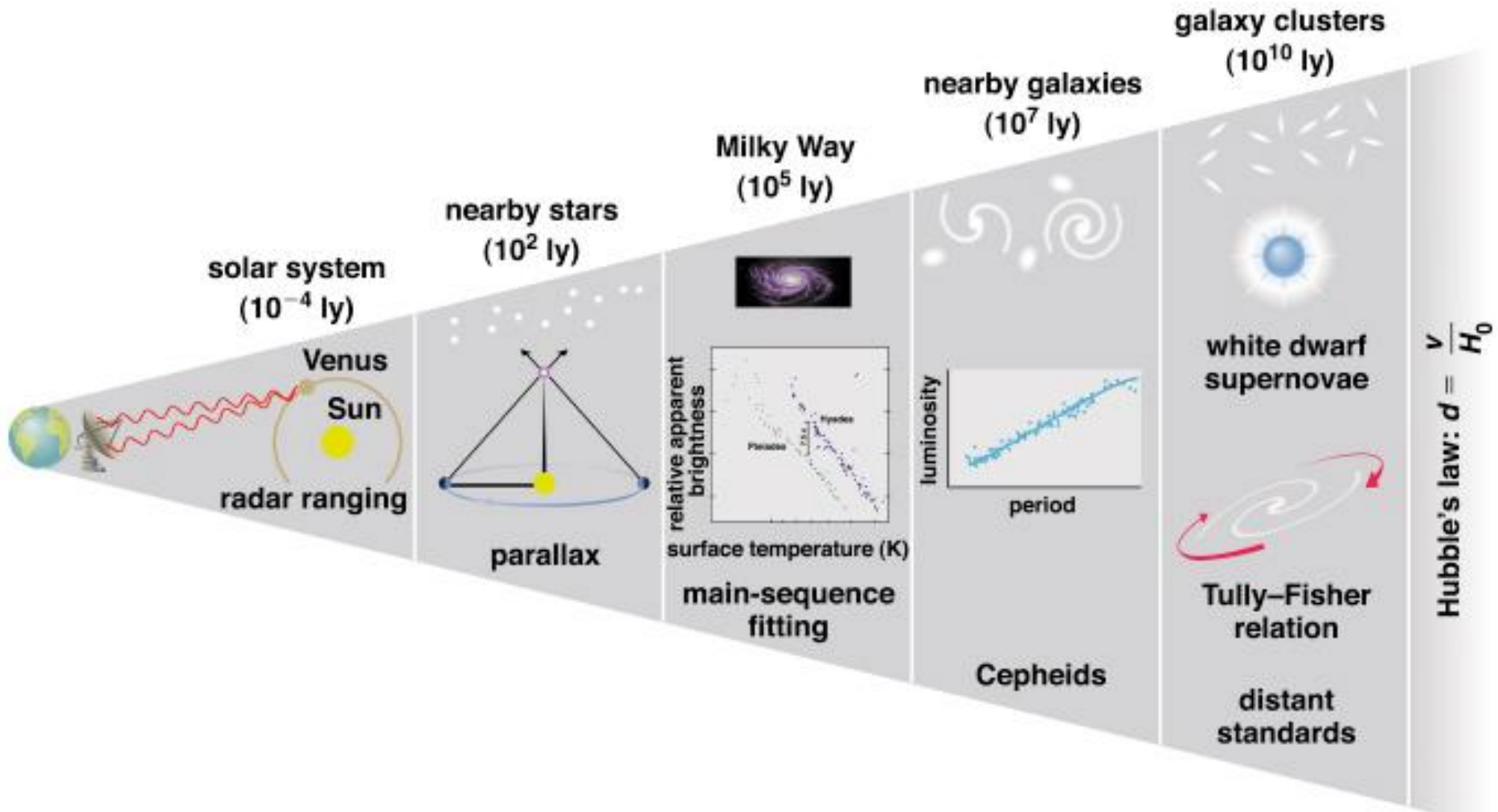
From “The Essential Cosmic Perspective”, Bennett et al.

The smaller distances are controlled by even smaller distances...



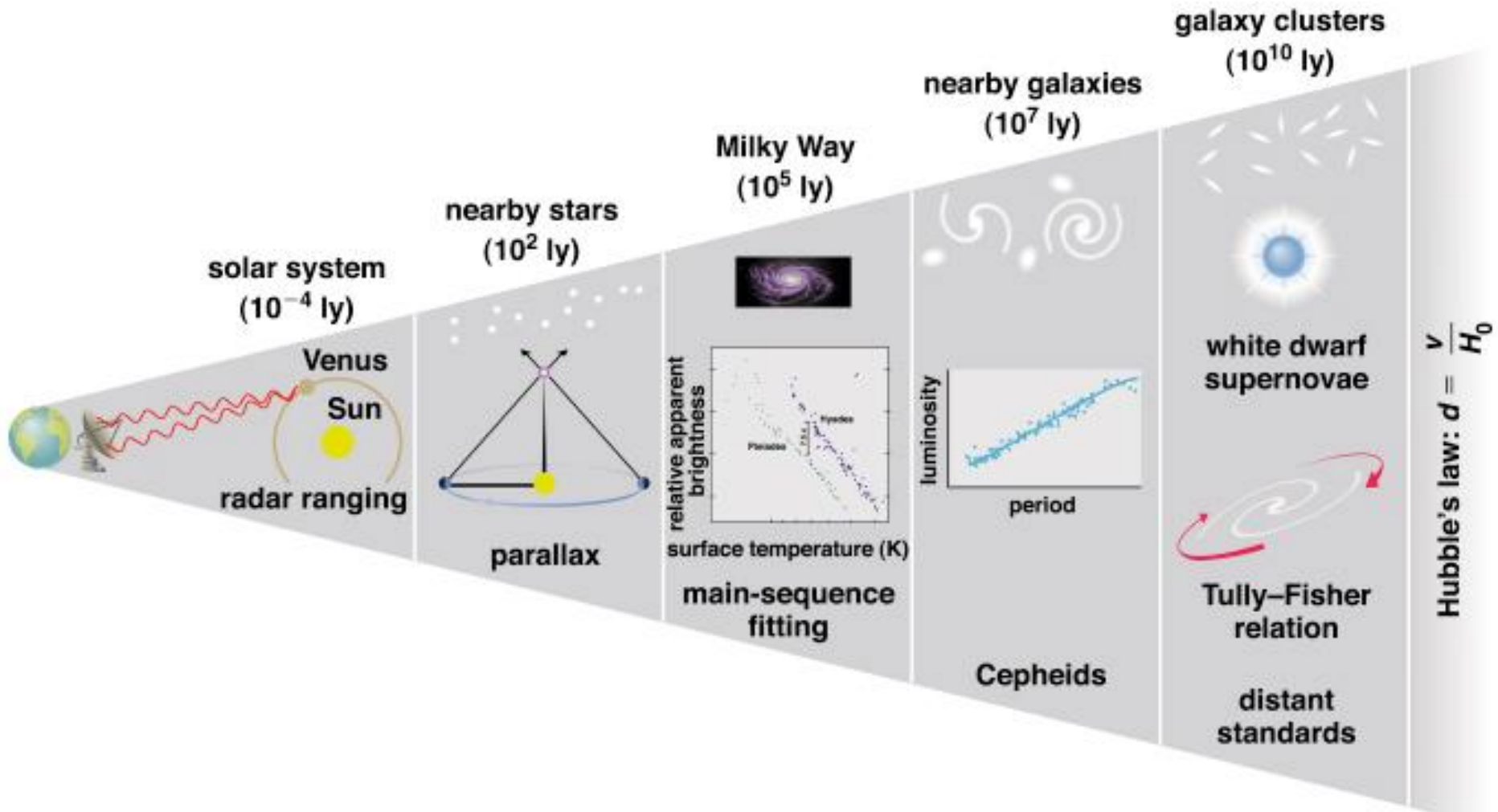
From “The Essential Cosmic Perspective”, Bennett et al.

... and so on, until one reaches distances that one can measure directly.



From “The Essential Cosmic Perspective”, Bennett et al.

This is the **cosmic distance ladder**.

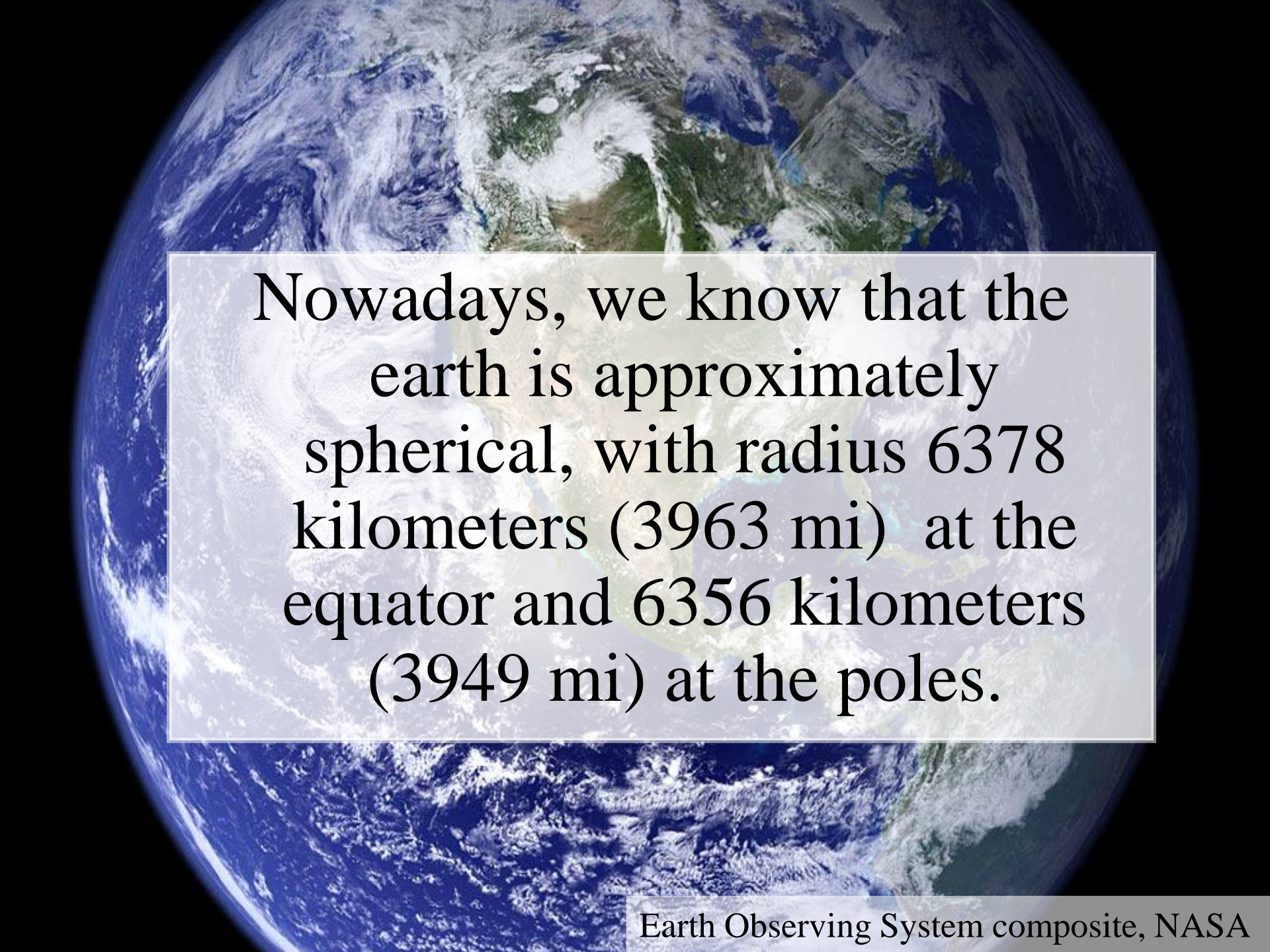


From “The Essential Cosmic Perspective”, Bennett et al.




1st rung: the Earth

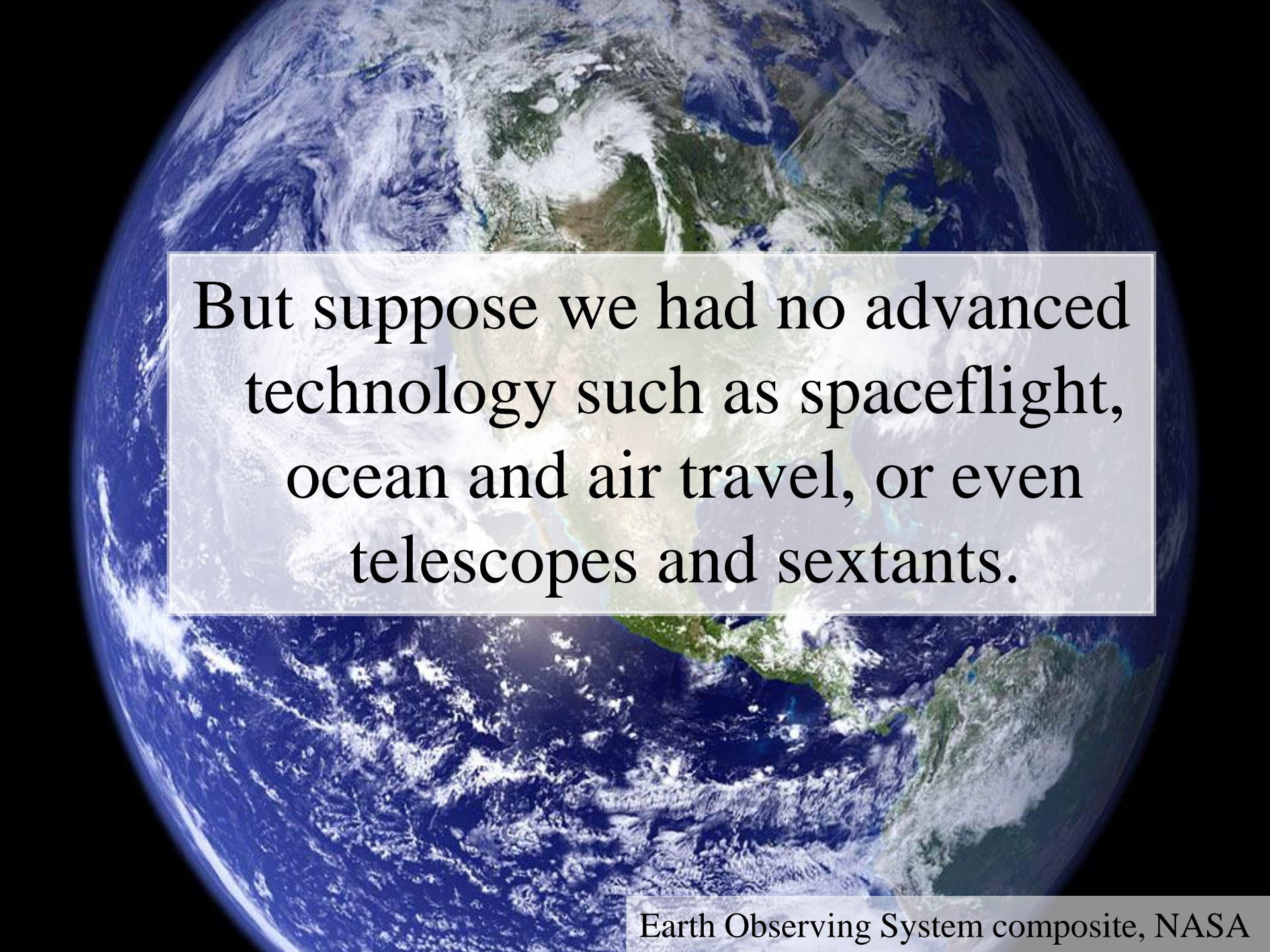
Earth Observing System composite, NASA



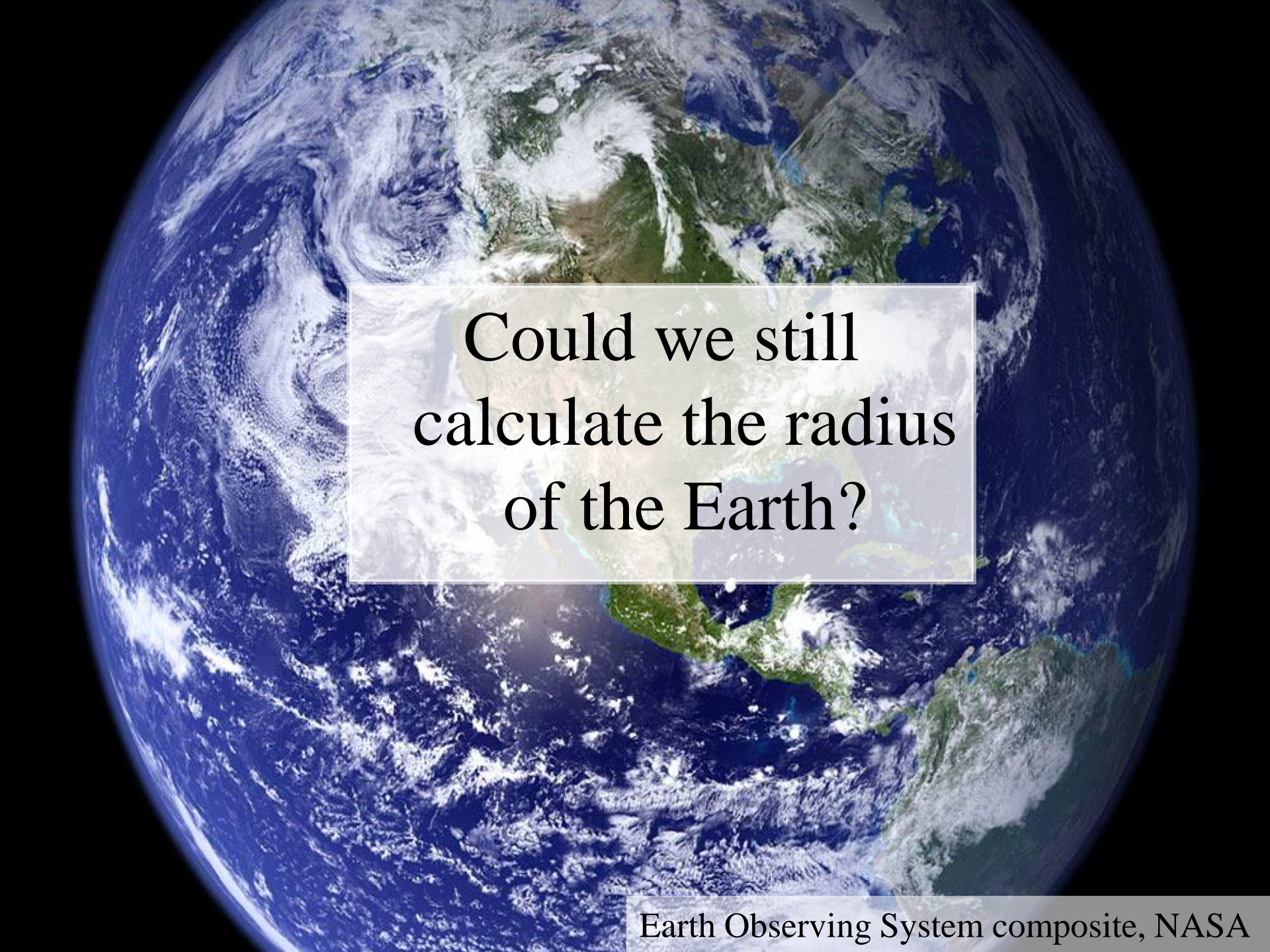
Nowadays, we know that the earth is approximately spherical, with radius 6378 kilometers (3963 mi) at the equator and 6356 kilometers (3949 mi) at the poles.




These values have now been
verified to great precision by
many means, including modern
satellites.



But suppose we had no advanced technology such as spaceflight, ocean and air travel, or even telescopes and sextants.



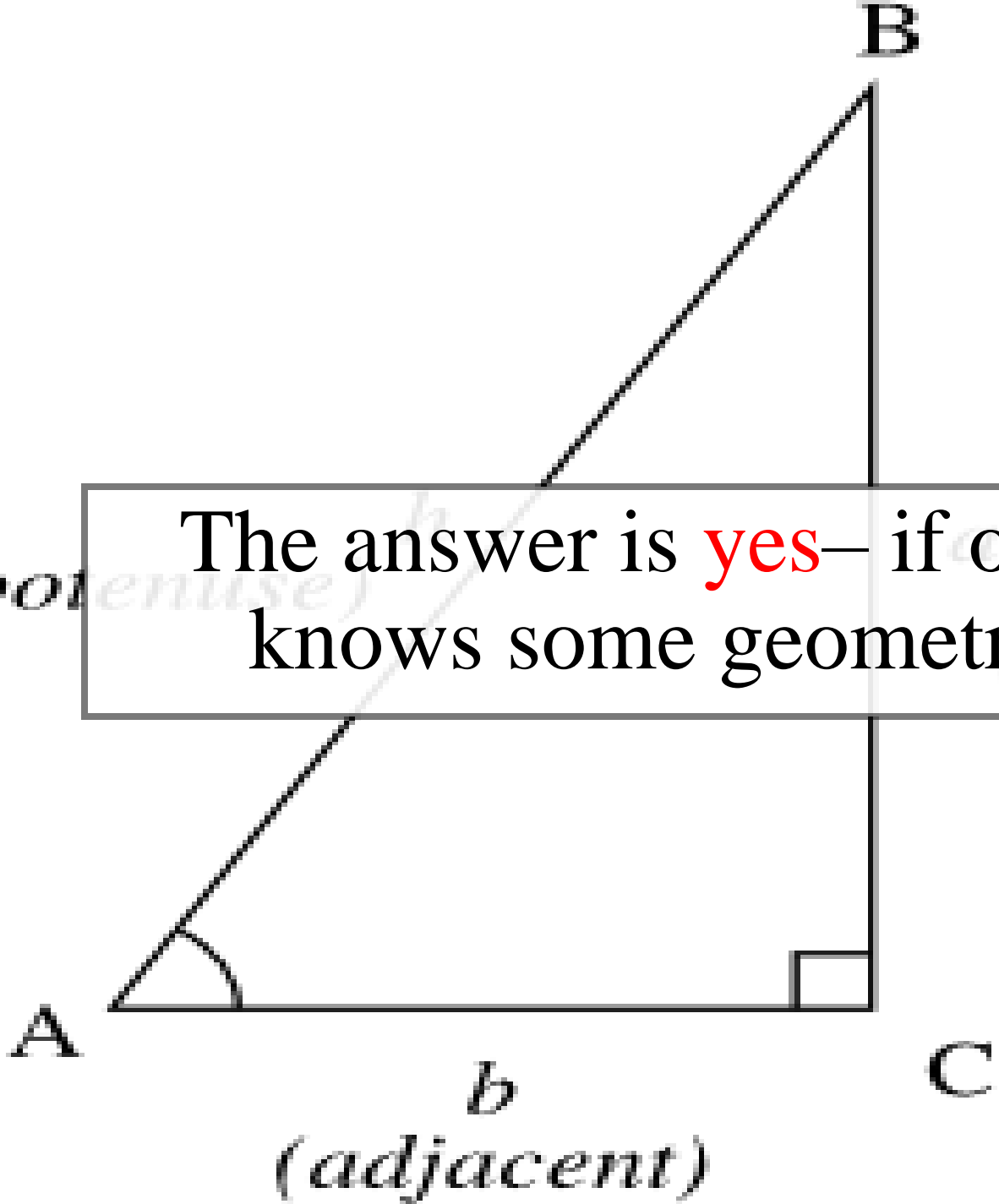
Could we still
calculate the radius
of the Earth?

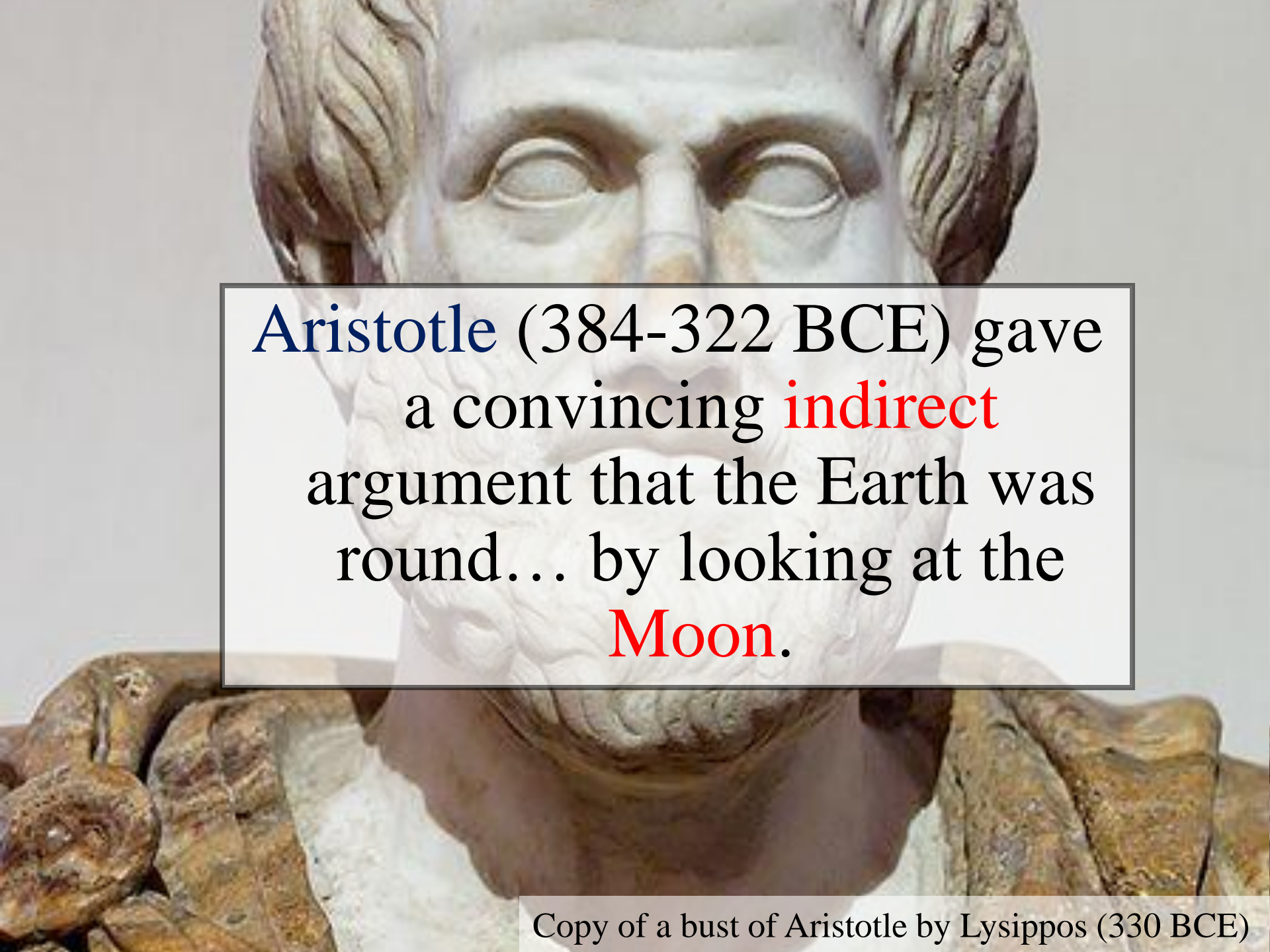
A composite satellite image of Earth from the Earth Observing System, showing the Americas and surrounding oceans with a text overlay.

Could we even tell
that the Earth was
round?

Earth Observing System composite, NASA


The answer is **yes**— if one knows some geometry!



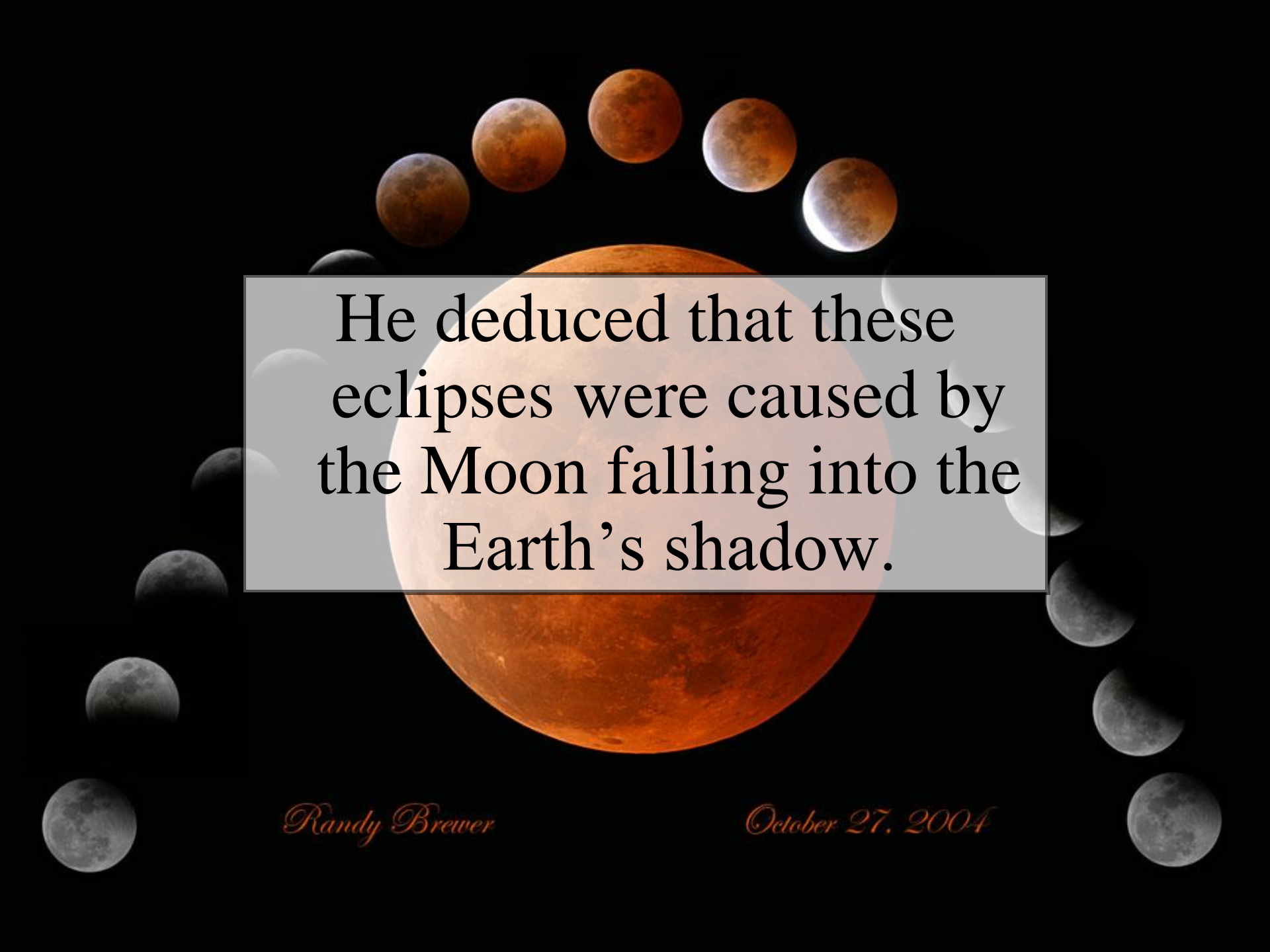
A marble bust of Aristotle, showing his face and curly hair. The bust is set against a light background. A white text box with a black border is overlaid on the bust, containing text about Aristotle's argument for a round Earth.

Aristotle (384-322 BCE) gave
a convincing **indirect**
argument that the Earth was
round... by looking at the
Moon.


Copy of a bust of Aristotle by Lysippos (330 BCE)




Aristotle knew that **lunar eclipses** only occurred when the Moon was directly opposite the Sun.



He deduced that these eclipses were caused by the Moon falling into the Earth's shadow.



But the shadow of the
Earth on the Moon in an
eclipse was always a
circular arc.

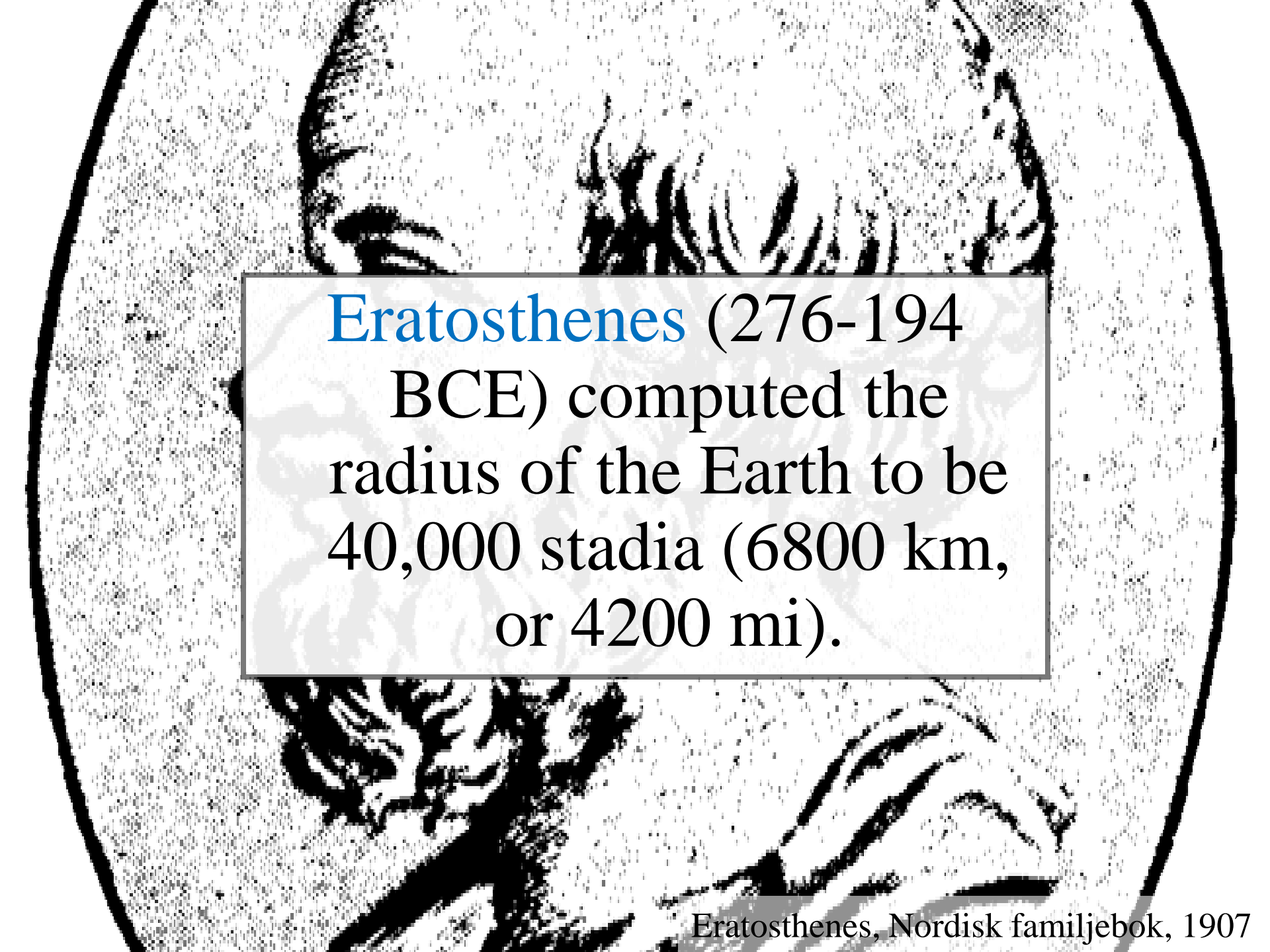


In order for Earth's shadows to always be circular, the Earth must be round.

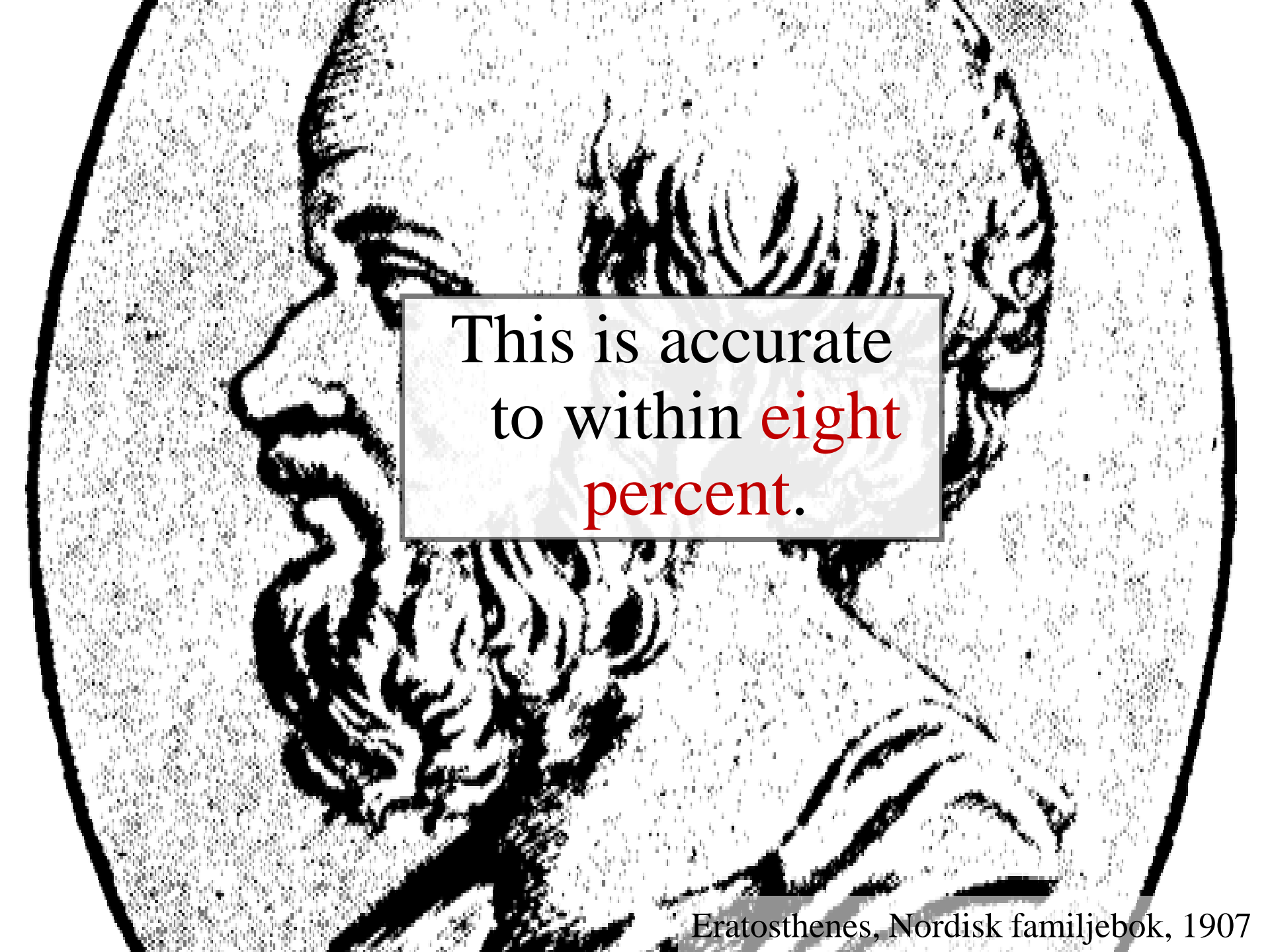
Aristotle also knew there
were stars one could see
in Egypt but not in
Greece.

He reasoned that this was due to the curvature of the Earth, so that its radius was finite.

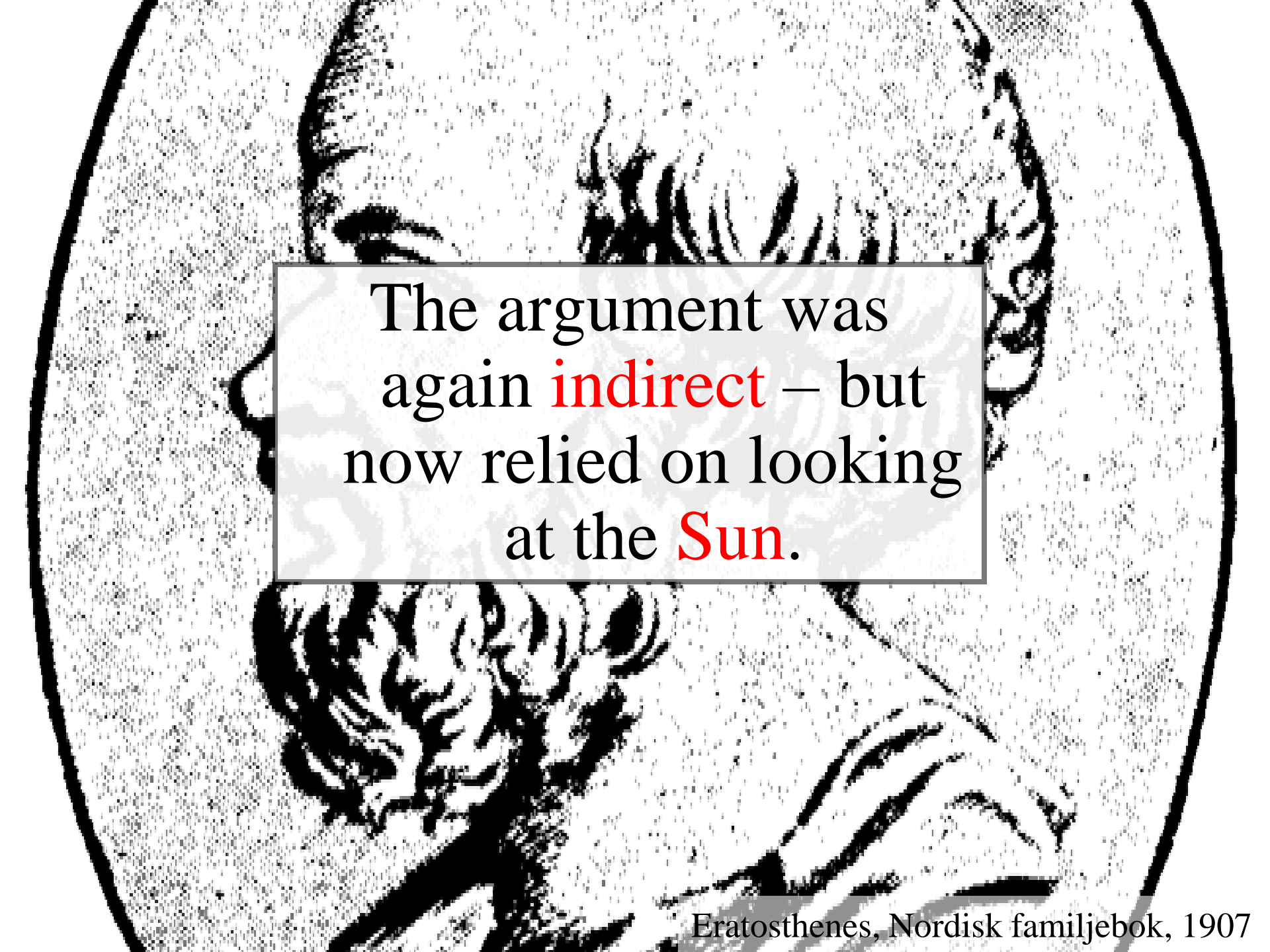
However, he was unable to
get an accurate
measurement of this
radius.



Eratosthenes (276-194
BCE) computed the
radius of the Earth to be
40,000 stadia (6800 km,
or 4200 mi).



This is accurate
to within **eight**
percent.

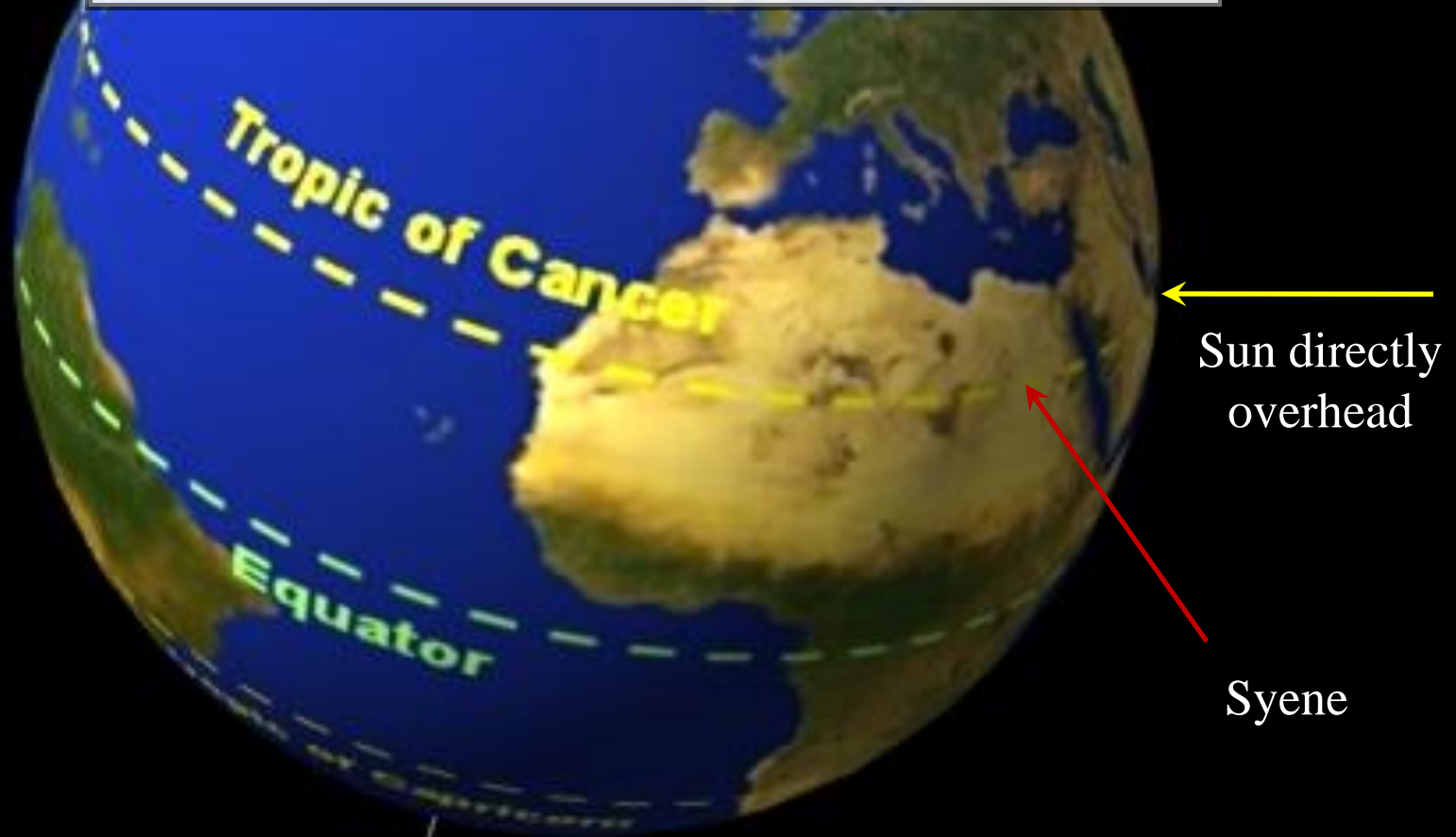


The argument was
again **indirect** – but
now relied on looking
at the **Sun**.

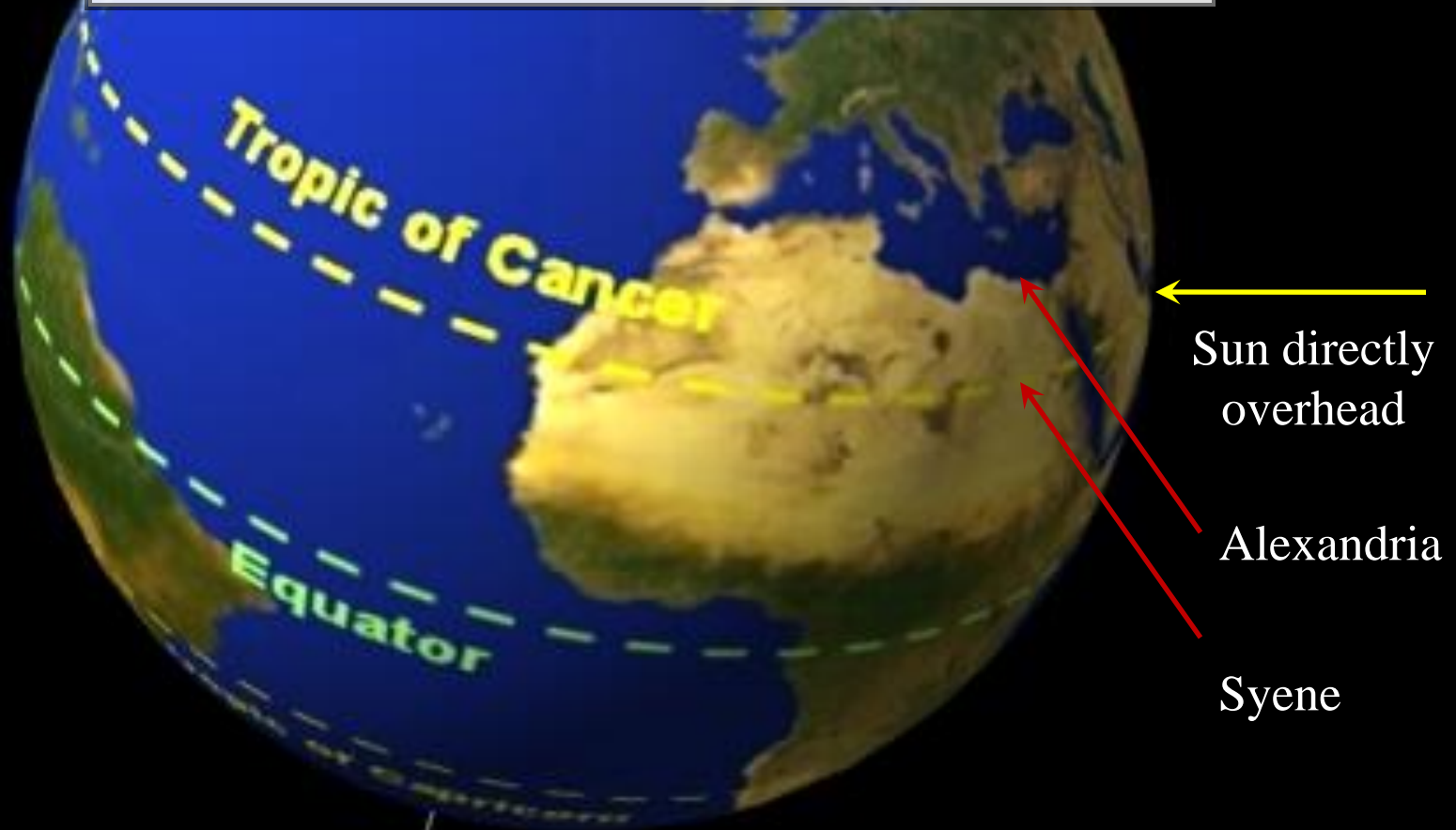
Eratosthenes read of a well in Syene, Egypt which at noon on the summer solstice (June 21) would reflect the overhead sun.



[This is because Syene lies almost directly on the
Tropic of Cancer.]



Eratosthenes tried the same experiment in his home city of Alexandria.



But on the solstice, the sun was at an angle and did not reflect from the bottom of the well.



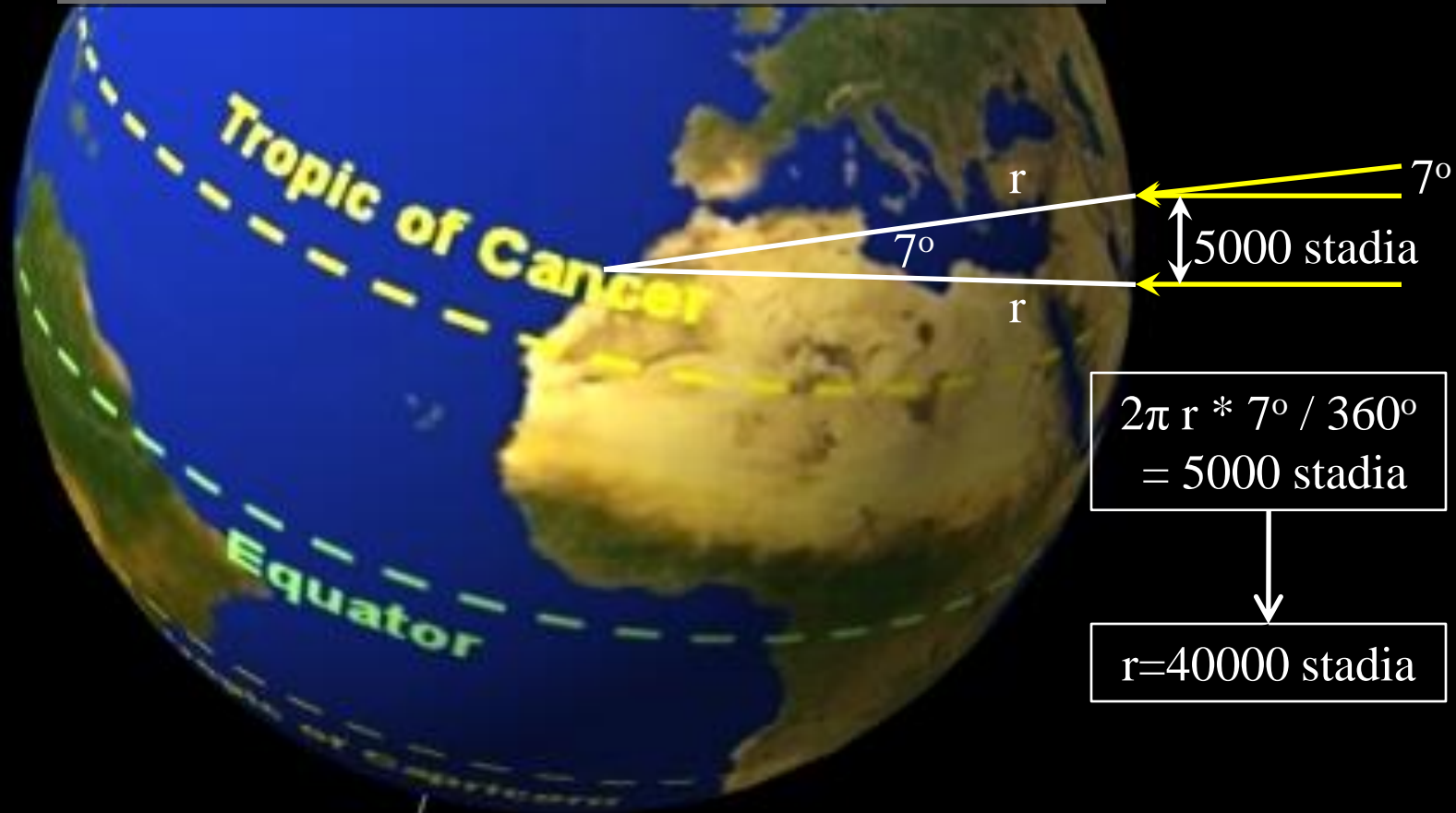
Using a **gnomon** (measuring stick), Eratosthenes measured the deviation of the sun from the vertical as 7° .



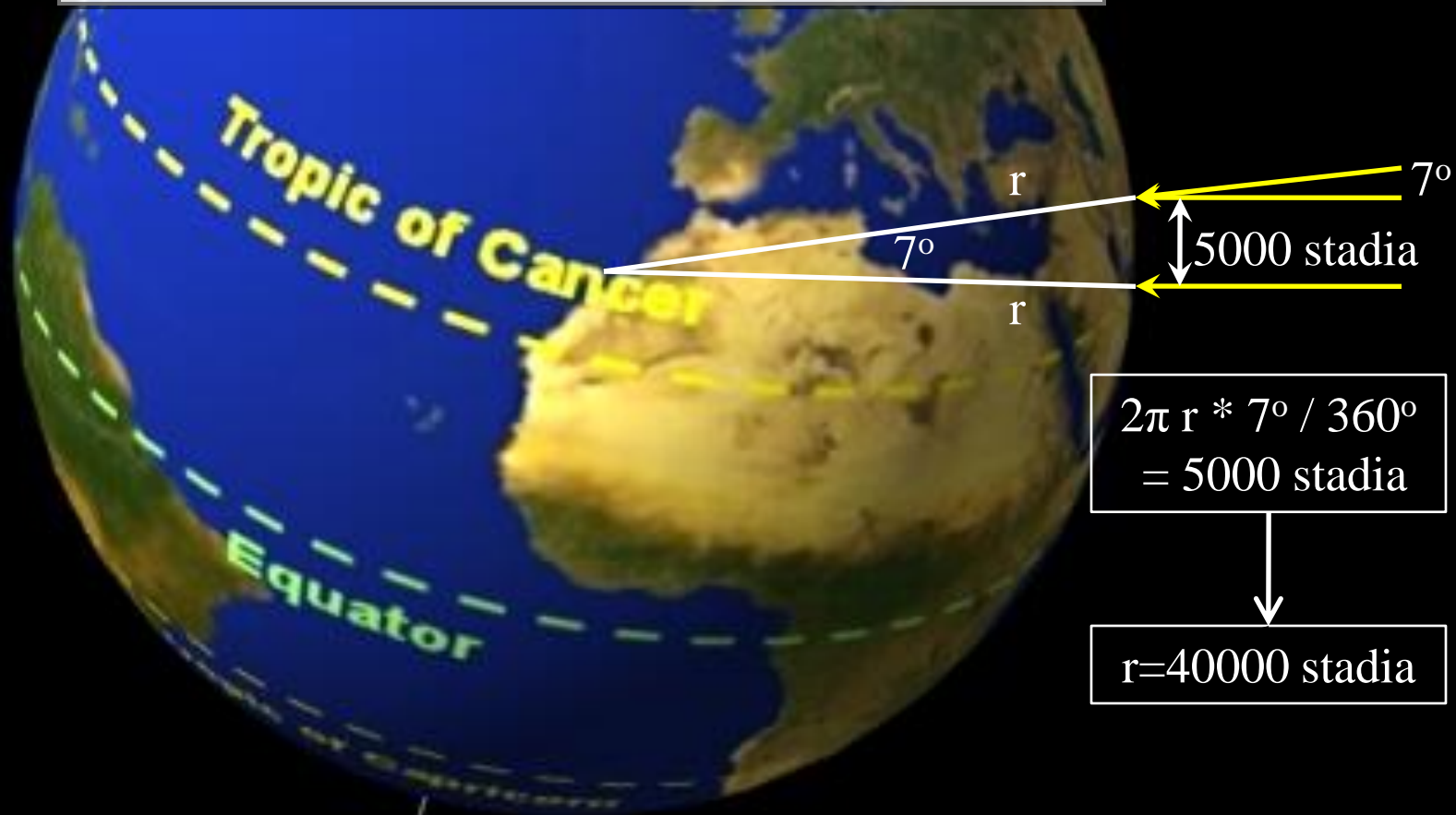
From trade caravans and other sources, Eratosthenes knew Syene to be 5,000 stadia (740 km) south of Alexandria.



This is enough information to compute the radius of the Earth.

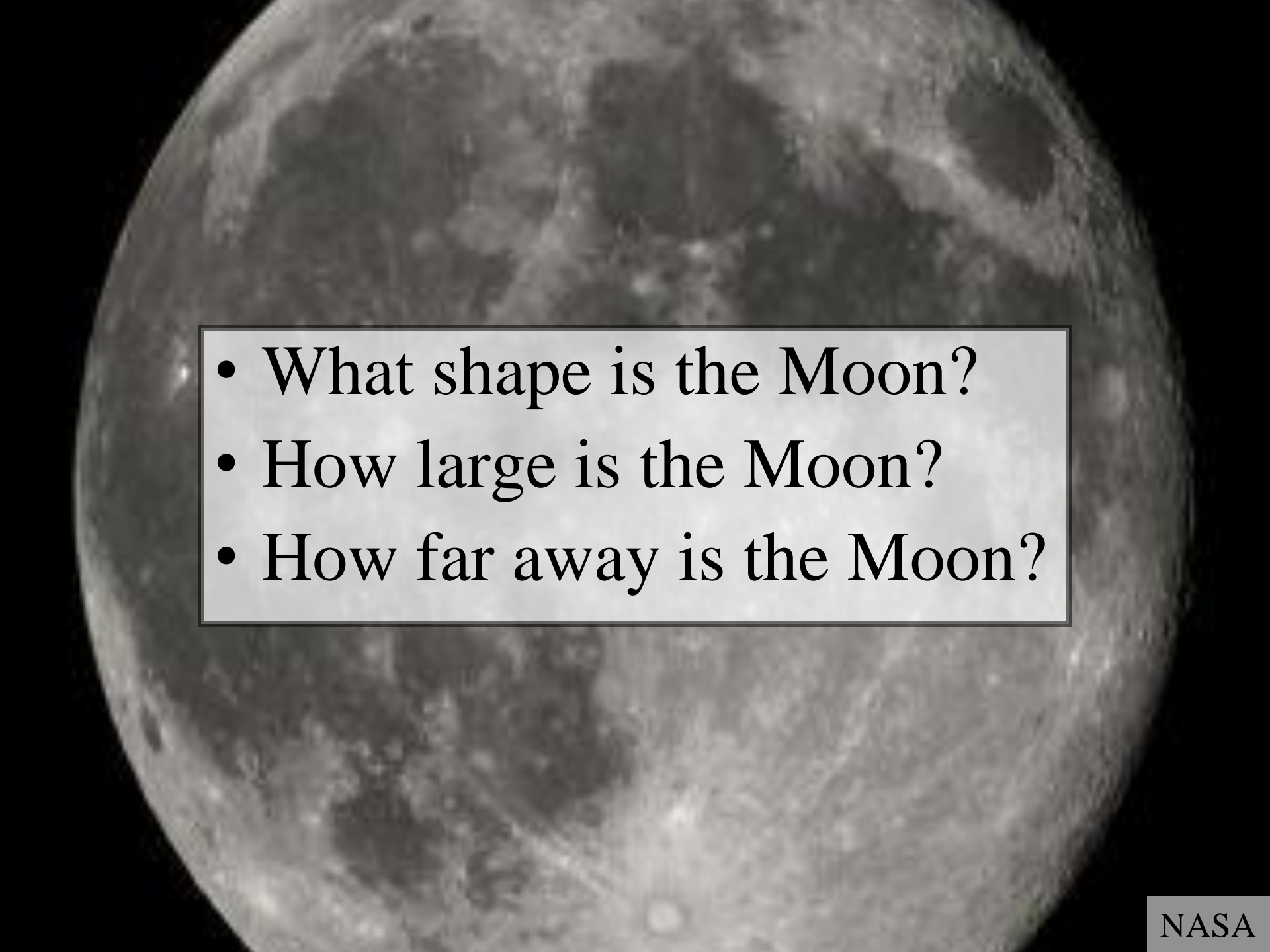



[This assumes that the Sun is quite far away, but more on this later.]





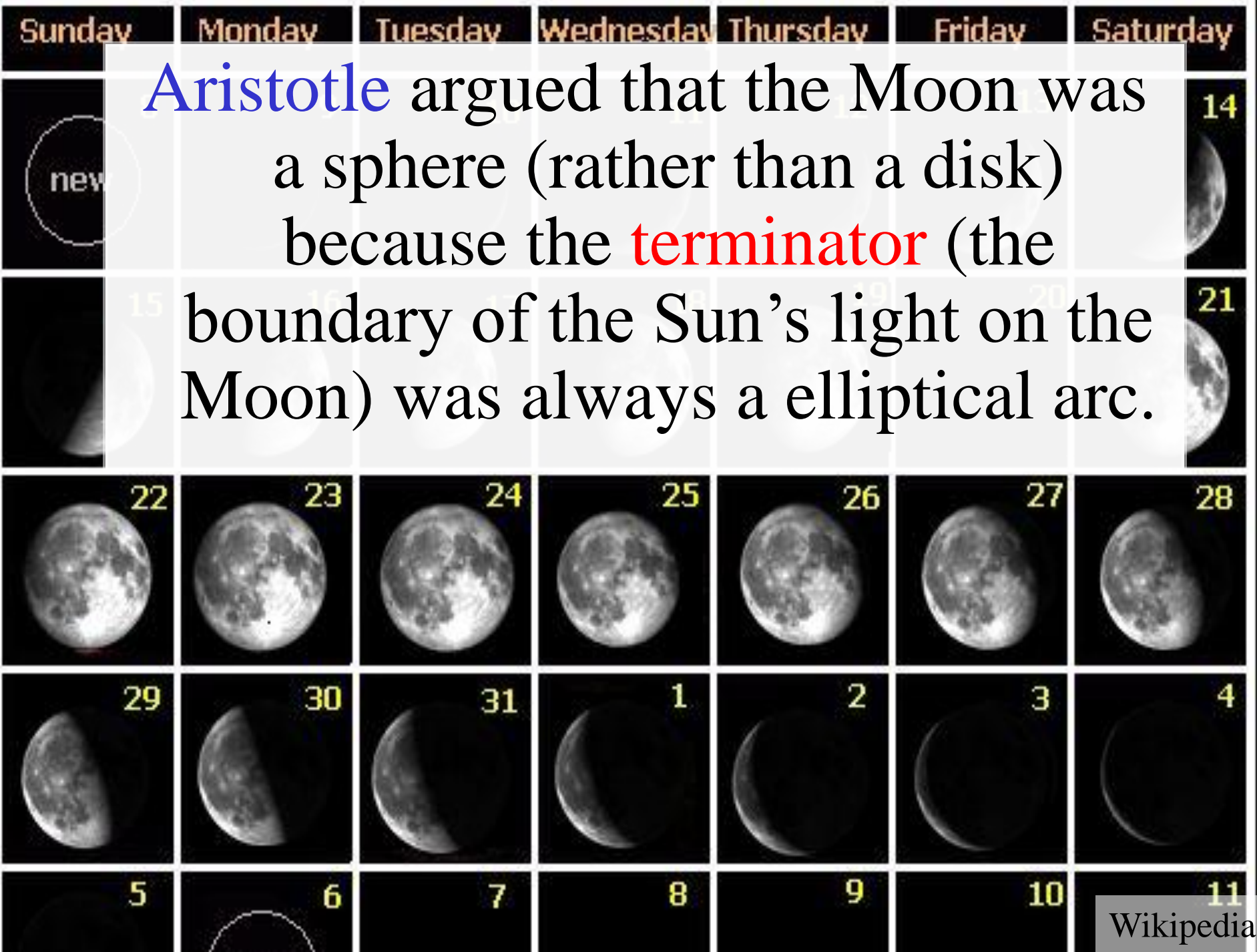
**2nd rung: the
Moon**

- 
- What shape is the Moon?
 - How large is the Moon?
 - How far away is the Moon?



The ancient Greeks
could answer these
questions also.

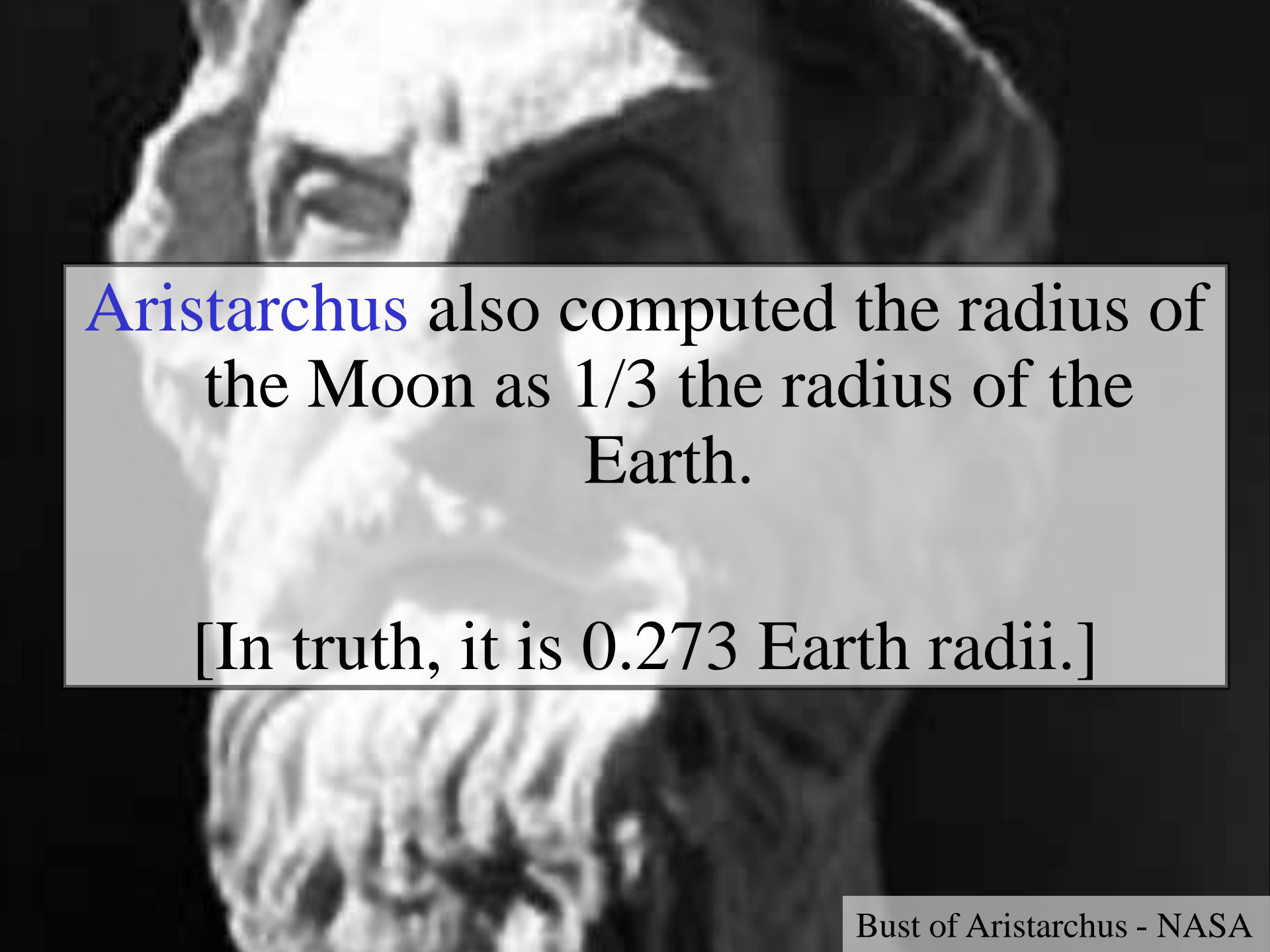
Aristotle argued that the Moon was a sphere (rather than a disk) because the **terminator** (the boundary of the Sun's light on the Moon) was always an elliptical arc.





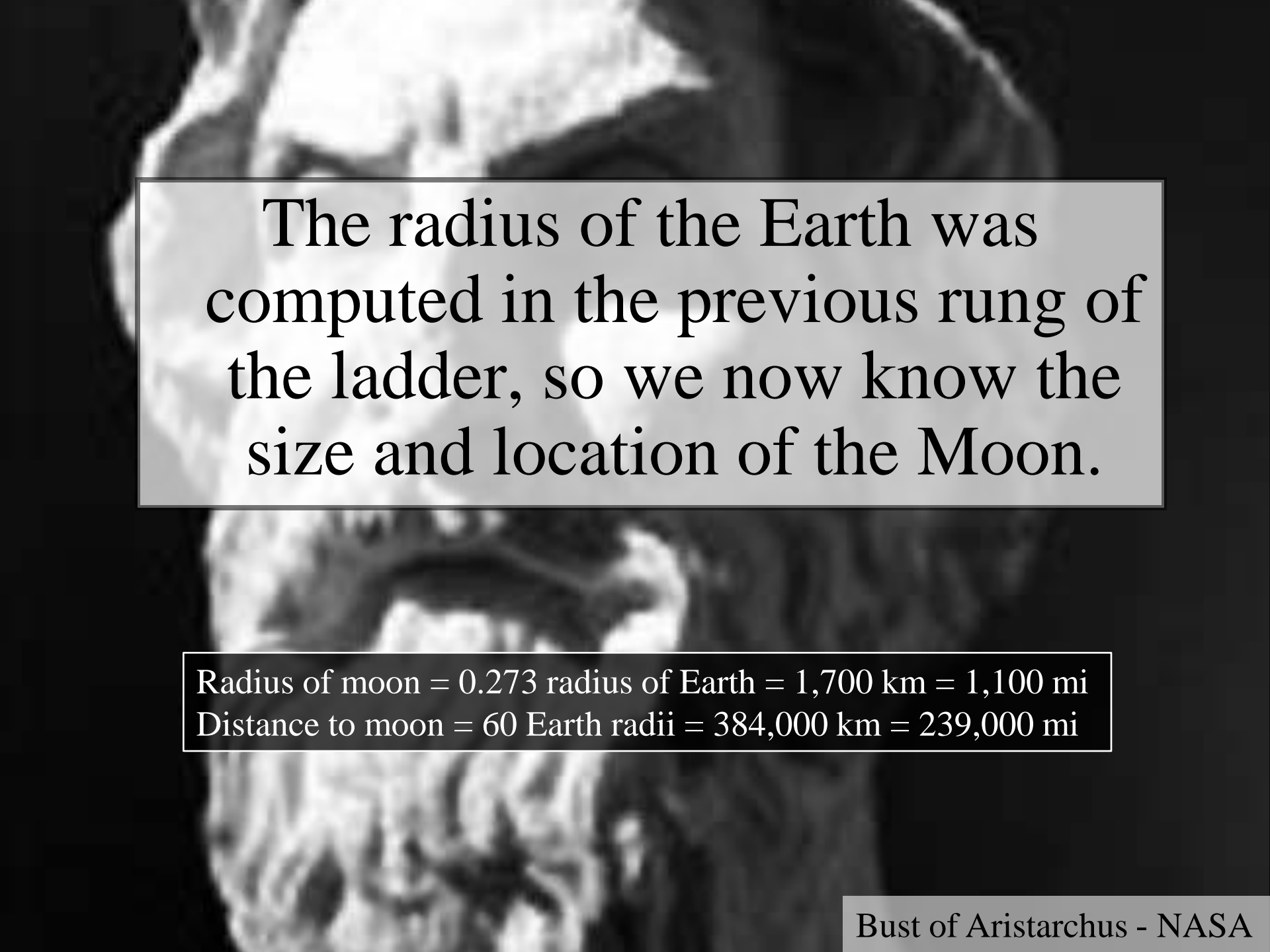
Aristarchus (310-230 BCE) computed
the distance of the Earth to the Moon
as about 60 Earth radii.

[In truth, it varies from 57 to 63 Earth
radii.]



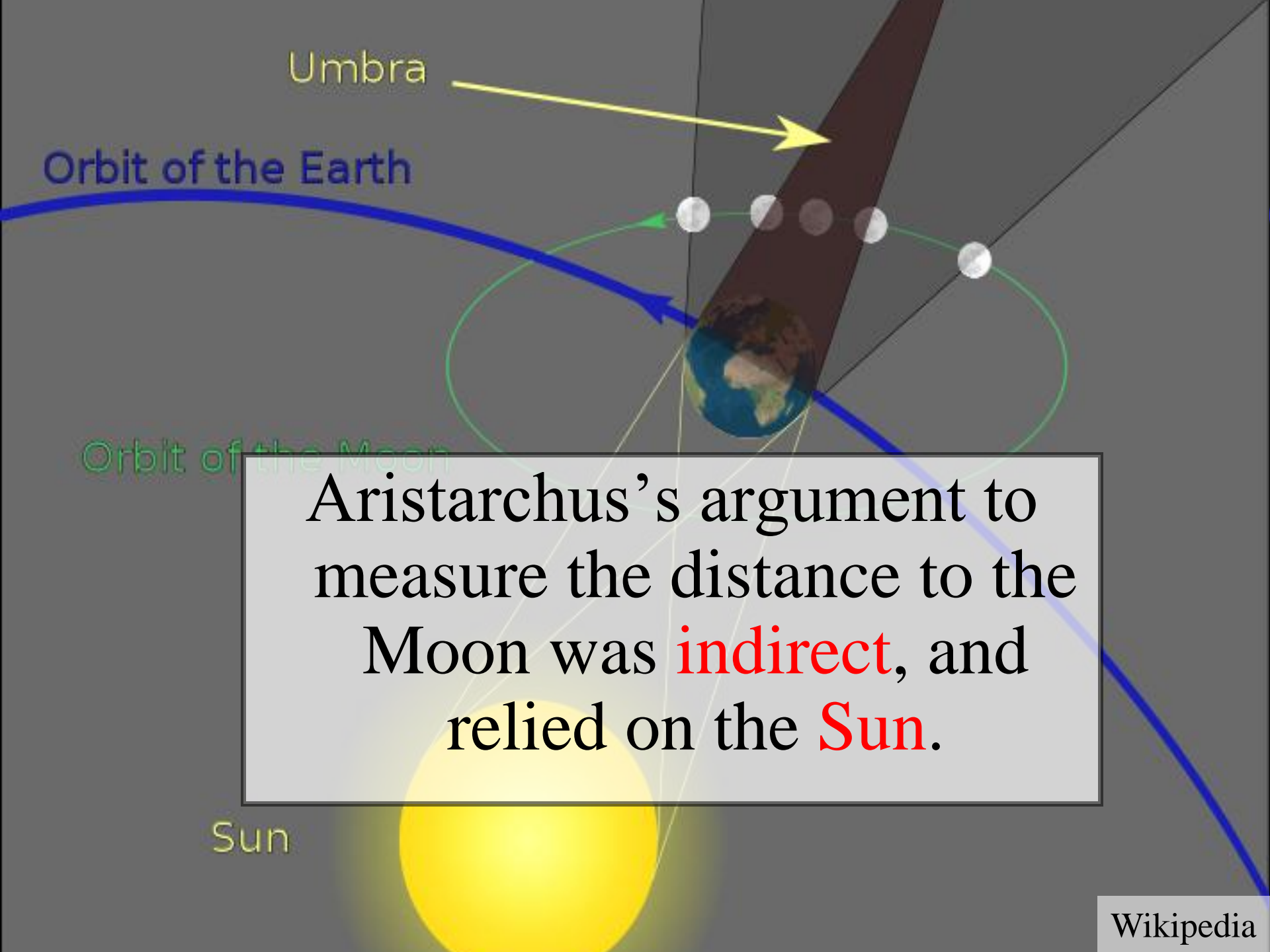
Aristarchus also computed the radius of the Moon as $\frac{1}{3}$ the radius of the Earth.

[In truth, it is 0.273 Earth radii.]



The radius of the Earth was computed in the previous rung of the ladder, so we now know the size and location of the Moon.

Radius of moon = 0.273 radius of Earth = 1,700 km = 1,100 mi
Distance to moon = 60 Earth radii = 384,000 km = 239,000 mi



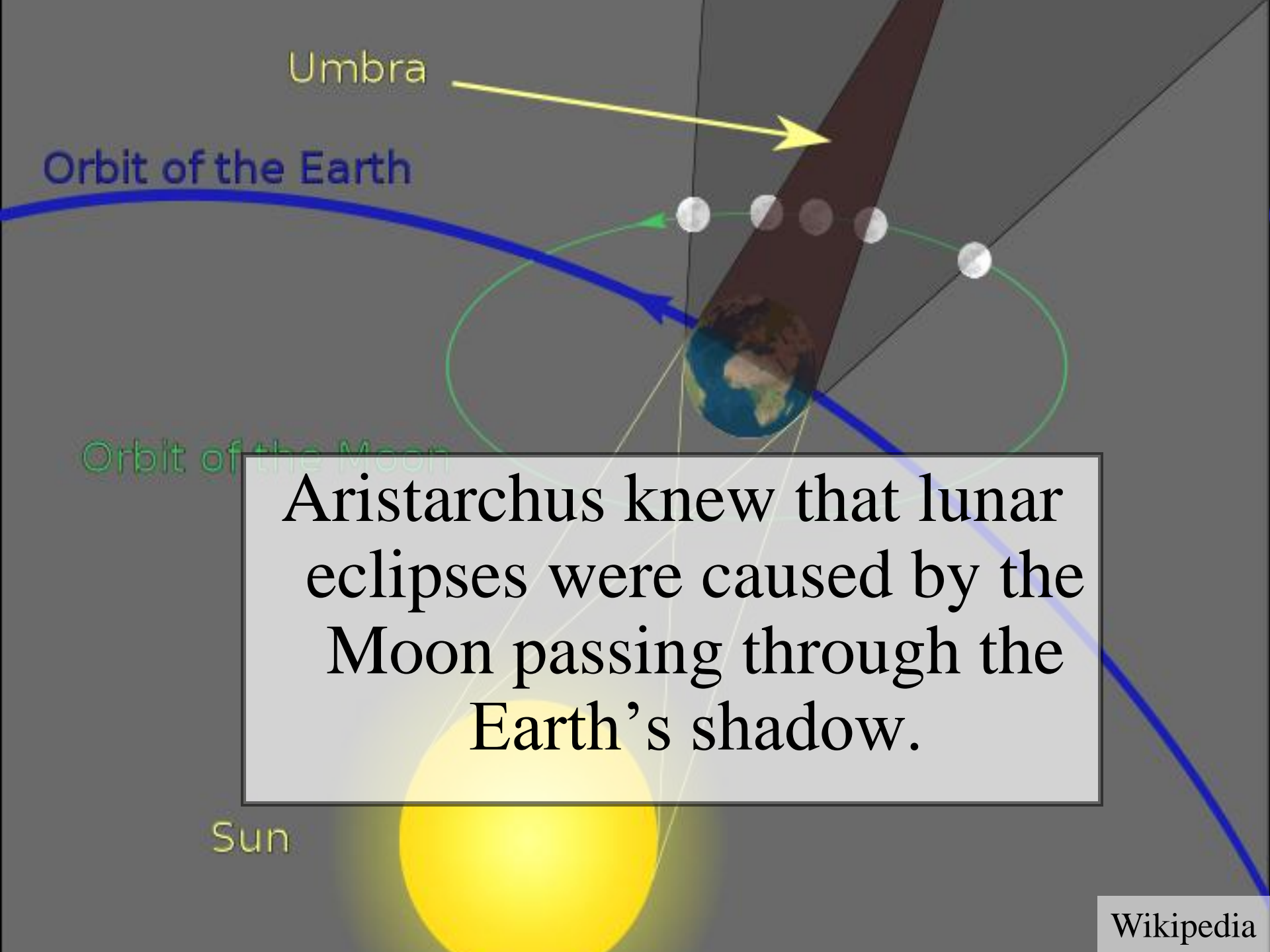
Umbra

Orbit of the Earth

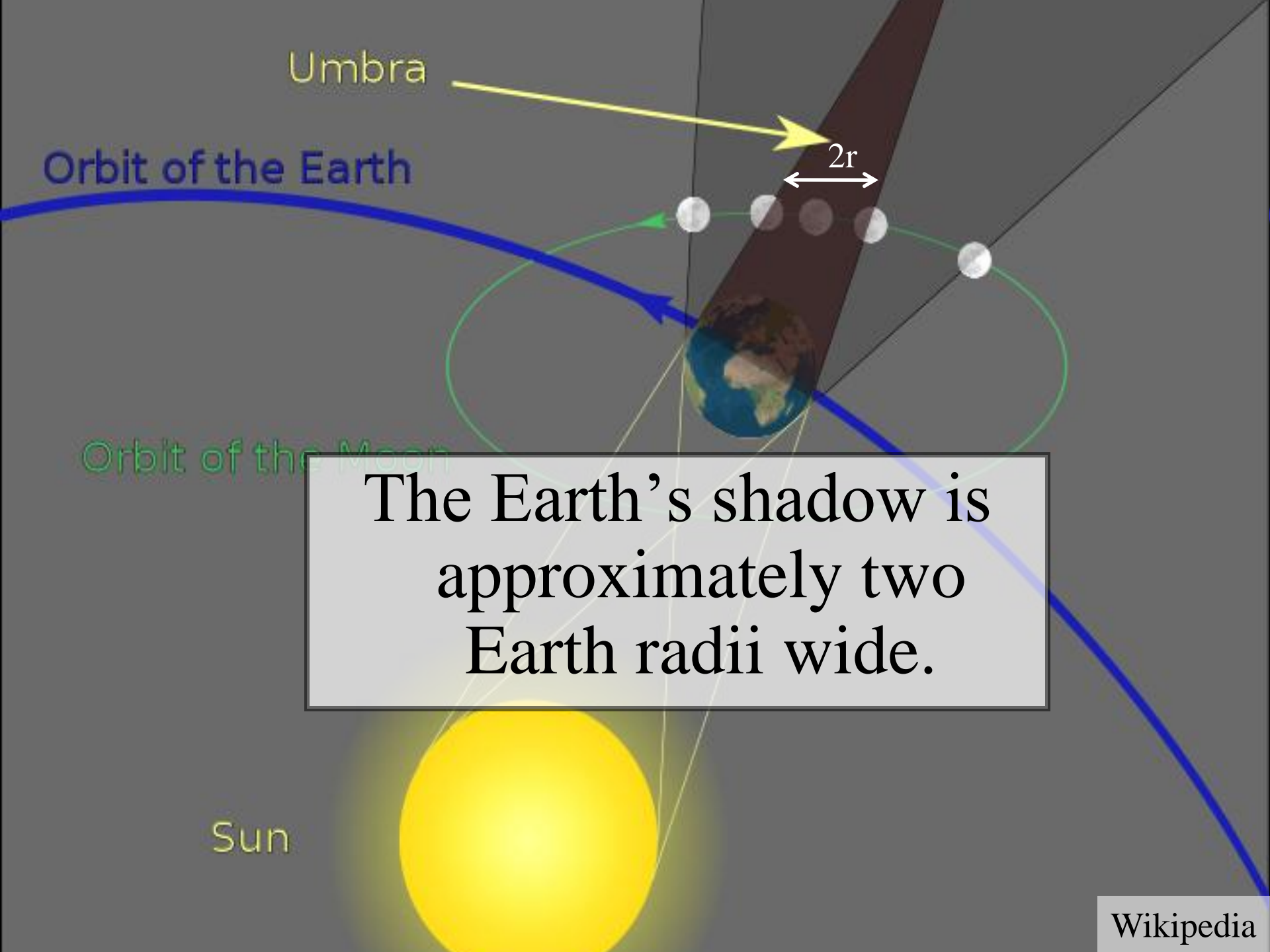
Orbit of the Moon

Sun

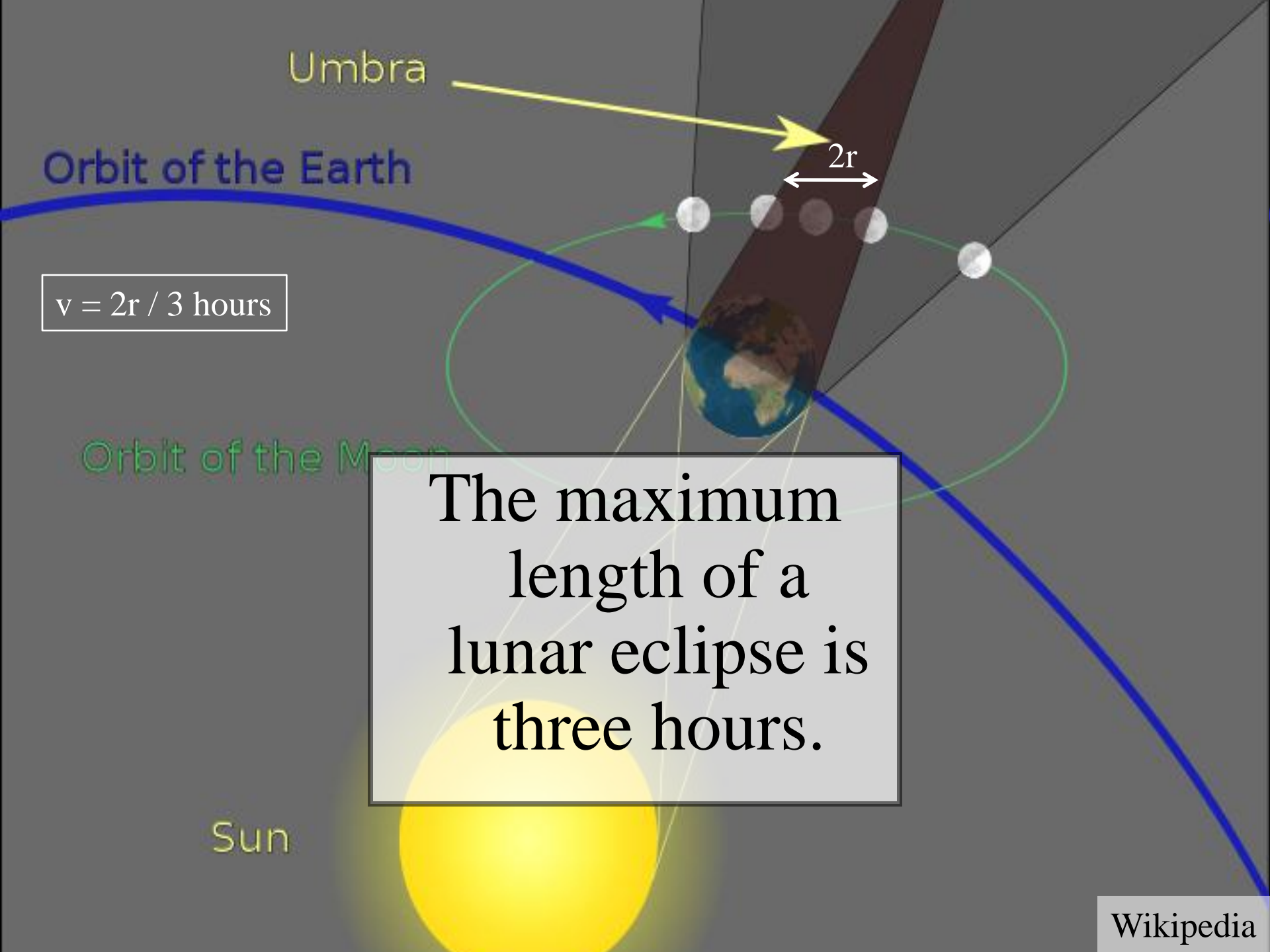
Aristarchus's argument to measure the distance to the Moon was **indirect**, and relied on the **Sun**.



Aristarchus knew that lunar eclipses were caused by the Moon passing through the Earth's shadow.



The Earth's shadow is approximately two Earth radii wide.



Umbra

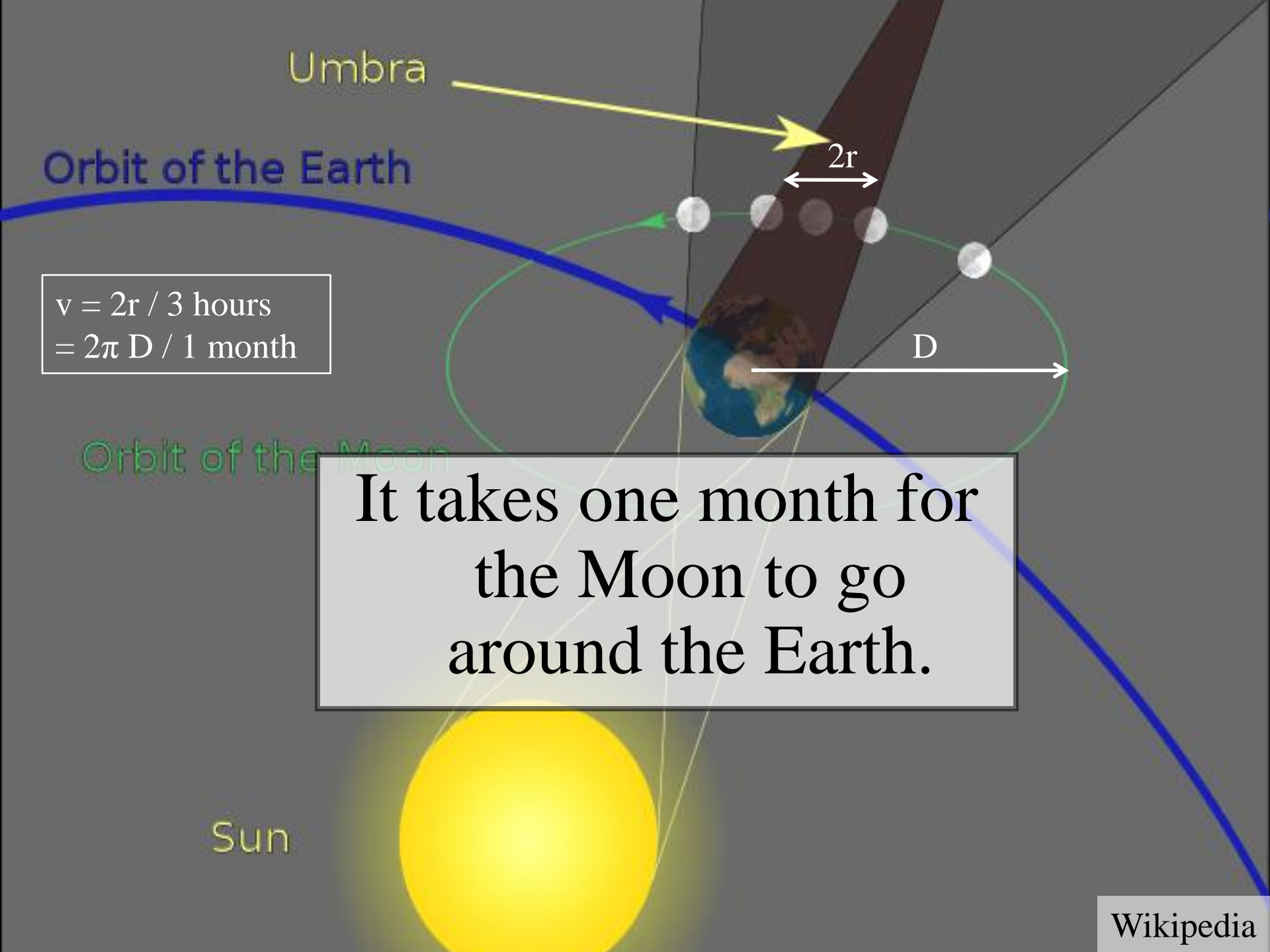
Orbit of the Earth

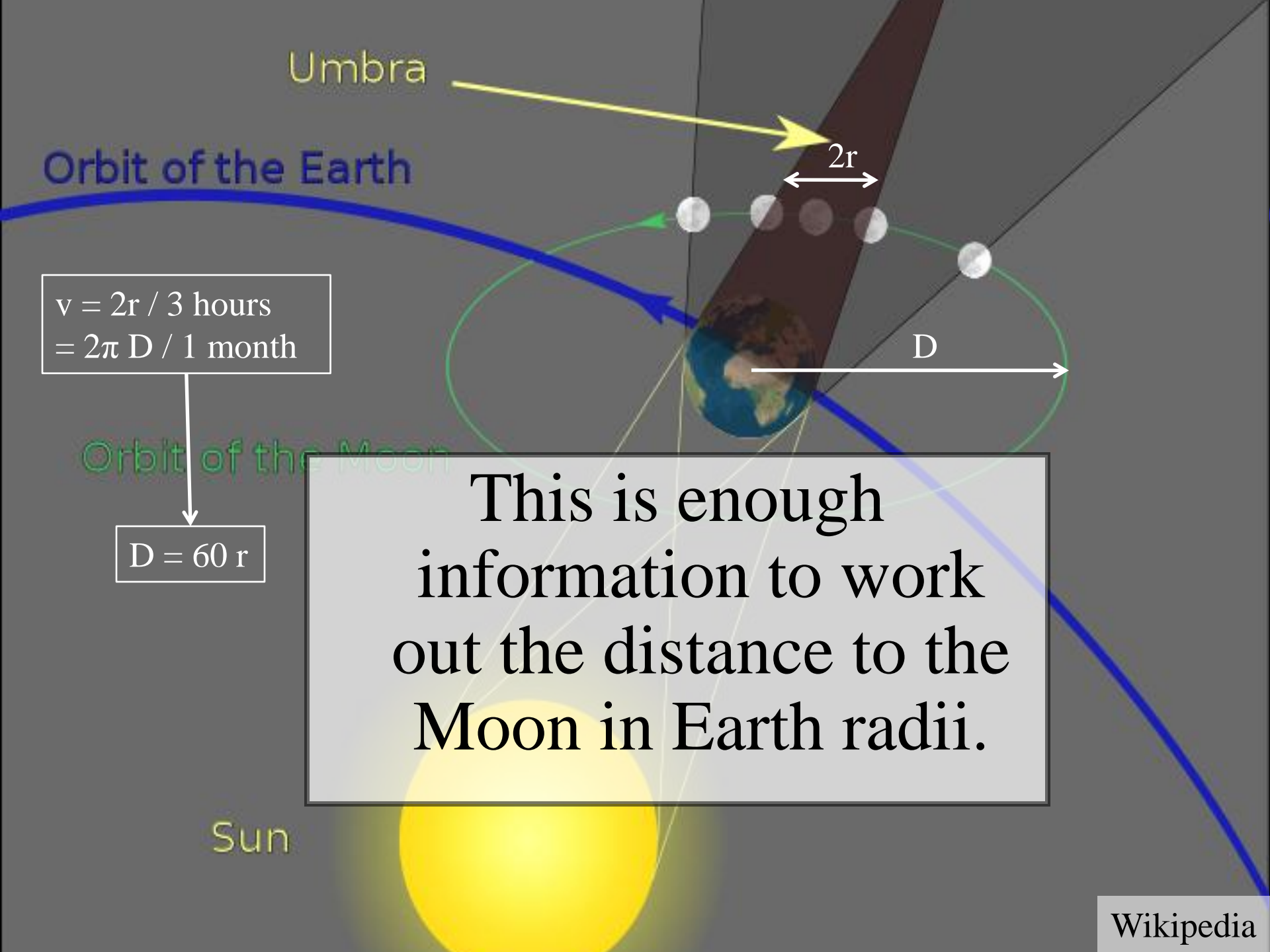
$$v = 2r / 3 \text{ hours}$$

Orbit of the Moon

The maximum length of a lunar eclipse is three hours.

Sun





Umbra

Orbit of the Earth

$$v = 2r / 3 \text{ hours}$$
$$= 2\pi D / 1 \text{ month}$$

$$D = 60 r$$

This is enough information to work out the distance to the Moon in Earth radii.

Orbit of the Moon

Sun

$$V = 2R / 2 \text{ min}$$



Also, the Moon takes
about 2 minutes to
set.

Moonset over the Colorado Rocky Mountains,
Sep 15 2008, Alek Kolmarnitsky www.ko

$$V = 2R / 2 \text{ min}$$
$$= 2\pi D / 24 \text{ hours}$$



The Moon takes 24 hours
to make a full (apparent)
rotation around the Earth.

Moonset over the Colorado Rocky Mountains,
Sep 15 2008, Alek Kolmarnitsky www.ko

$$V = 2R / 2 \text{ min}$$
$$= 2\pi D / 24 \text{ hours}$$

$$R = D / 180$$

$2R$



This is enough information to determine the radius of the Moon, in terms of the distance to the Moon...

$$V = 2R / 2 \text{ min} \\ = 2\pi D / 24 \text{ hours}$$

$$R = D / 180 \\ = r / 3$$

$2R$

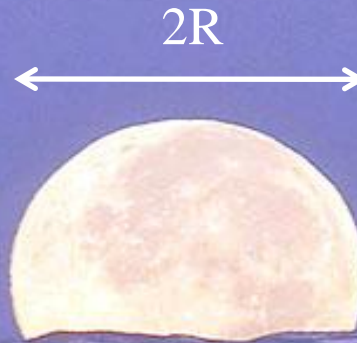


... which we have
just computed.

Moonset over the Colorado Rocky Mountains,
Sep 15 2008, Alek Kolmarnitsky www.ko

$$V = 2R / 2 \text{ min} \\ = 2\pi D / 24 \text{ hours}$$


$$R = D / 180 \\ = r / 3$$




[Aristarchus, by the way, was handicapped by not having an accurate value of π , which had to wait until [Archimedes](#) (287-212BCE) some decades later!]




3rd rung: the Sun

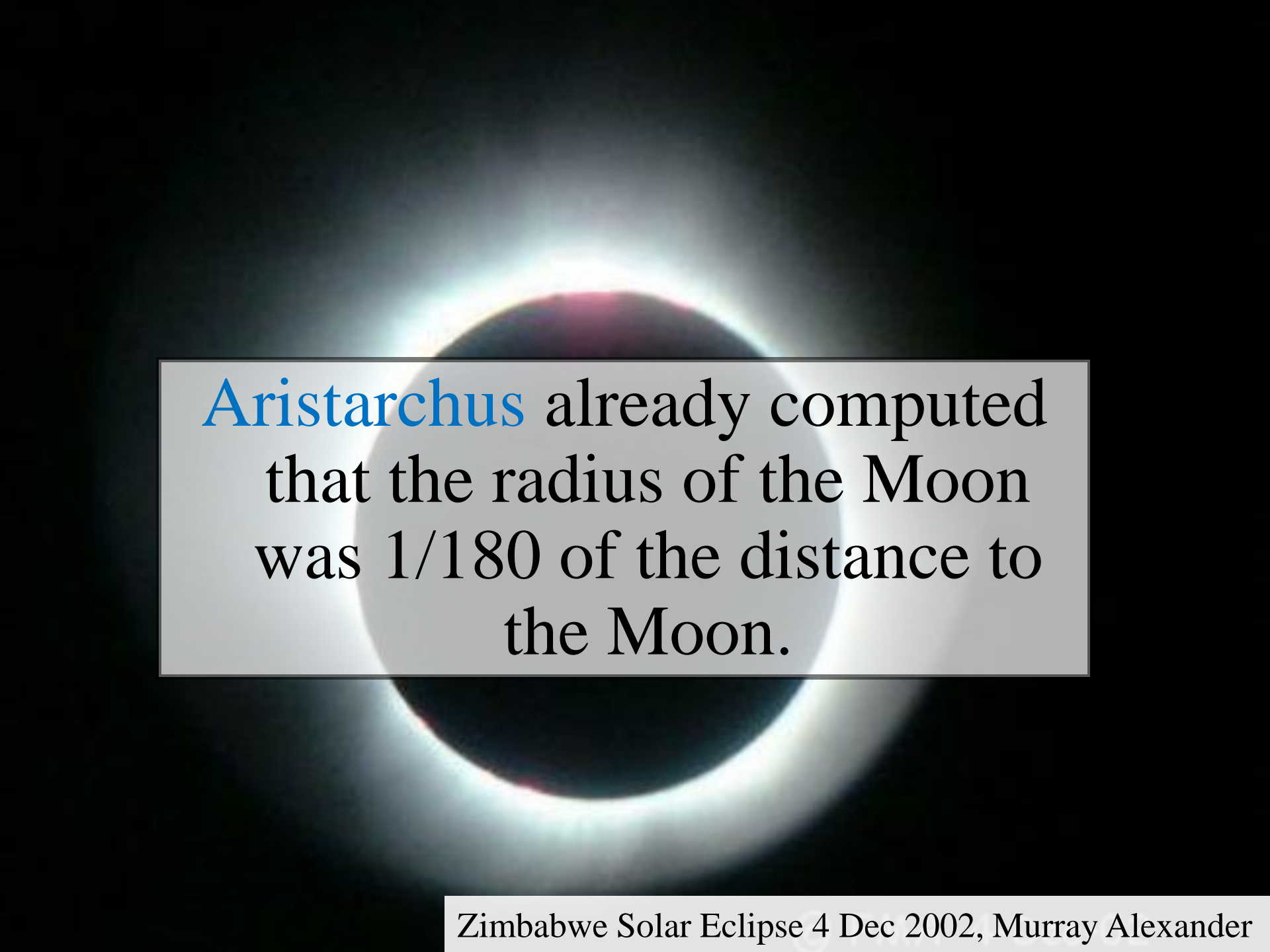
- 
- How large is the Sun?
 - How far away is the Sun?




Once again, the ancient Greeks
could answer these questions
(but with imperfect accuracy).




Their methods were **indirect**,
and relied on the **Moon**.




Aristarchus already computed
that the radius of the Moon
was $1/180$ of the distance to
the Moon.




He also knew that during a solar eclipse, the Moon covered the Sun almost perfectly.



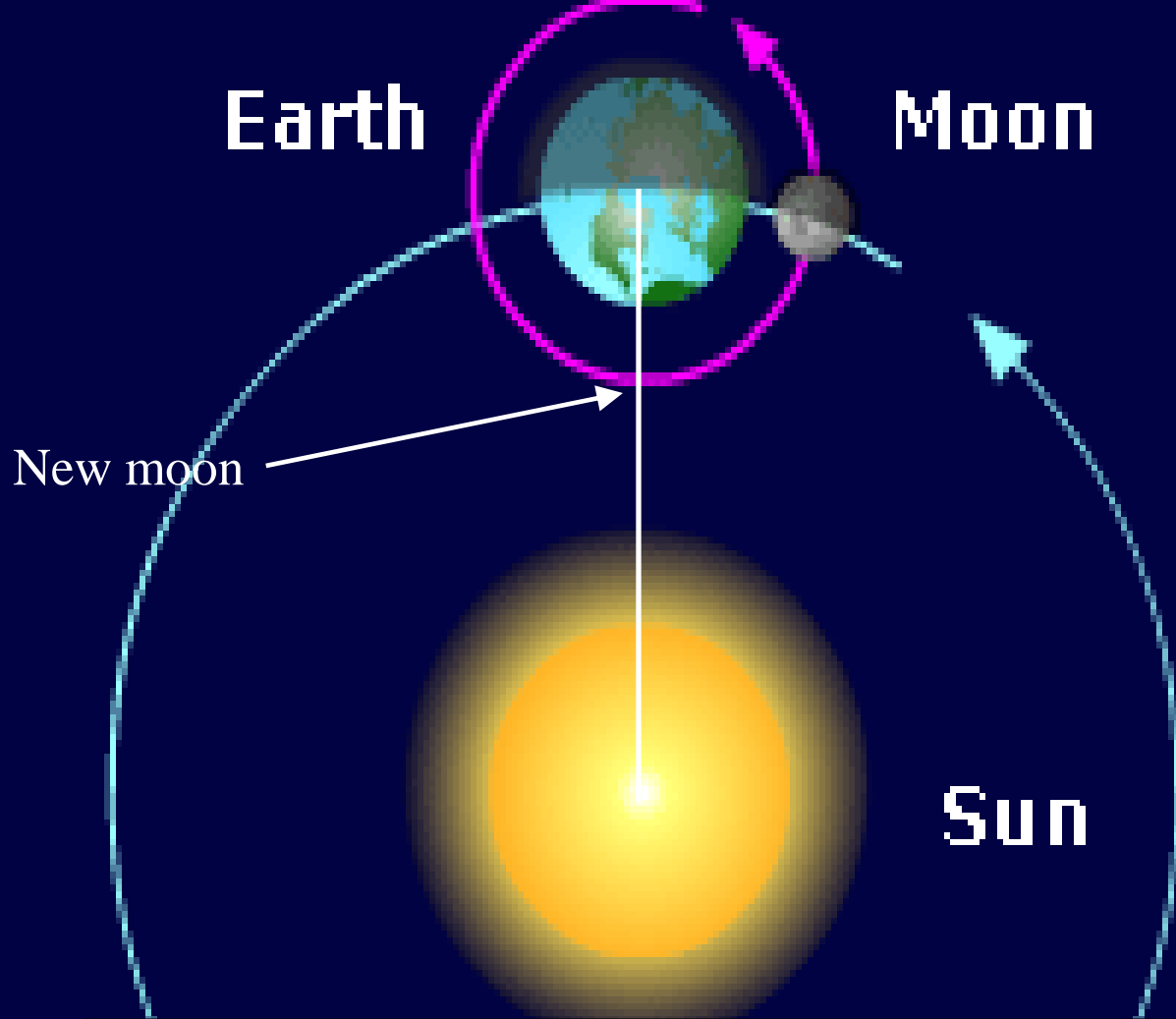
Using **similar triangles**, he concluded that the radius of the Sun was also $1/180$ of the distance to the Sun.



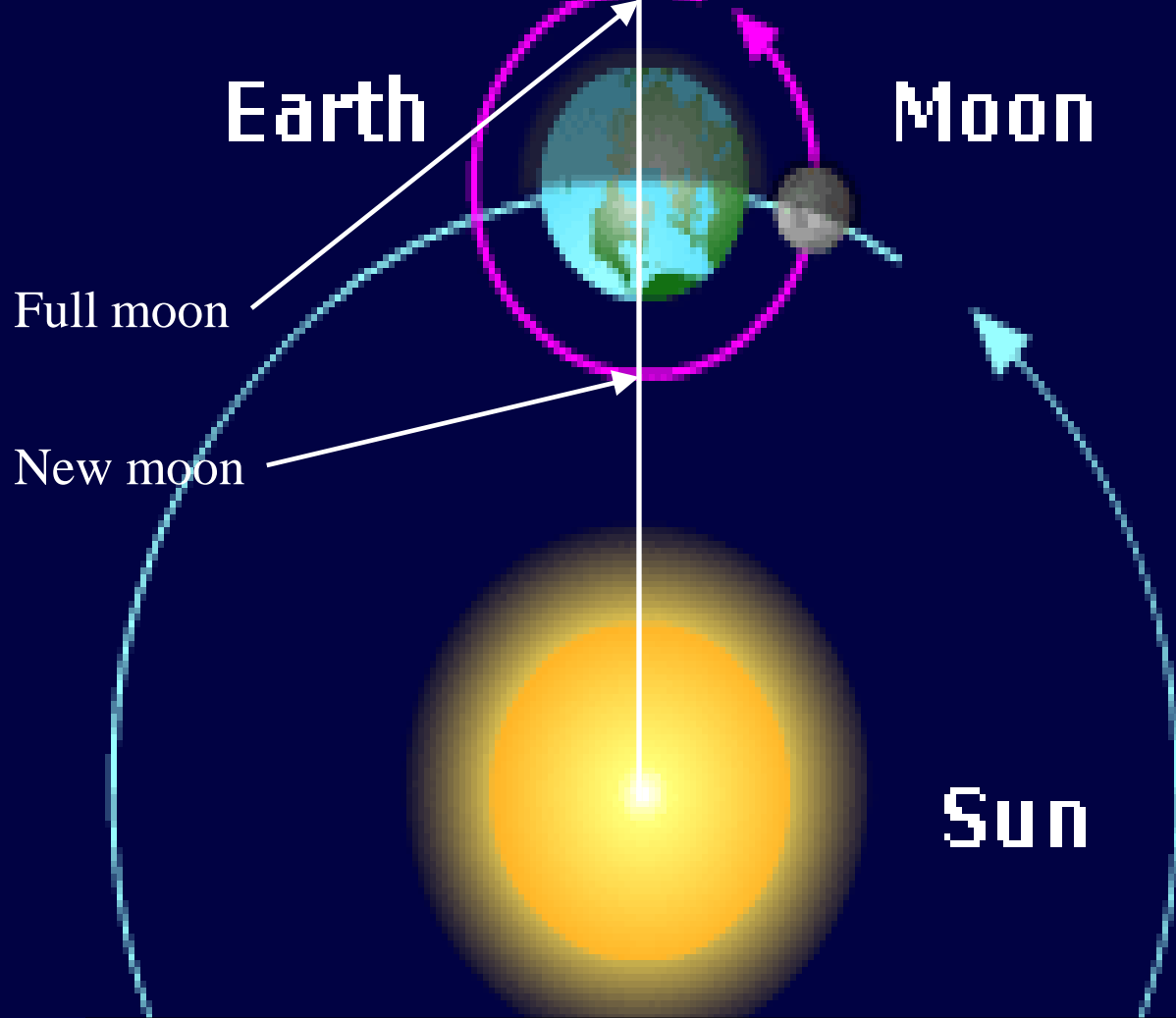
So his next task was to
compute the distance
to the Sun.

A photograph of a total solar eclipse. The sun is completely obscured by the dark disk of the moon, creating a dark central shadow surrounded by a bright, glowing white ring of light (the corona). The background is a dark, clear sky. A semi-transparent white rectangular box with a thin black border is centered over the eclipse, containing text.

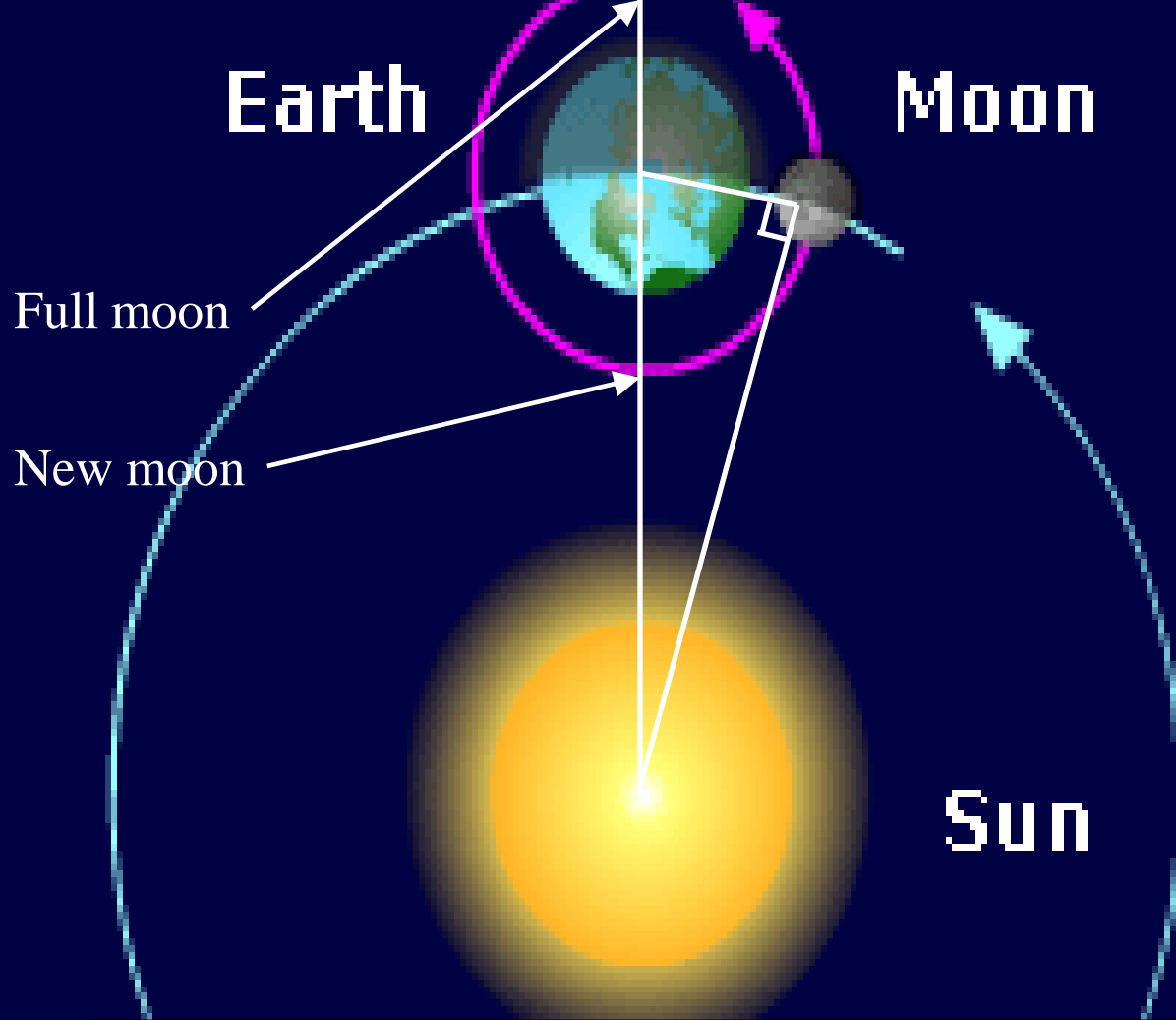
For this, he turned to
the Moon again for
help.



He knew that new Moons occurred when the Moon was between the Earth and Sun...

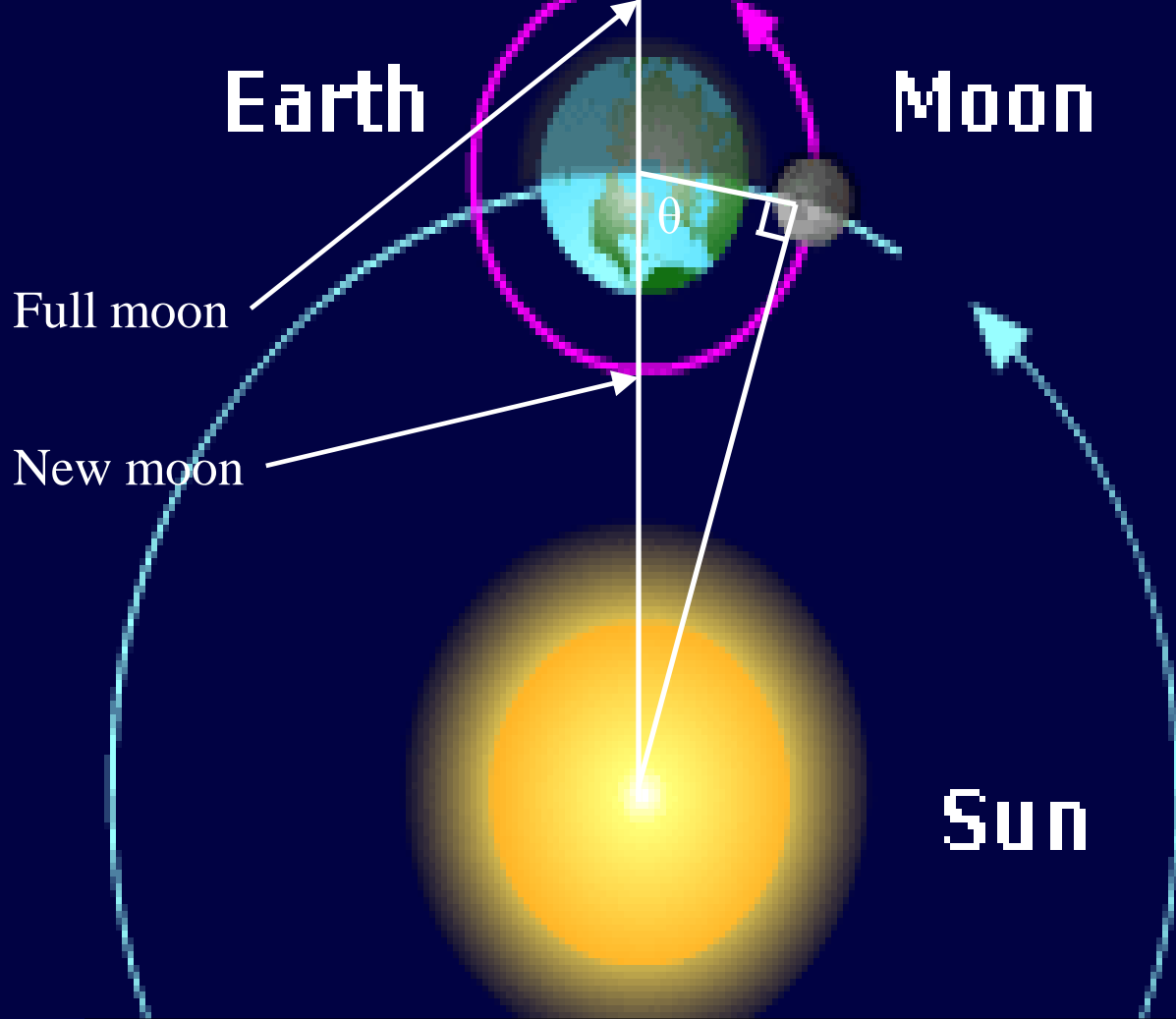


... full Moons occurred when the Moon was directly opposite the Sun...



... and half Moons occurred when the Moon made a right angle between Earth and Sun.

$$\theta < \pi/2$$

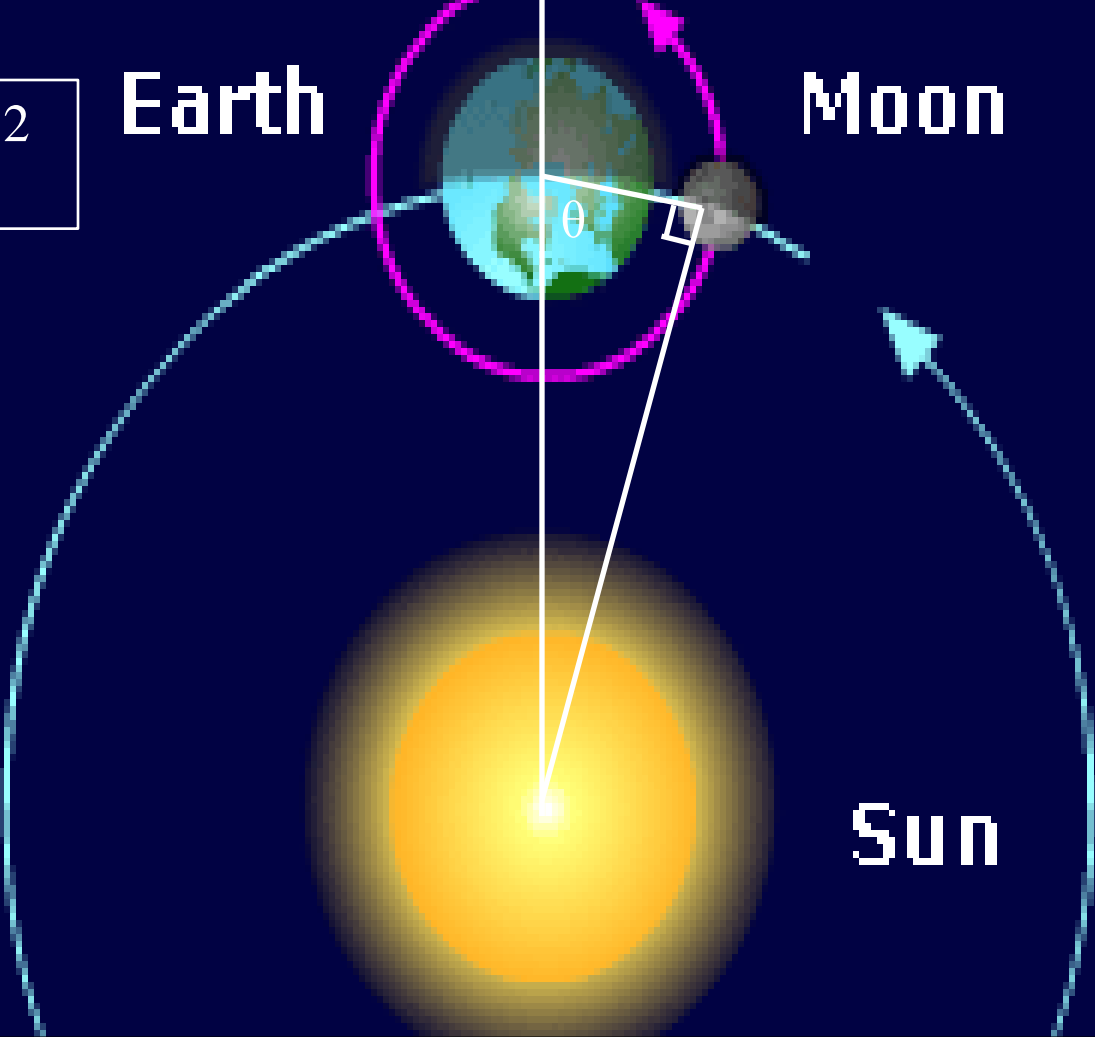


This implies that half Moons occur slightly closer to new Moons than to full Moons.

$$\theta = \pi/2 - 2\pi * 12 \text{ hours/1 month}$$

Earth

Moon

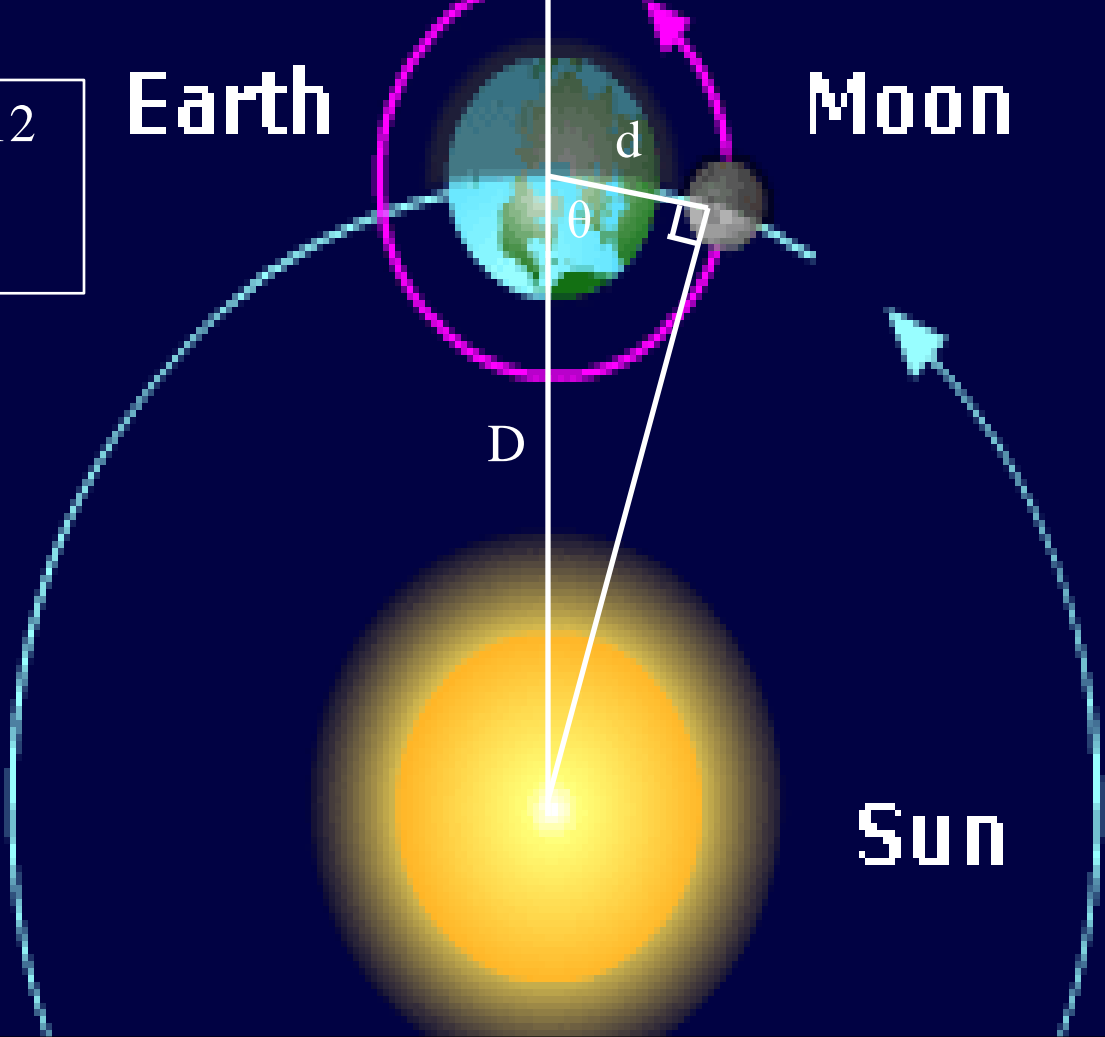


Sun

Aristarchus thought that half Moons occurred 12 hours before the midpoint of a new and full Moon.

$$\theta = \pi/2 - 2\pi * 12 \text{ hours/1 month}$$
$$\cos \theta = d/D$$

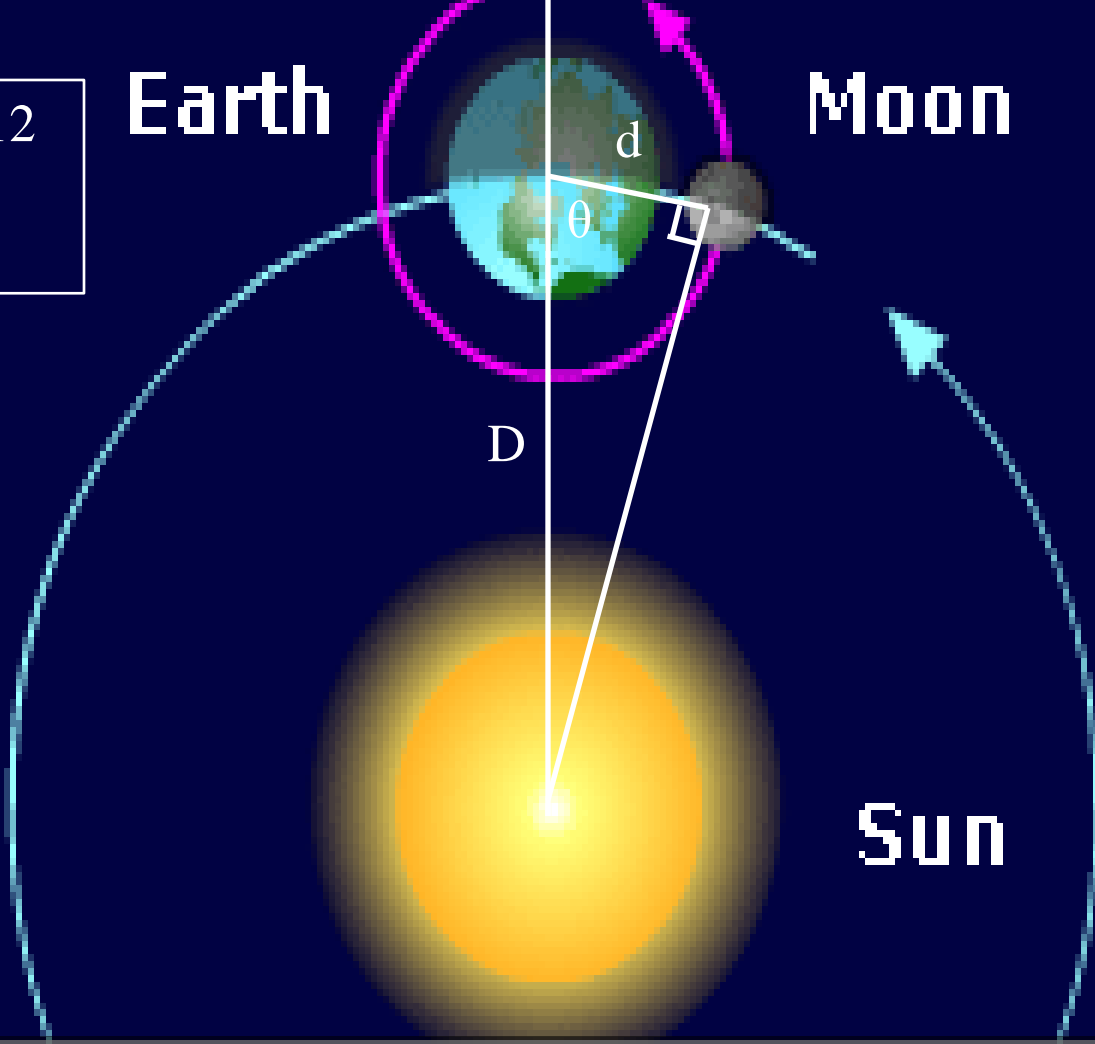
$$D = 20 d$$



From this and trigonometry, he concluded that the Sun was 20 times further away than the Moon.

$$\theta = \pi/2 - 2\pi * 12 \text{ hours} / 1 \text{ month}$$
$$\cos \theta = d/D$$

$$D = 20 d$$



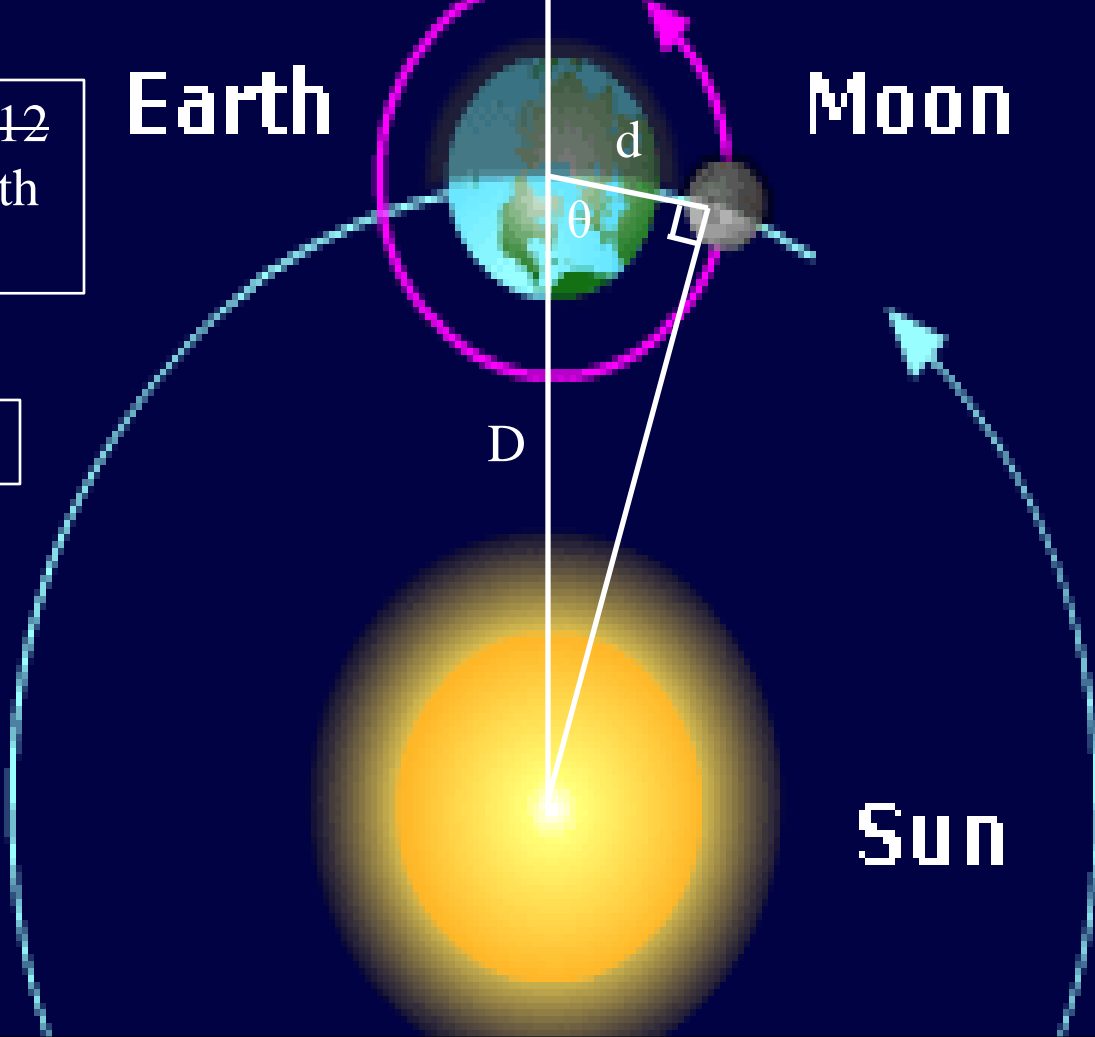
Unfortunately, with ancient Greek technology it was hard to time a new Moon perfectly.

$$\theta = \pi/2 - 2\pi * 12$$

$$0.5 \text{ hour}/1 \text{ month}$$

$$\cos \theta = d/D$$

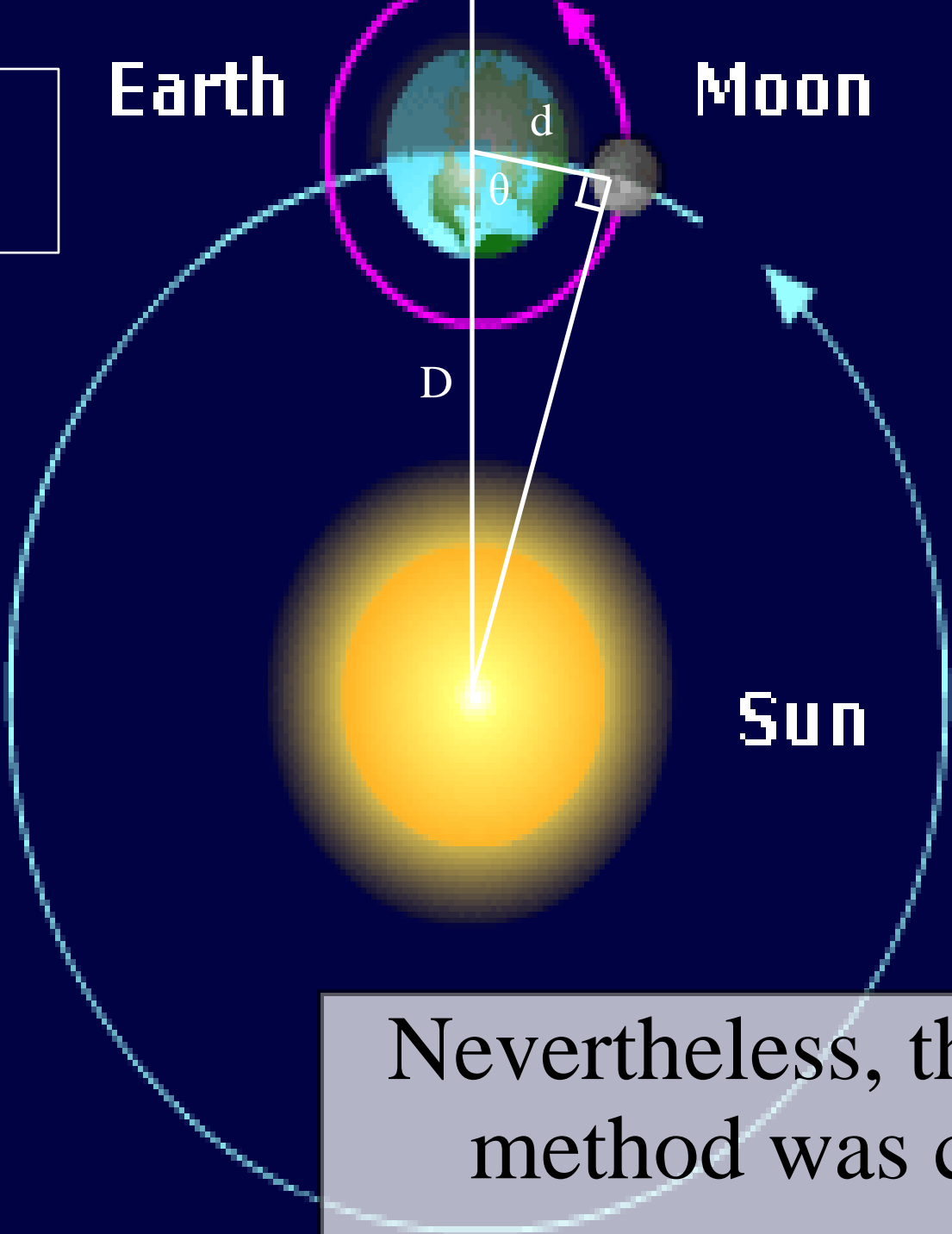
$$D = 20 \cdot 390 \cdot d$$



The true time discrepancy is 1/2 hour (not 12 hours), and the Sun is 390 times further away (not 20 times).

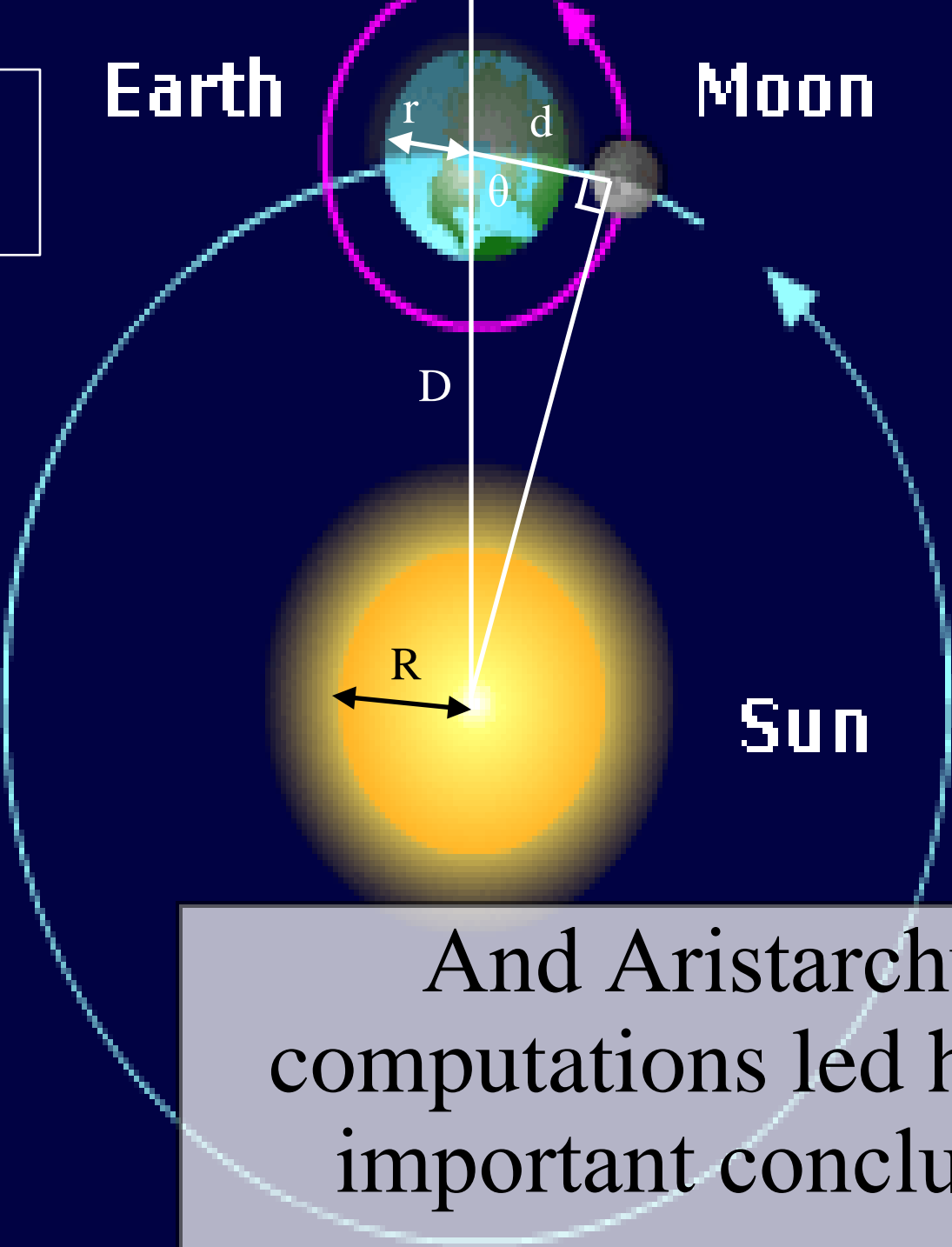
$\theta = \pi/2 - 2\pi / 2$
hour/1 month
 $\cos \theta = d/D$

$D = 390 d$



Nevertheless, the basic method was correct.

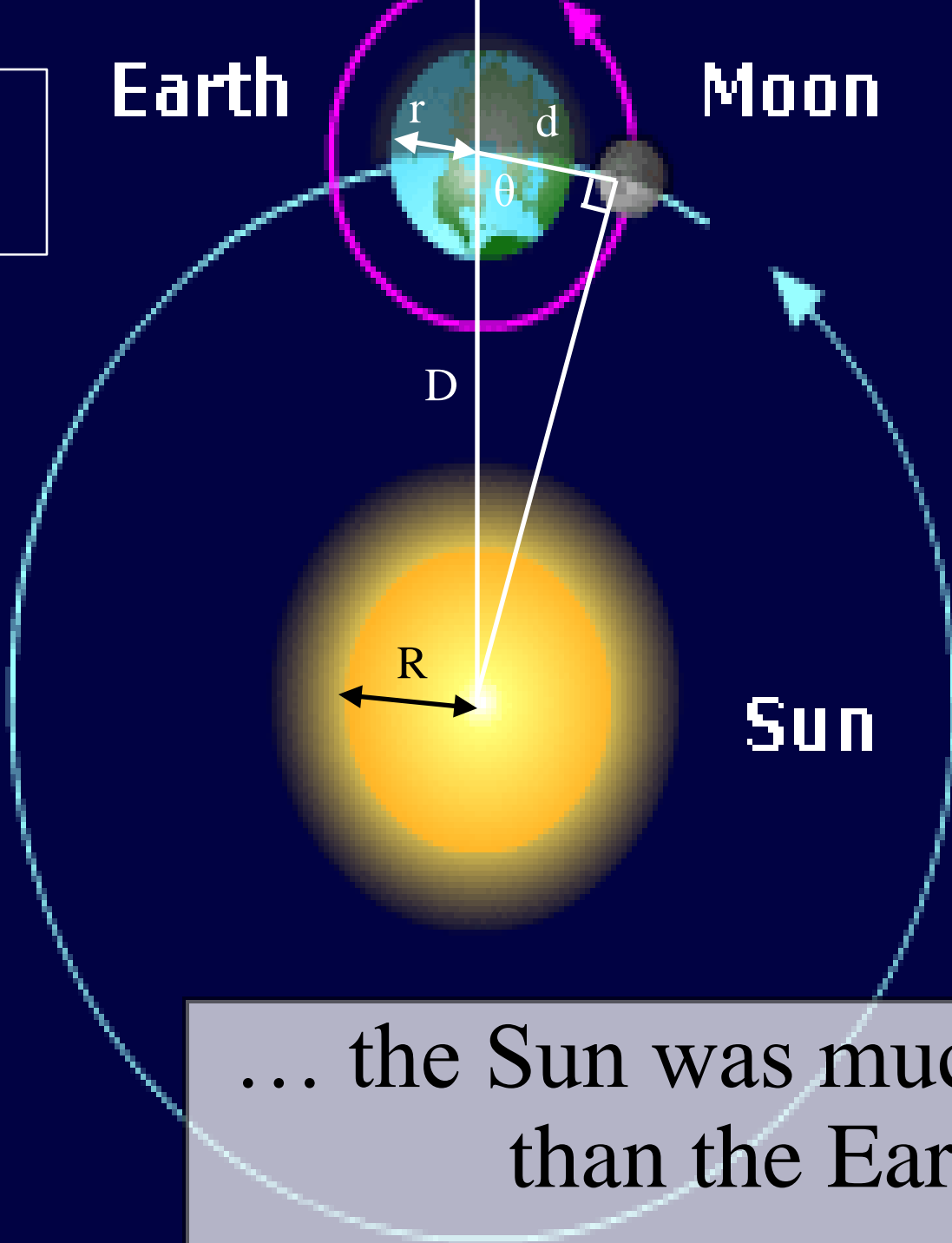
$d = 60 r$
 $D/d = 20$
 $R/D = 1/180$



And Aristarchus' computations led him to an important conclusion...

$d = 60 r$
 $D/d = 20$
 $R/D = 1/180$

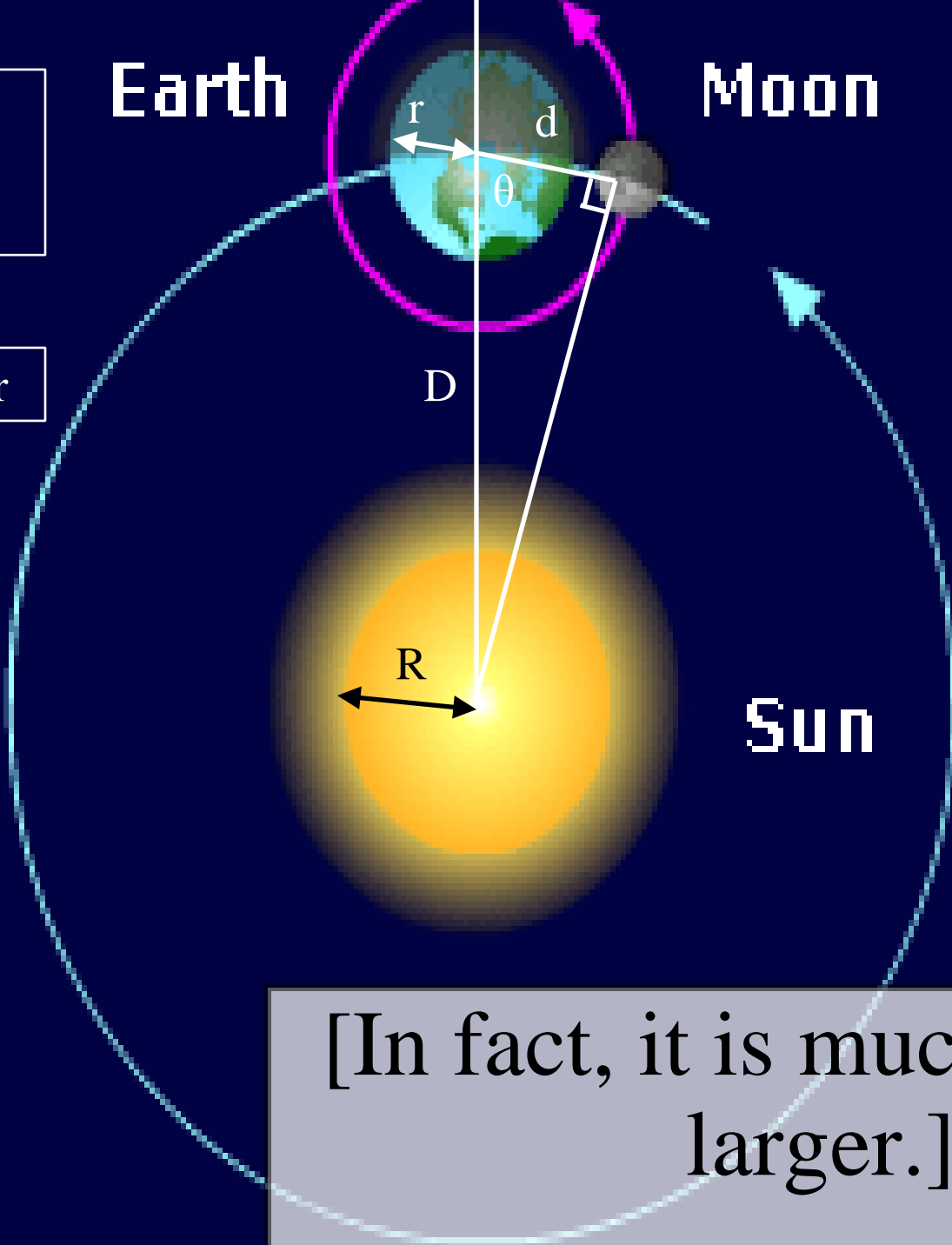
$R \sim 7 r$



... the Sun was much larger than the Earth.

$d = 60 r$
 $D/d = 20\ 390$
 $R/D = 1/180$

$R = 7\ 109 r$




[In fact, it is much, much larger.]

Earth radius = 6371 km = 3959 mi

Sun radius = 695,500 km = 432,200 mi


He then concluded it was
absurd to think the Sun
went around the Earth...

 ← **Approx. size of Earth**

Earth radius = 6371 km = 3959 mi

Sun radius = 695,500 km = 432,200 mi

... and was the first to propose the **heliocentric model** that the Earth went around the Sun.

 ← **Approx. size of Earth**

Earth radius = 6371 km = 3959 mi

Sun radius = 695,500 km = 432,200 mi


[1700 years later,
Copernicus would credit
Aristarchus for this
idea.]

 ← **Approx. size of Earth**

Earth radius = 6371 km = 3959 mi

Sun radius = 695,500 km = 432,200 mi


Ironically, Aristarchus' theory was not accepted by the other ancient Greeks...

 ← **Approx. size of Earth**

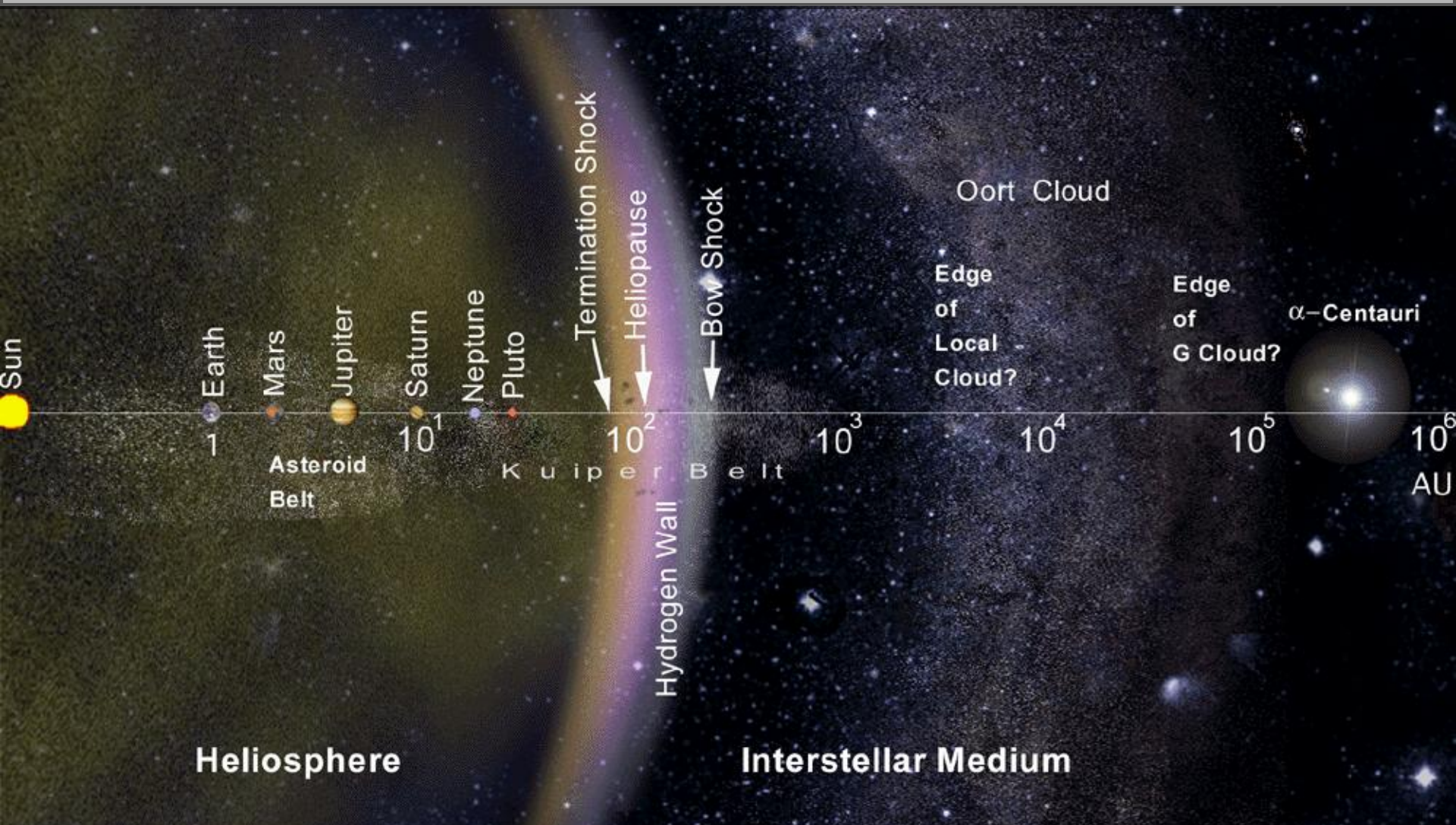
Earth radius = 6371 km = 3959 mi

Sun radius = 695,500 km = 432,200 mi

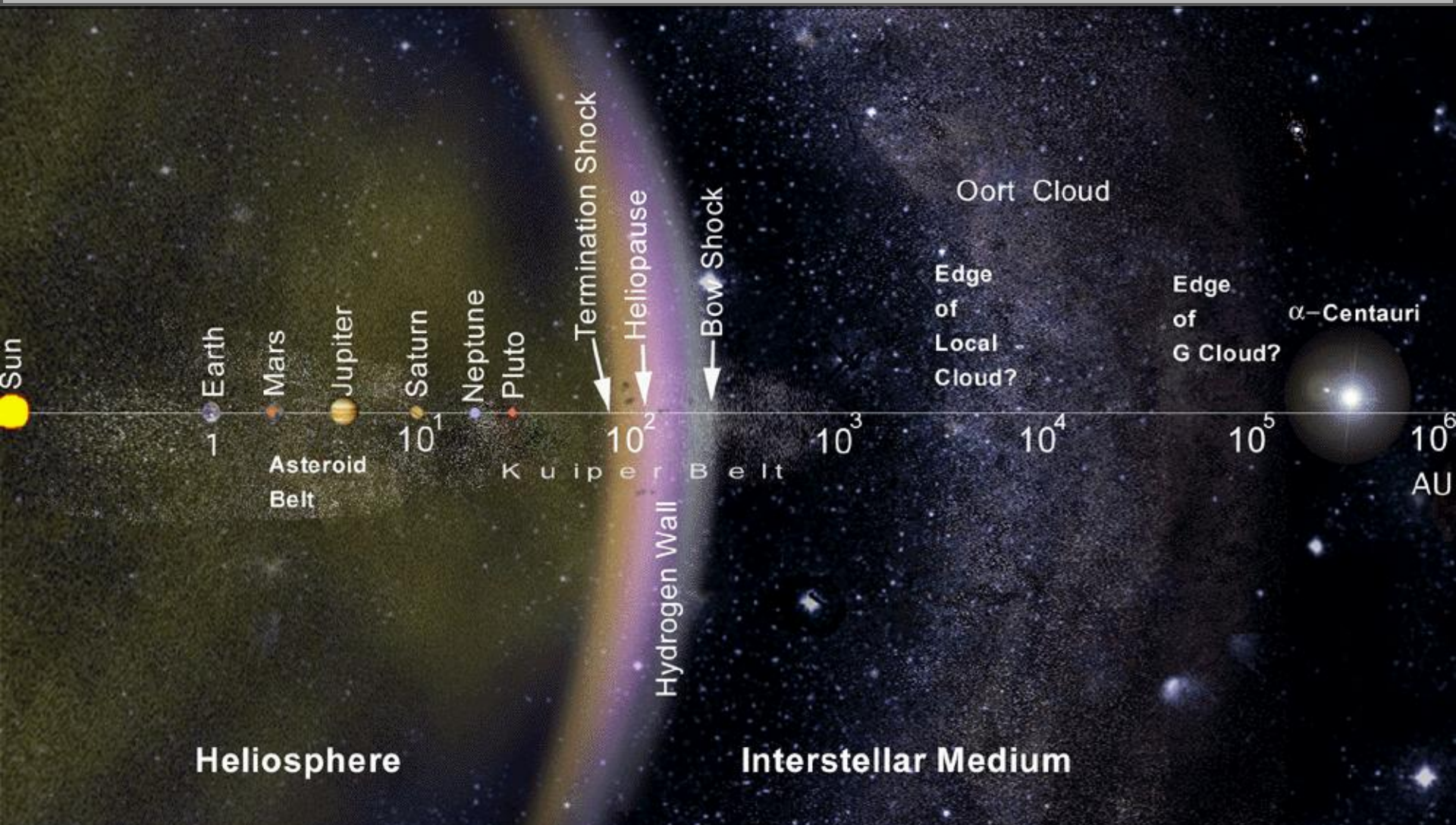
... but we'll explain
why later.

 ← **Approx. size of Earth**

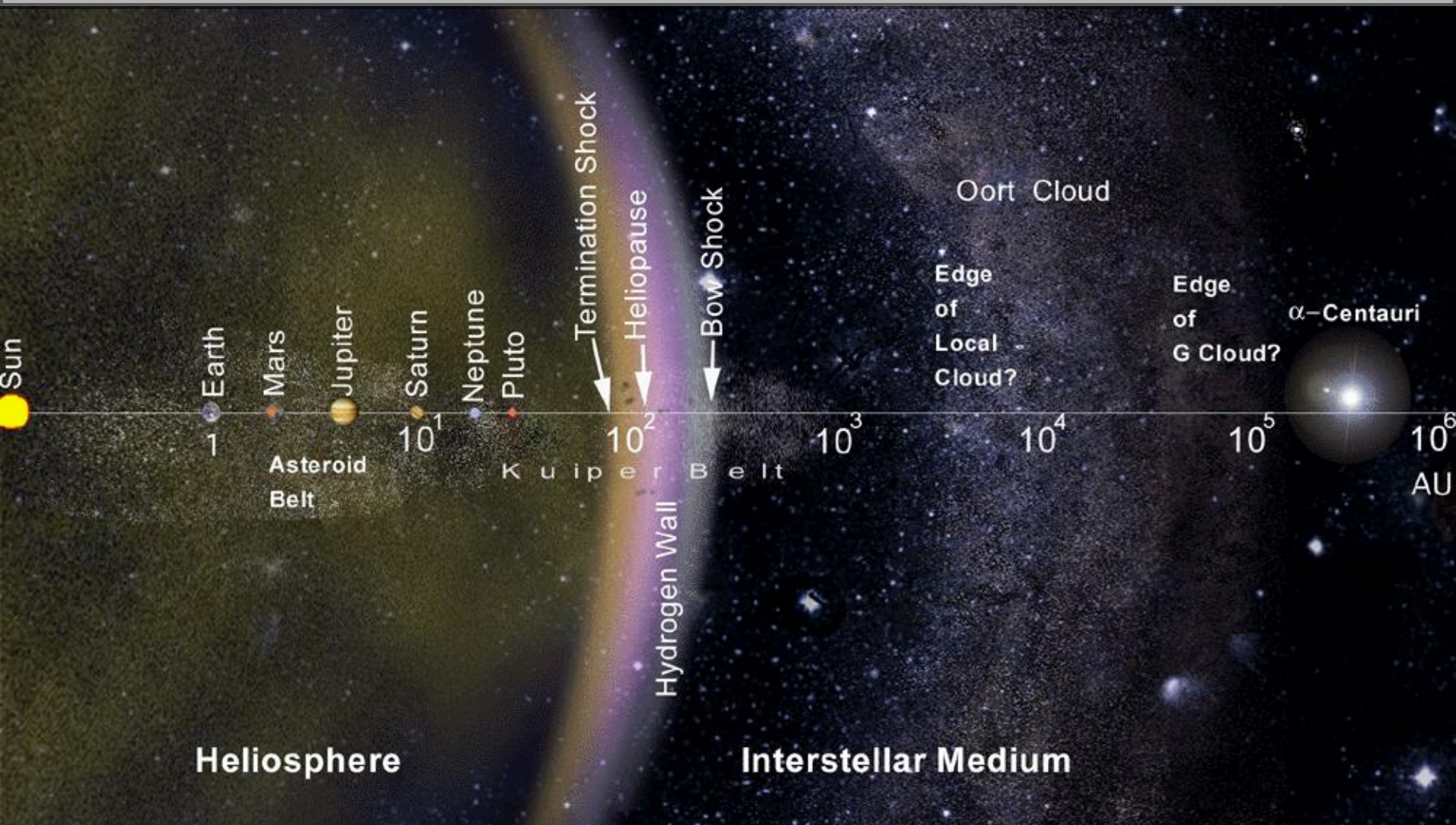
The distance from the Earth to the Sun is known as the Astronomical Unit (AU).



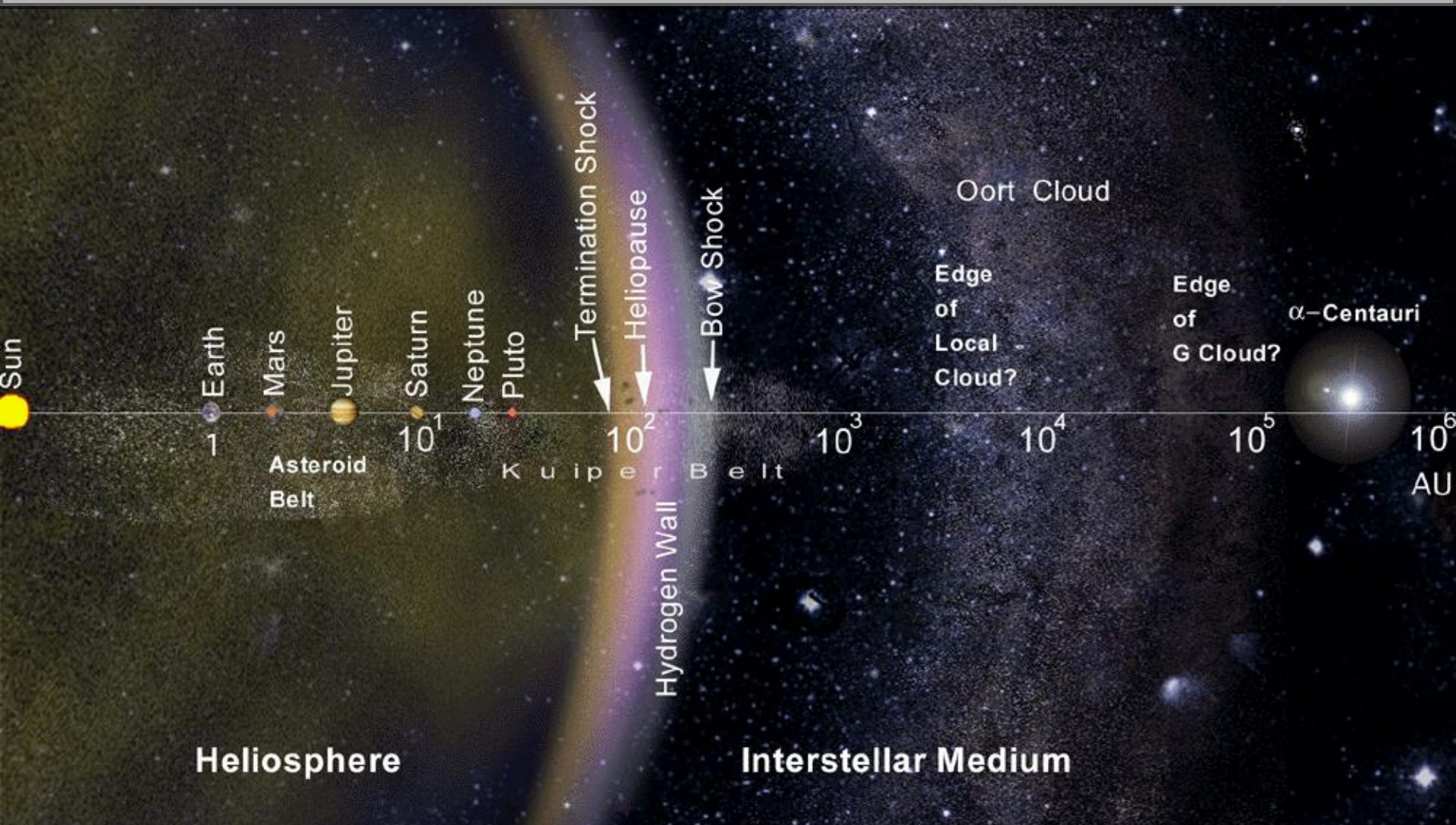
It is an extremely important rung in the cosmic distance ladder.



Aristarchus' original estimate of the AU was inaccurate...

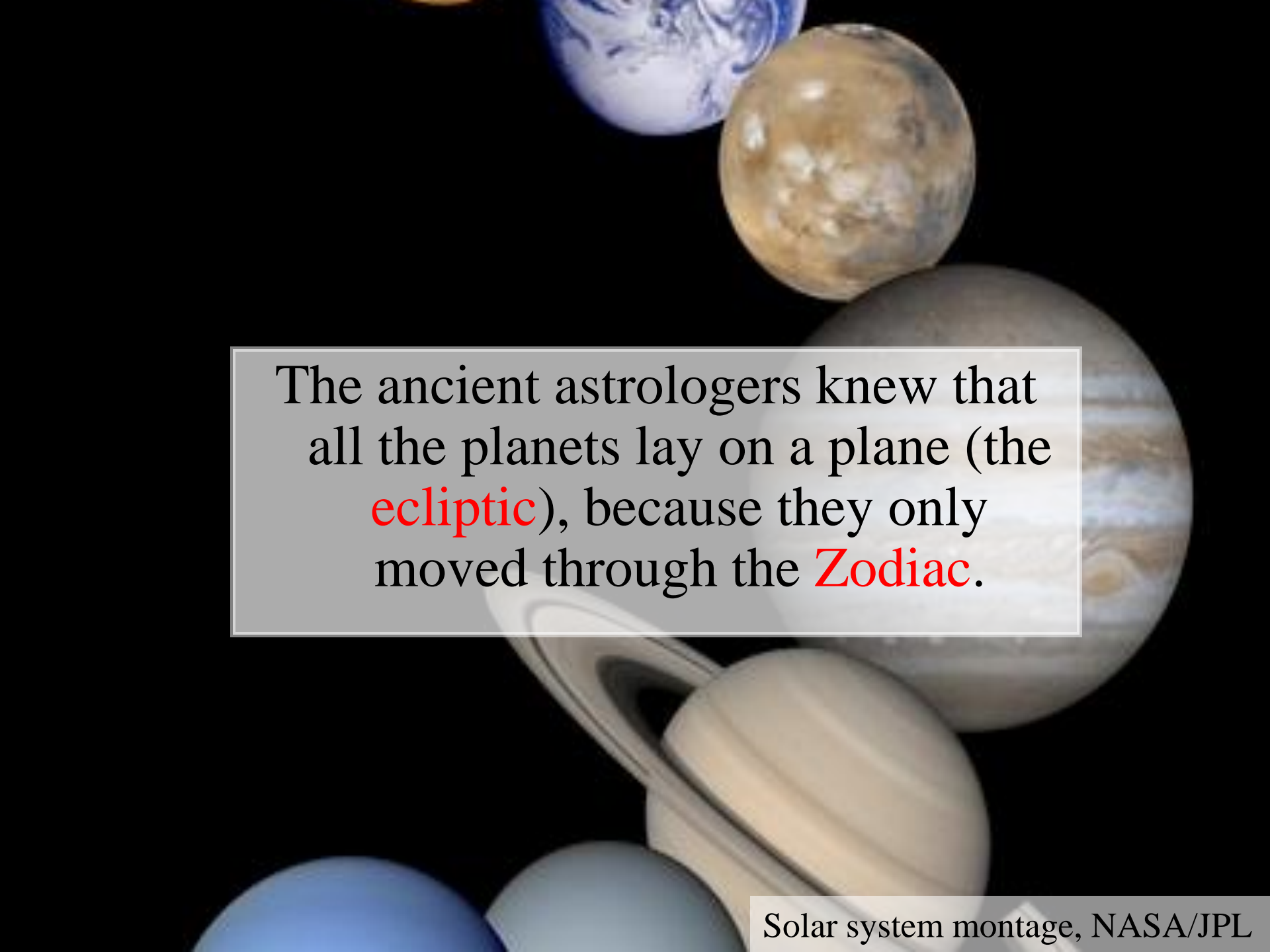


... but we'll see much more accurate ways to measure the AU later on.






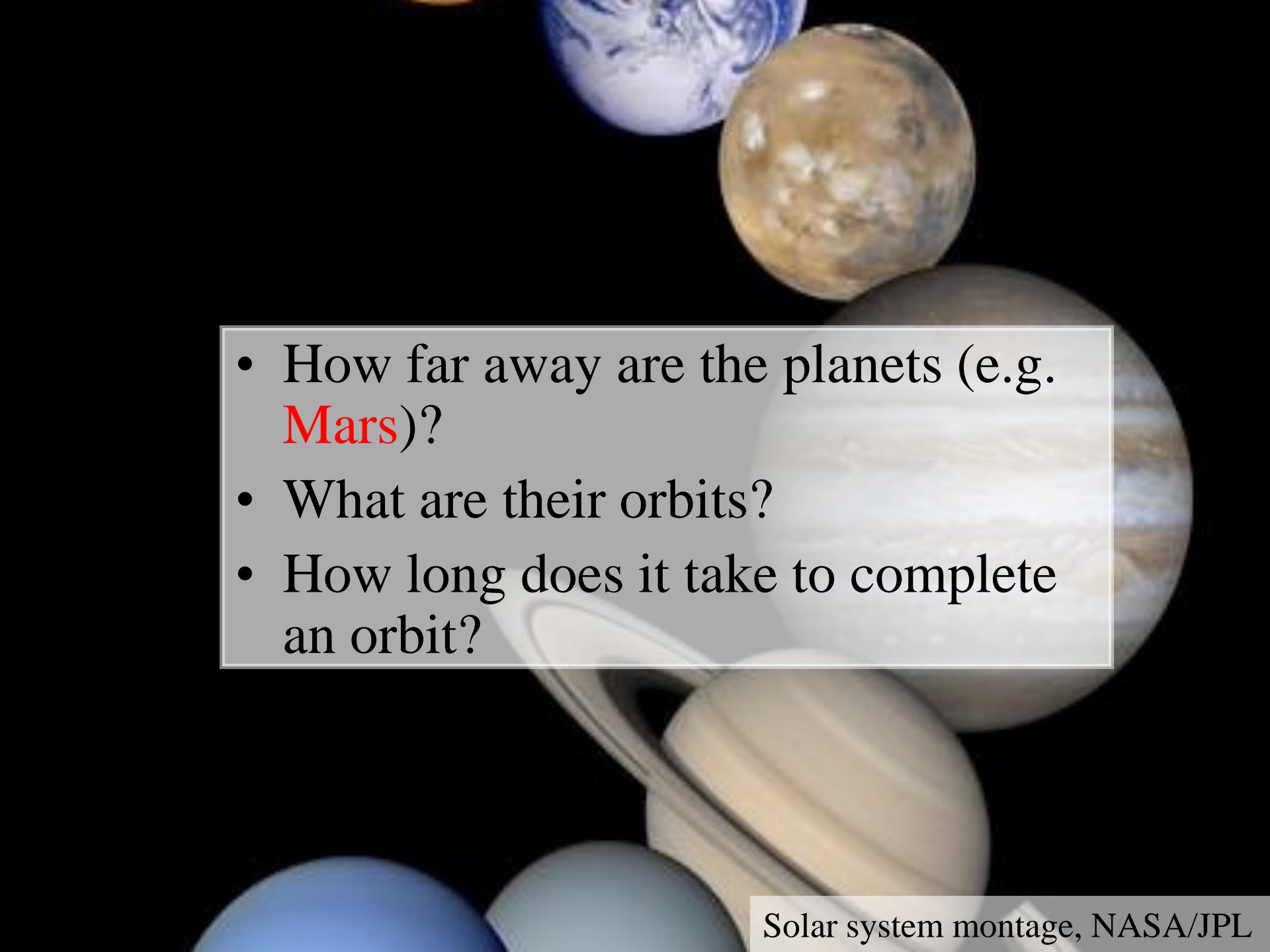
**4th rung: the
planets**

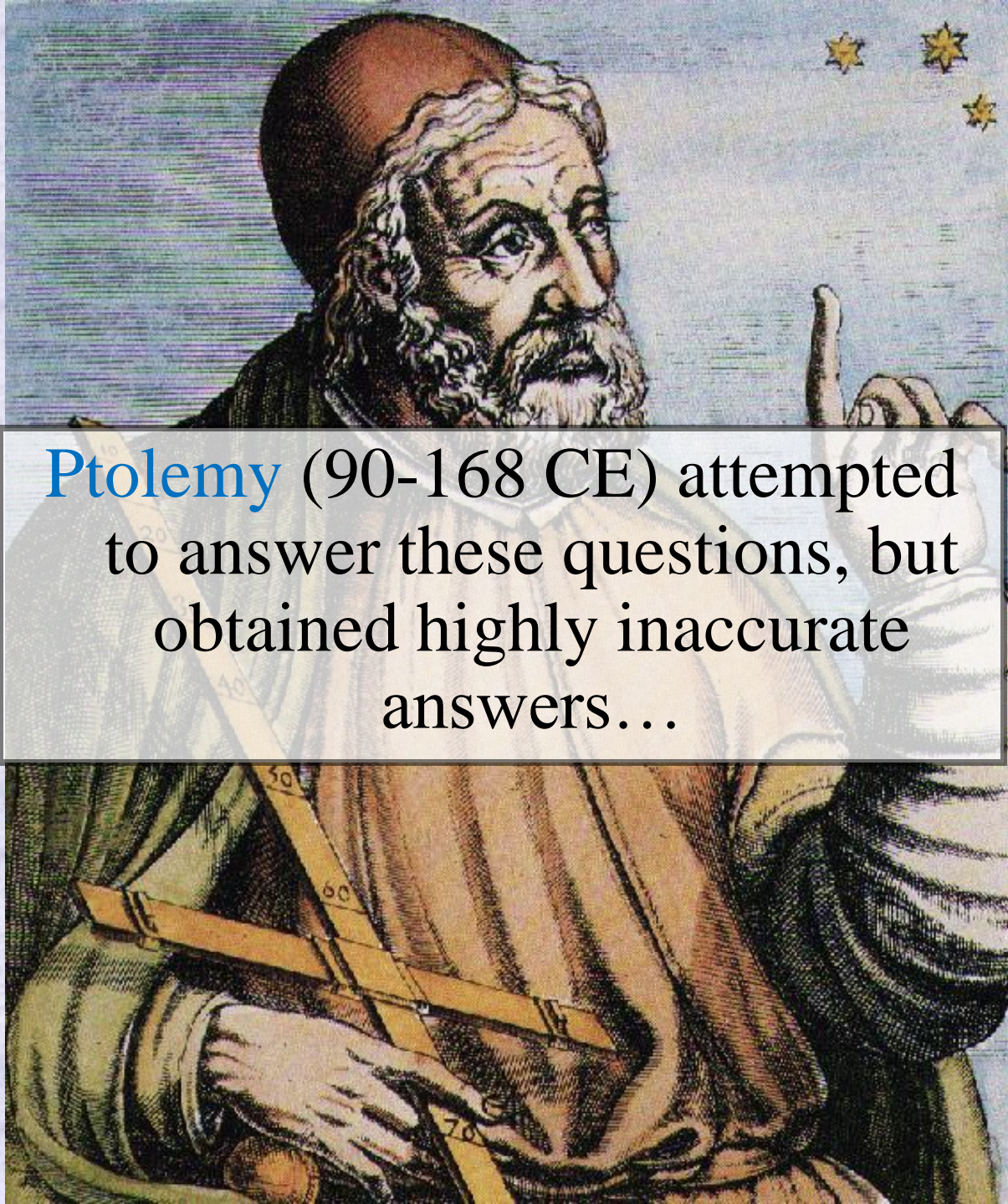


The ancient astrologers knew that all the planets lay on a plane (the **ecliptic**), because they only moved through the **Zodiac**.

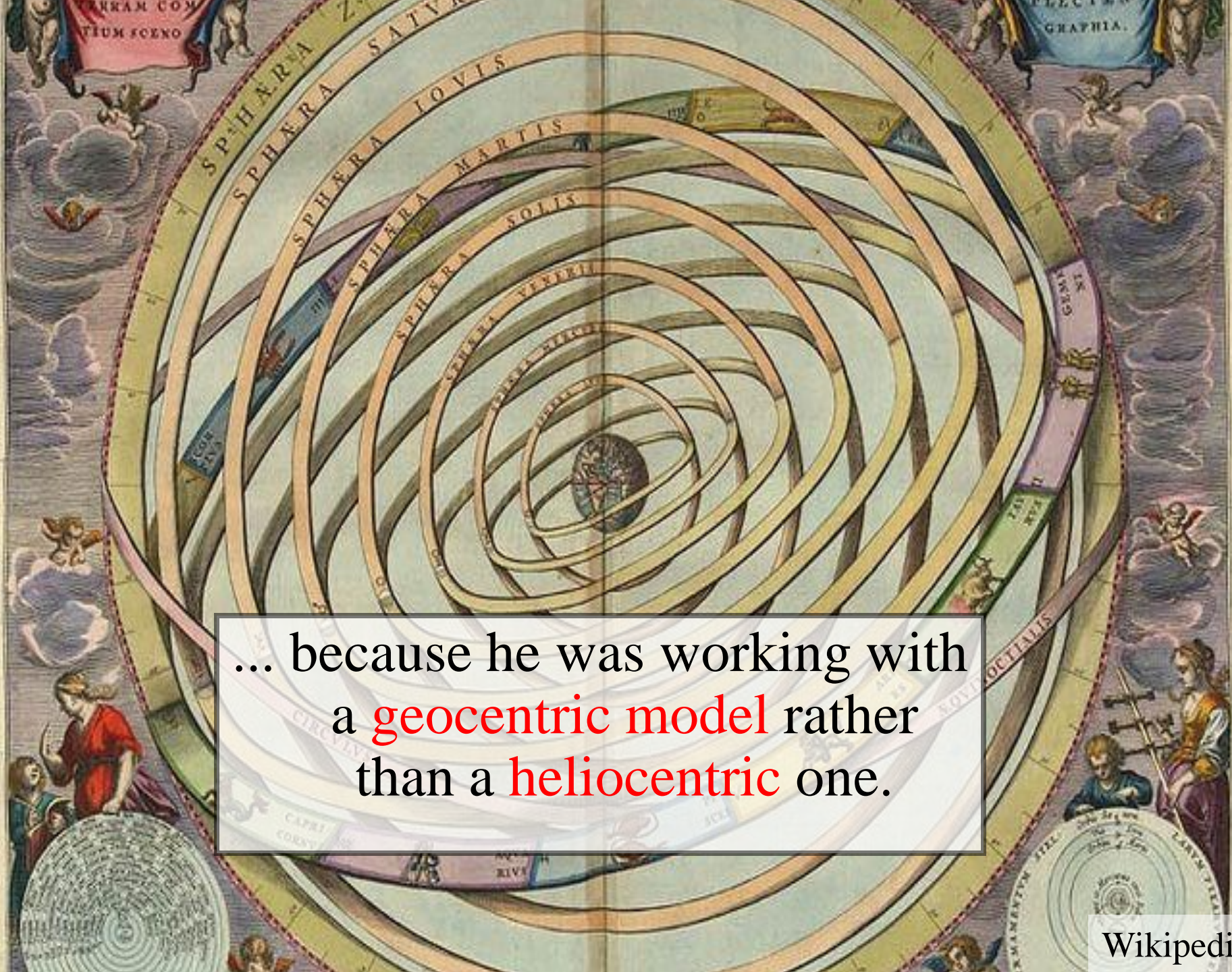
A solar system montage featuring Earth, Mars, Jupiter, Saturn, and Uranus against a black background. The planets are arranged in a descending diagonal from top-left to bottom-right. Earth is at the top left, followed by Mars, then Jupiter, Saturn, and Uranus at the bottom left. A semi-transparent grey box with a white border is centered over the image, containing the text "But this still left many questions unanswered:".

But this still left many
questions unanswered:

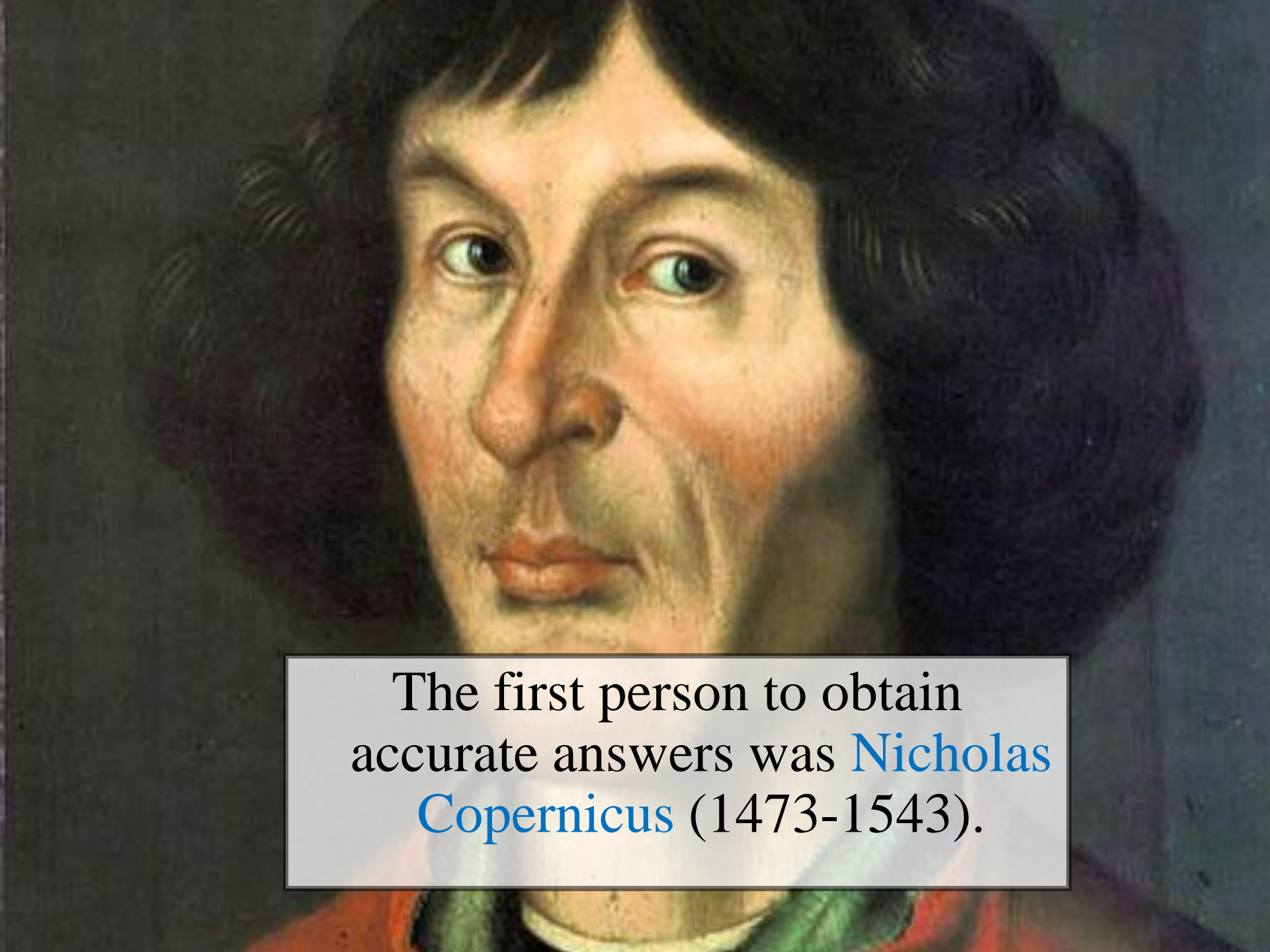
- 
- How far away are the planets (e.g. **Mars**)?
 - What are their orbits?
 - How long does it take to complete an orbit?



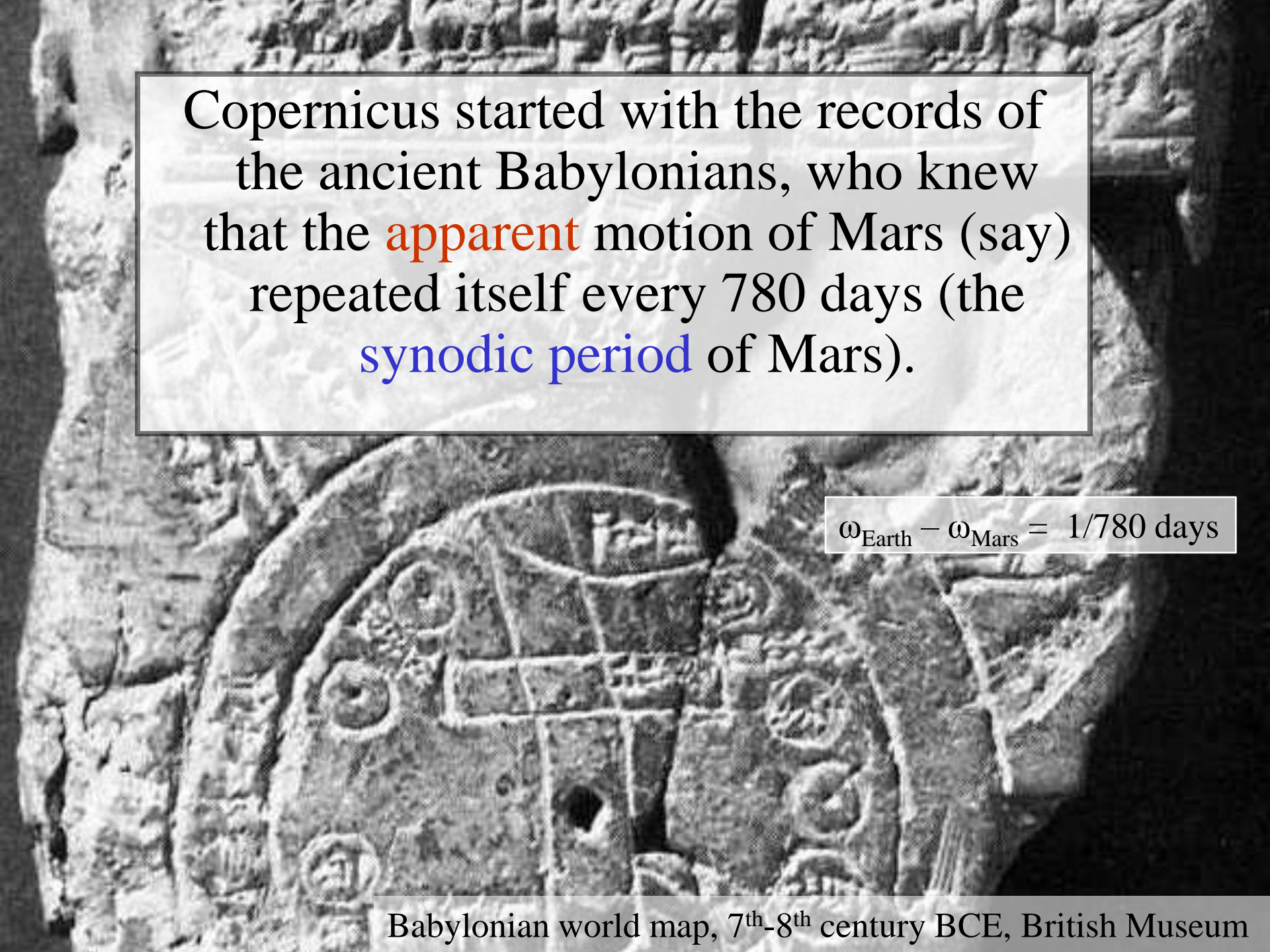
Ptolemy (90-168 CE) attempted to answer these questions, but obtained highly inaccurate answers...



... because he was working with a **geocentric model** rather than a **heliocentric** one.

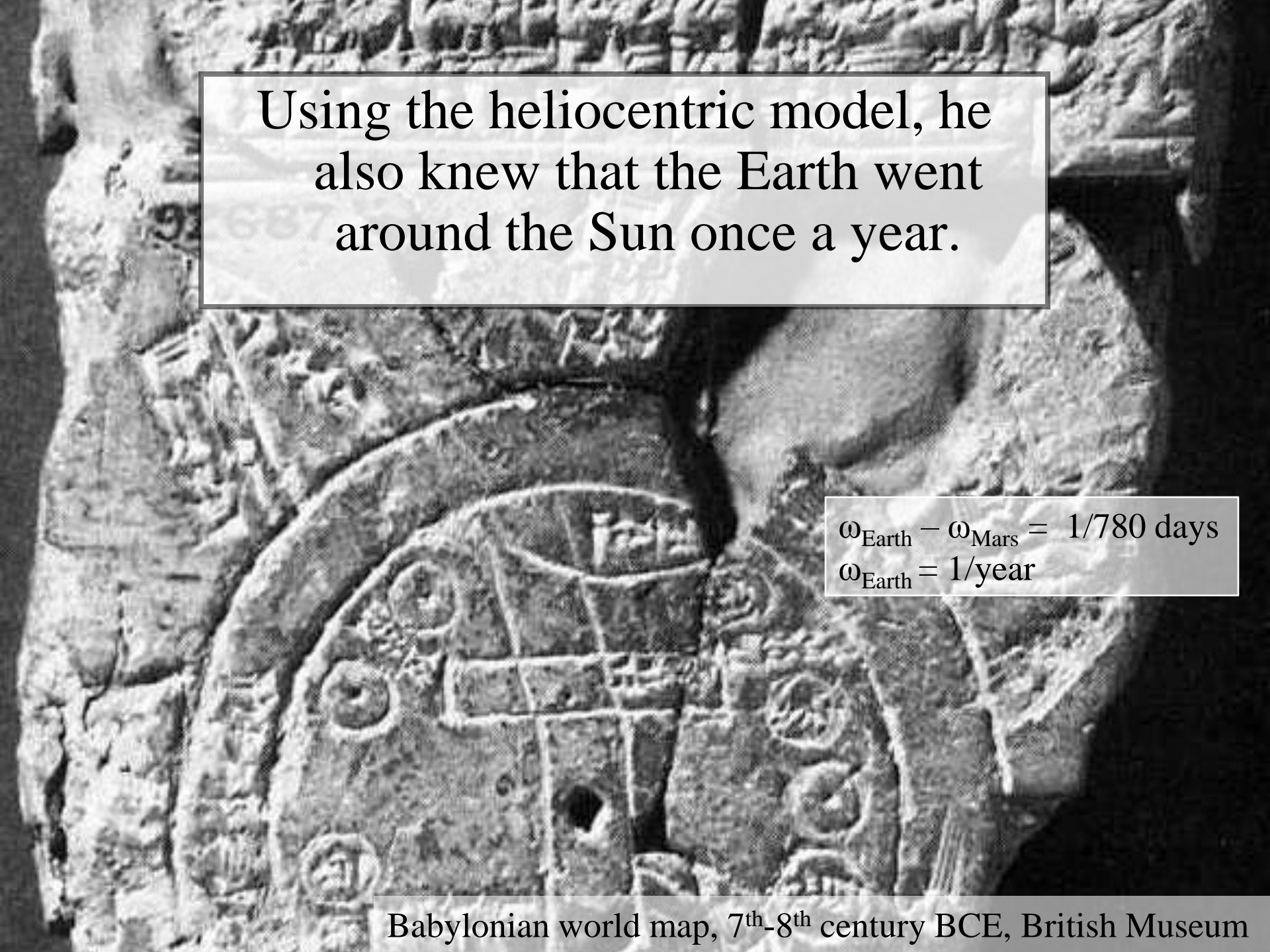
A detailed oil painting portrait of Nicholas Copernicus, showing him from the chest up. He has dark, wavy hair and is looking slightly to the left of the viewer. He is wearing a red garment with a green collar. The background is dark and indistinct.

The first person to obtain accurate answers was **Nicholas Copernicus** (1473-1543).



Copernicus started with the records of the ancient Babylonians, who knew that the **apparent** motion of Mars (say) repeated itself every 780 days (the **synodic period** of Mars).

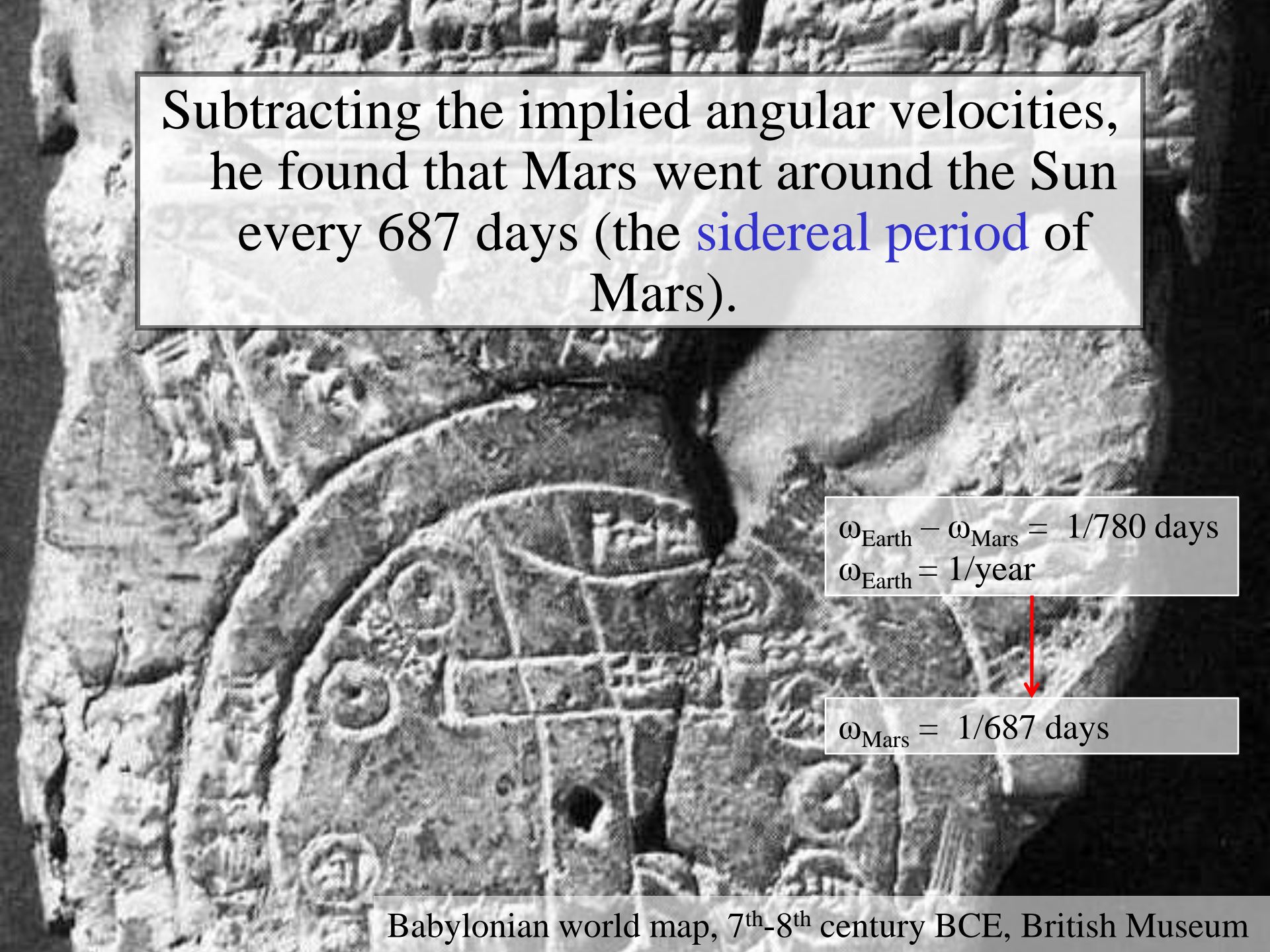
$$\omega_{\text{Earth}} - \omega_{\text{Mars}} = 1/780 \text{ days}$$



Using the heliocentric model, he also knew that the Earth went around the Sun once a year.


$$\omega_{\text{Earth}} - \omega_{\text{Mars}} = 1/780 \text{ days}$$
$$\omega_{\text{Earth}} = 1/\text{year}$$

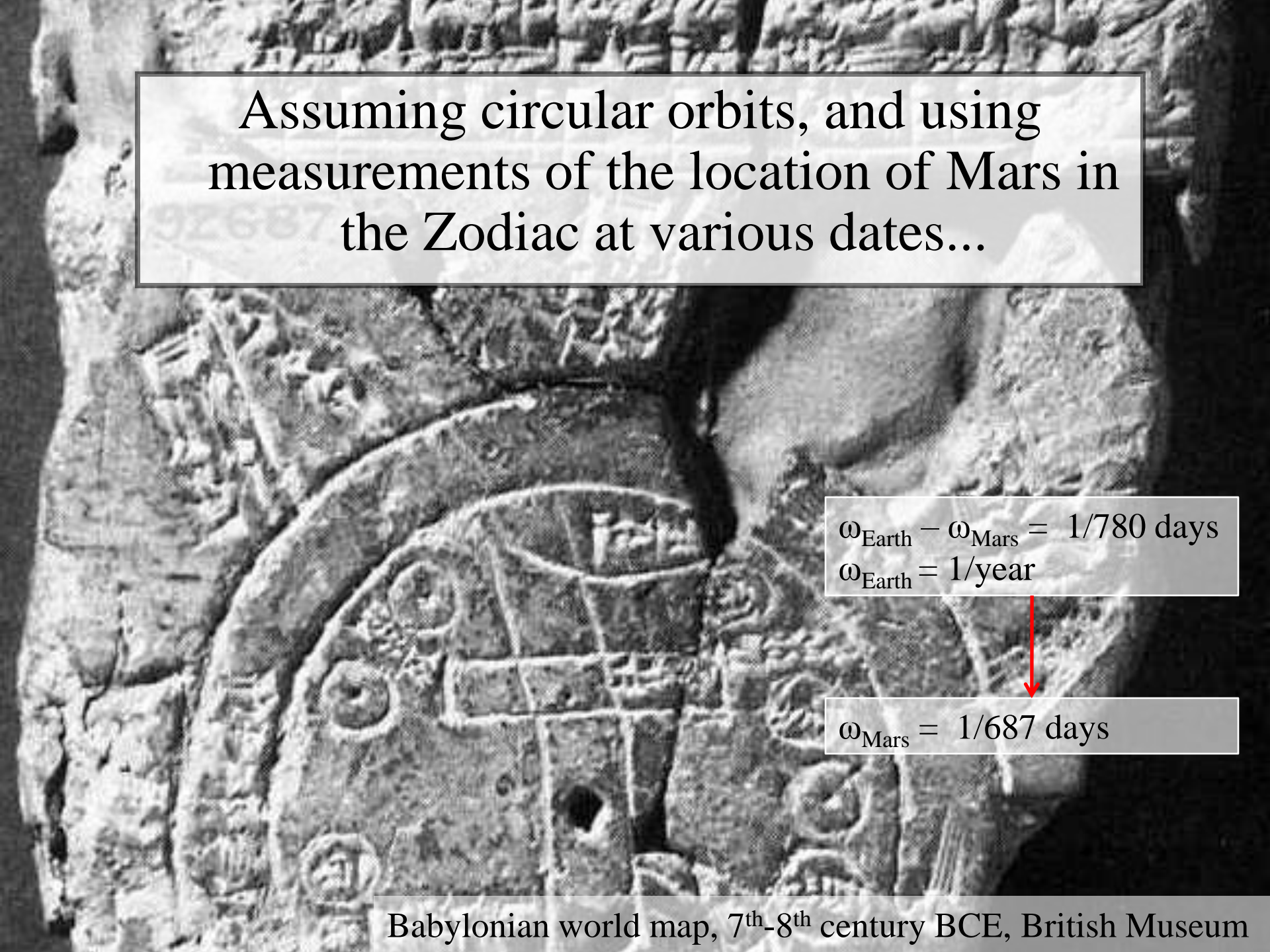
Babylonian world map, 7th-8th century BCE, British Museum



Subtracting the implied angular velocities, he found that Mars went around the Sun every 687 days (the **sidereal period** of Mars).

$$\omega_{\text{Earth}} - \omega_{\text{Mars}} = 1/780 \text{ days}$$
$$\omega_{\text{Earth}} = 1/\text{year}$$


$$\omega_{\text{Mars}} = 1/687 \text{ days}$$

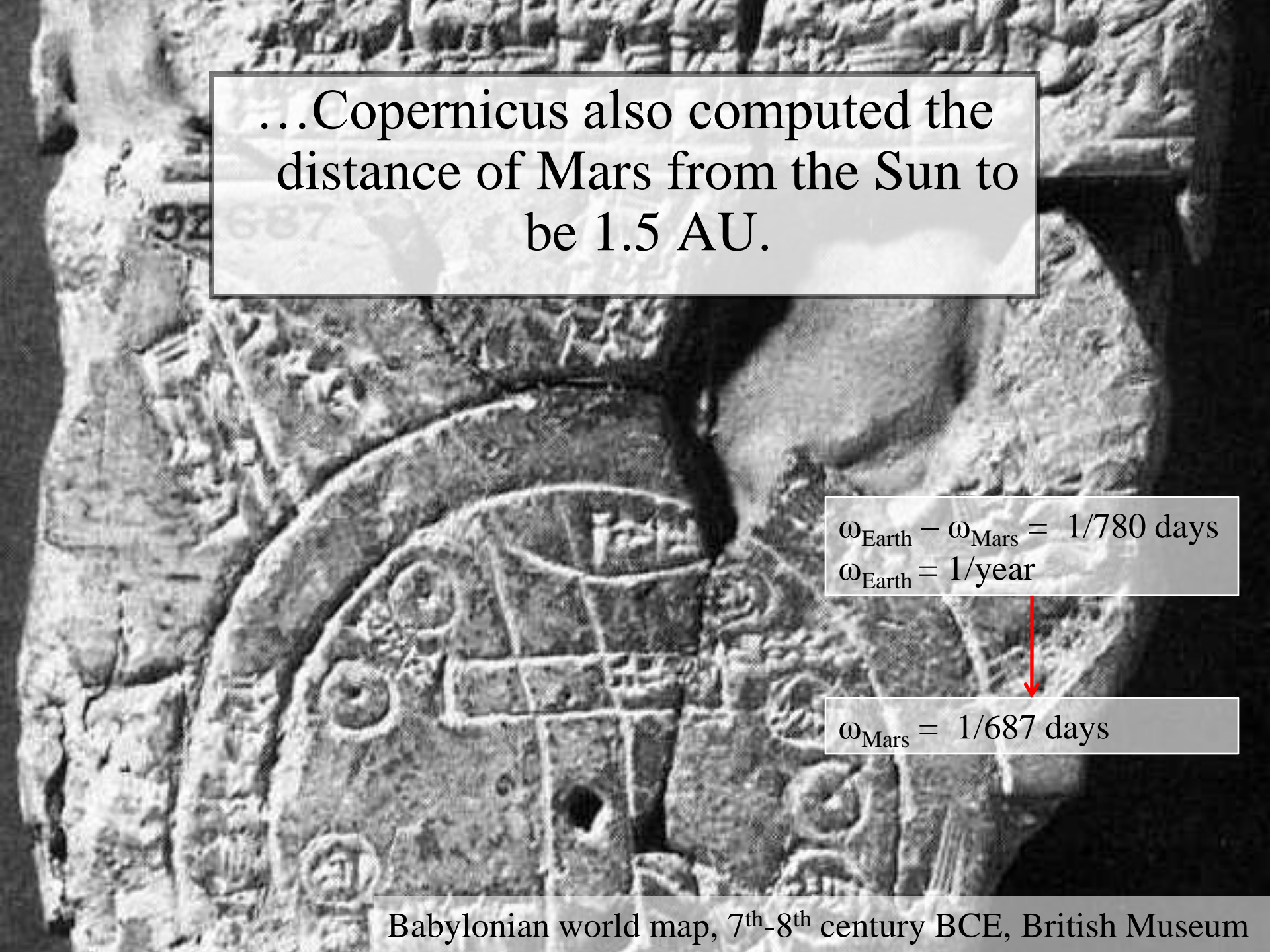


Assuming circular orbits, and using measurements of the location of Mars in the Zodiac at various dates...

$$\omega_{\text{Earth}} - \omega_{\text{Mars}} = 1/780 \text{ days}$$
$$\omega_{\text{Earth}} = 1/\text{year}$$



$$\omega_{\text{Mars}} = 1/687 \text{ days}$$

Babylonian world map, 7th-8th century BCE, British Museum

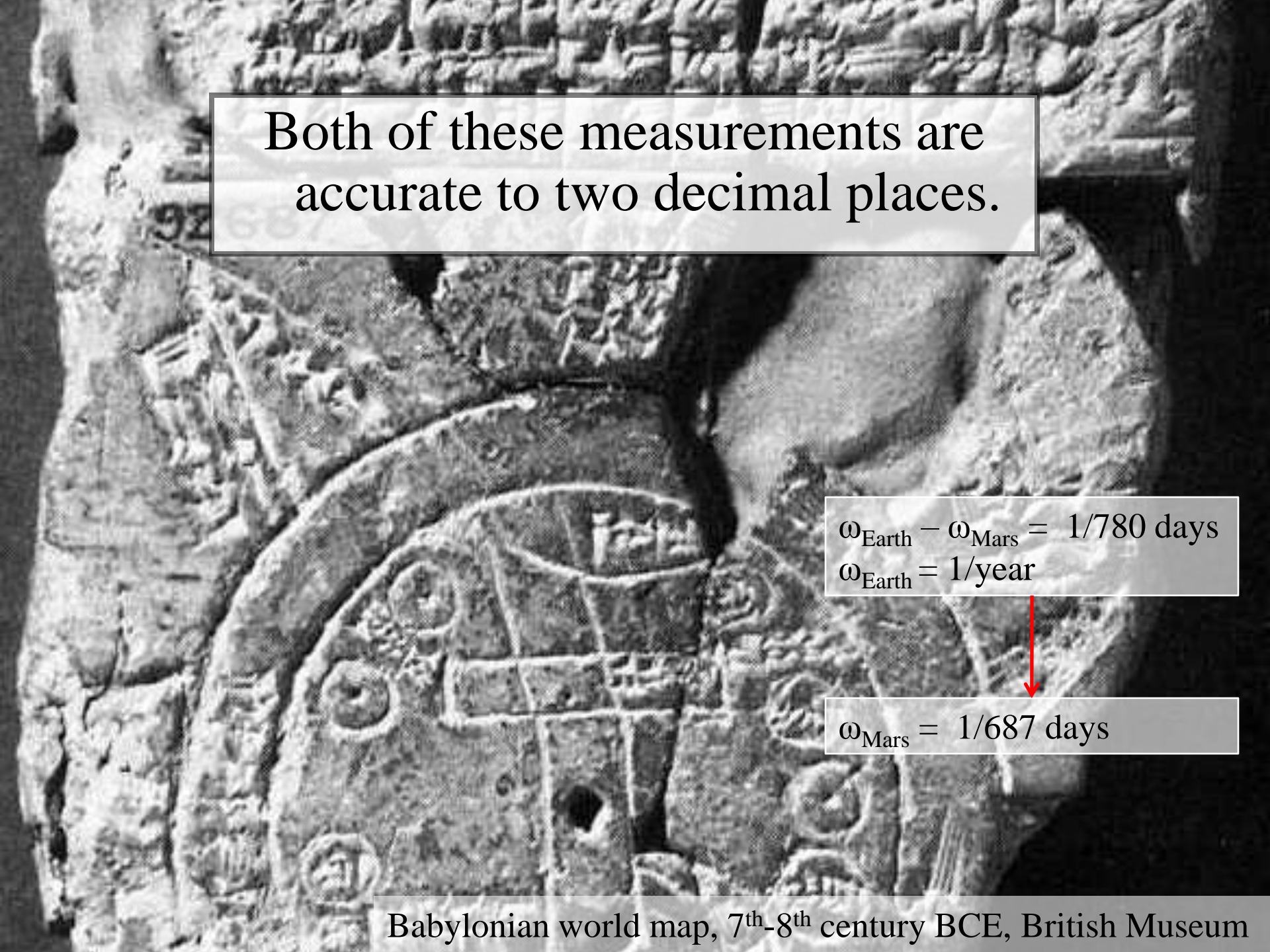


...Copernicus also computed the distance of Mars from the Sun to be 1.5 AU.

$$\omega_{\text{Earth}} - \omega_{\text{Mars}} = 1/780 \text{ days}$$
$$\omega_{\text{Earth}} = 1/\text{year}$$



$$\omega_{\text{Mars}} = 1/687 \text{ days}$$

Babylonian world map, 7th-8th century BCE, British Museum



Both of these measurements are accurate to two decimal places.

$$\omega_{\text{Earth}} - \omega_{\text{Mars}} = 1/780 \text{ days}$$
$$\omega_{\text{Earth}} = 1/\text{year}$$


$$\omega_{\text{Mars}} = 1/687 \text{ days}$$

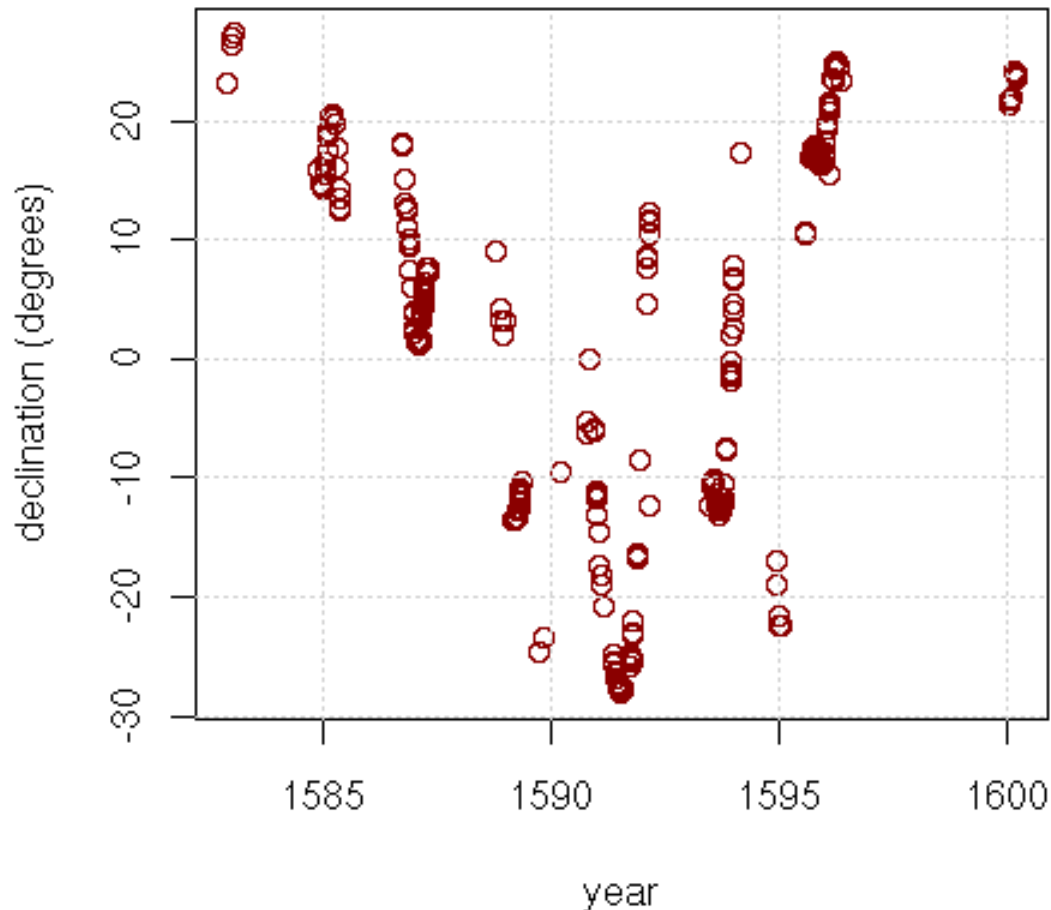
Babylonian world map, 7th-8th century BCE, British Museum



Tycho Brahe (1546-1601) made extremely detailed and long-term measurements of the position of Mars and other planets.

Unfortunately, his data deviated slightly from the predictions of the Copernican model.

Tycho Brahe's Mars Observations

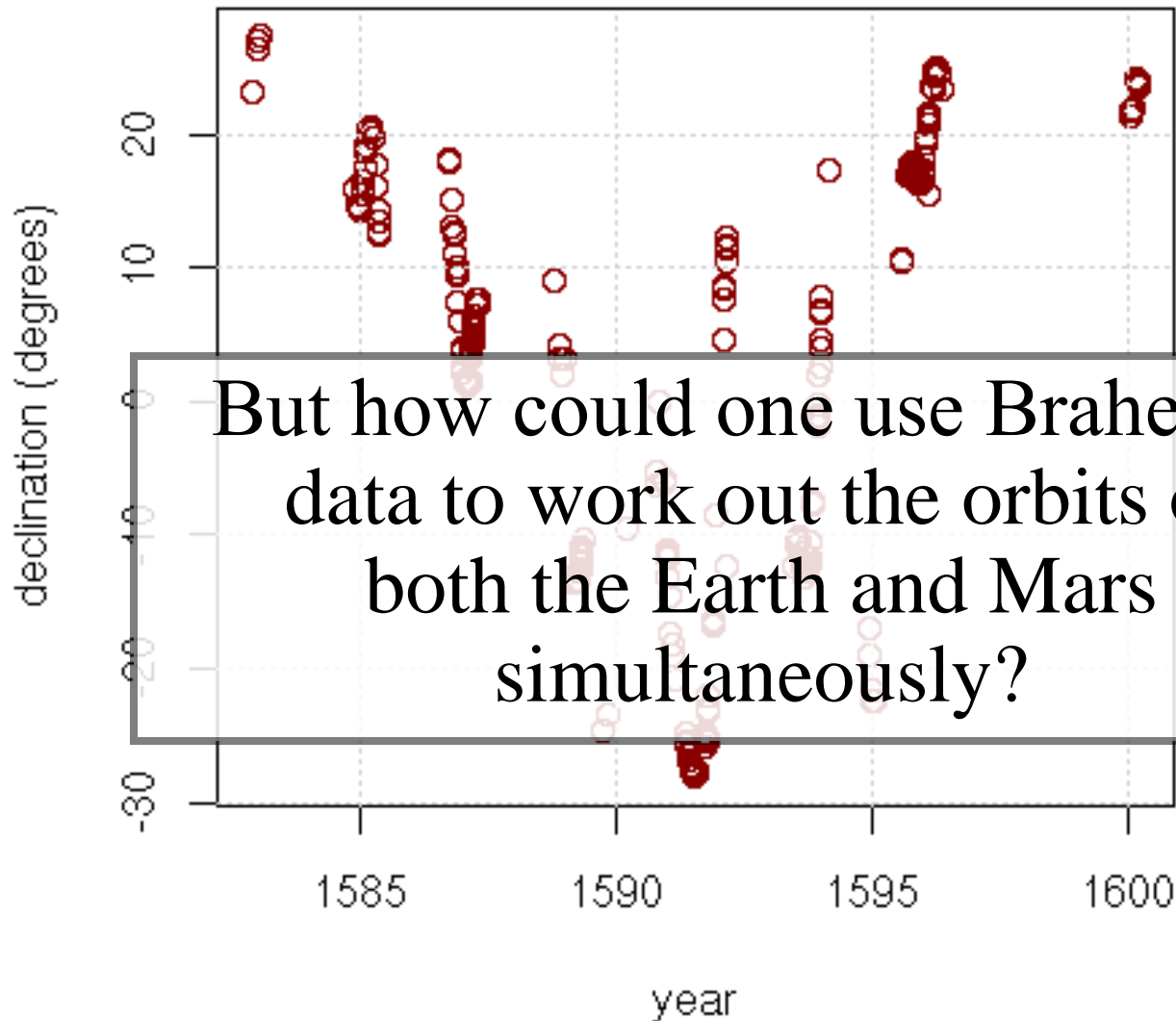


source: Tychonis Brahe Dani Opera Omnia



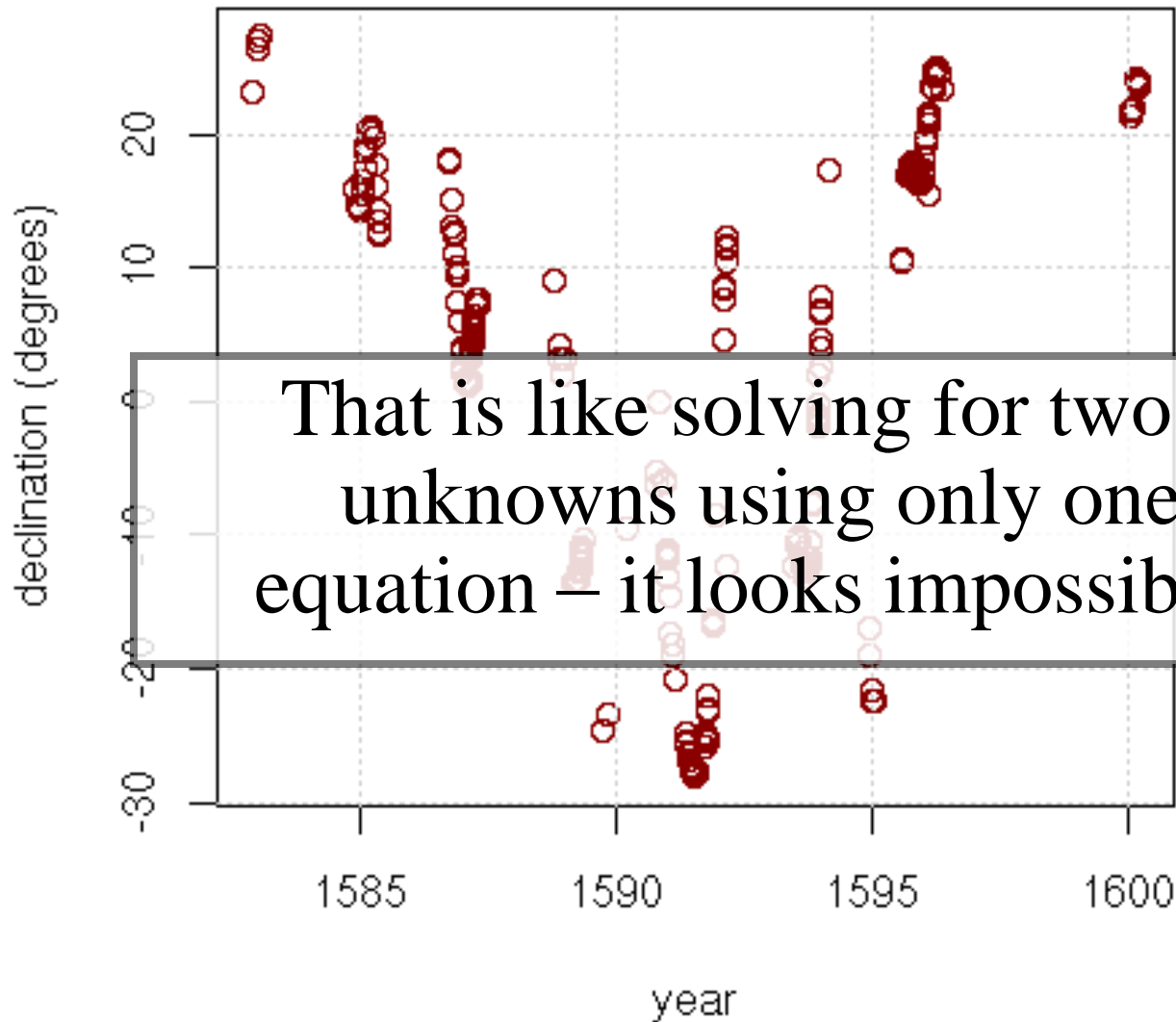
Johannes Kepler (1571-1630)
reasoned that this was because
the orbits of the Earth and Mars
were not quite circular.

Tycho Brahe's Mars Observations



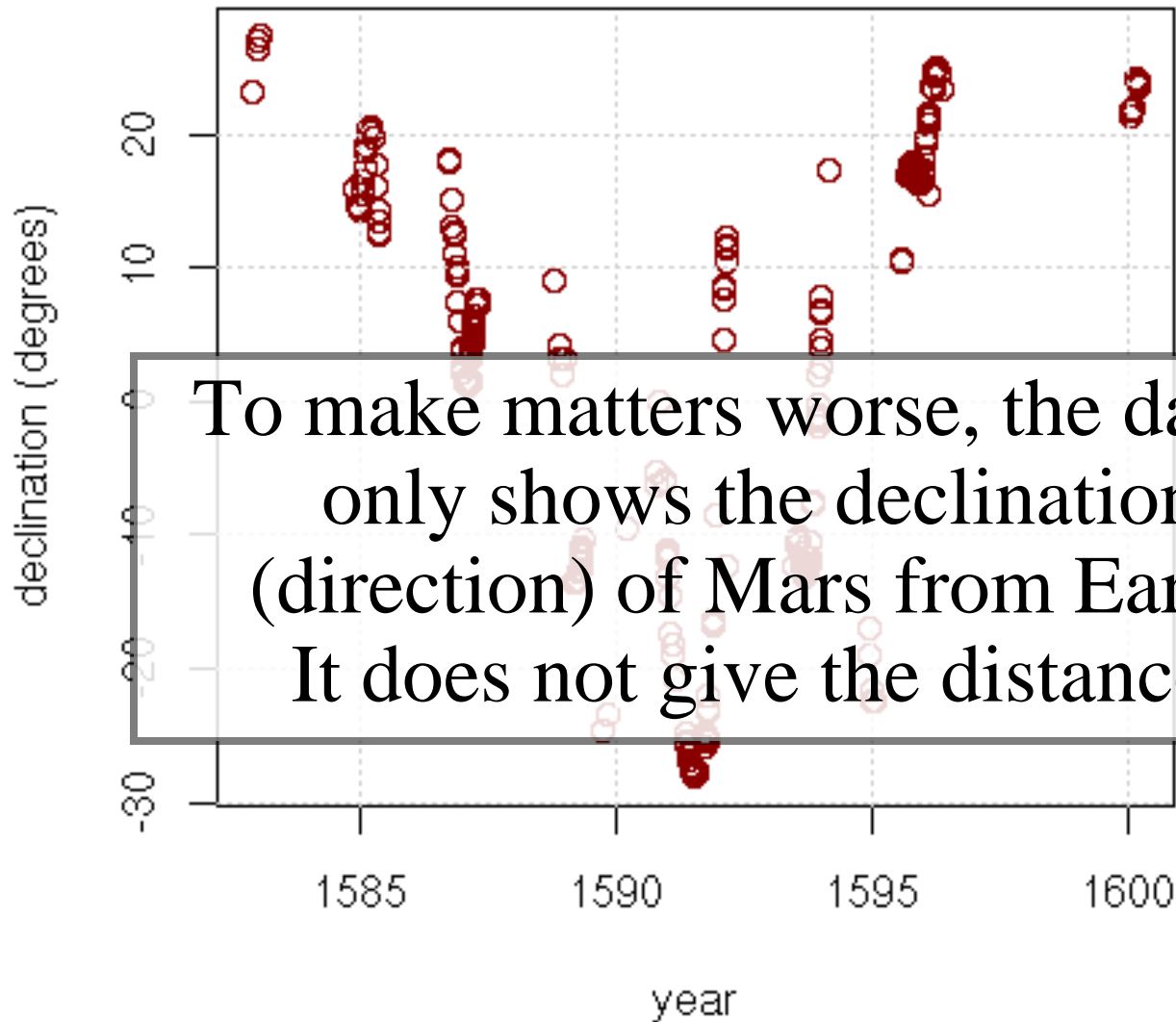
source: Tychonis Brahe Dani Opera Omnia

Tycho Brahe's Mars Observations



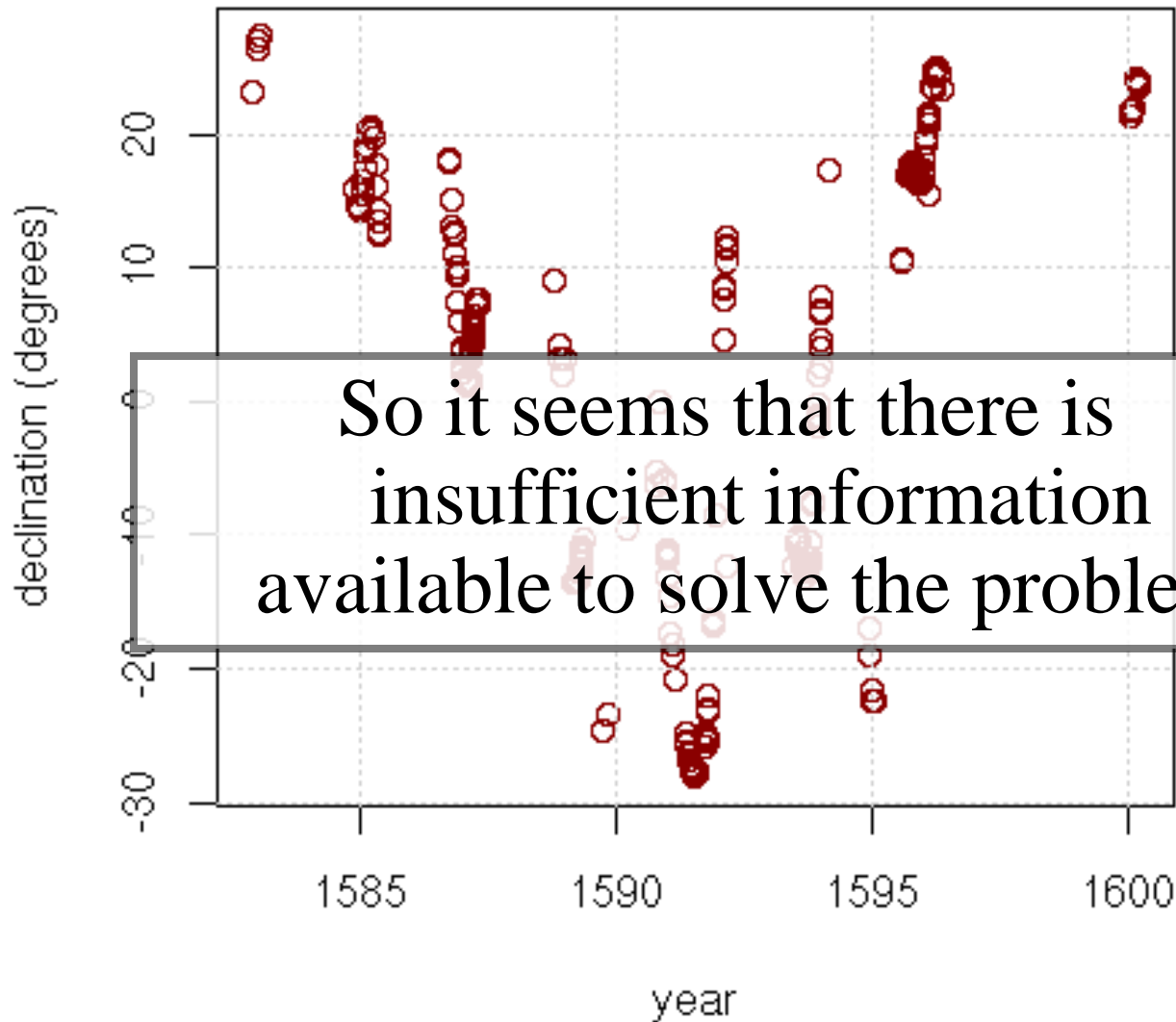
source: Tychonis Brahe Dani Opera Omnia

Tycho Brahe's Mars Observations



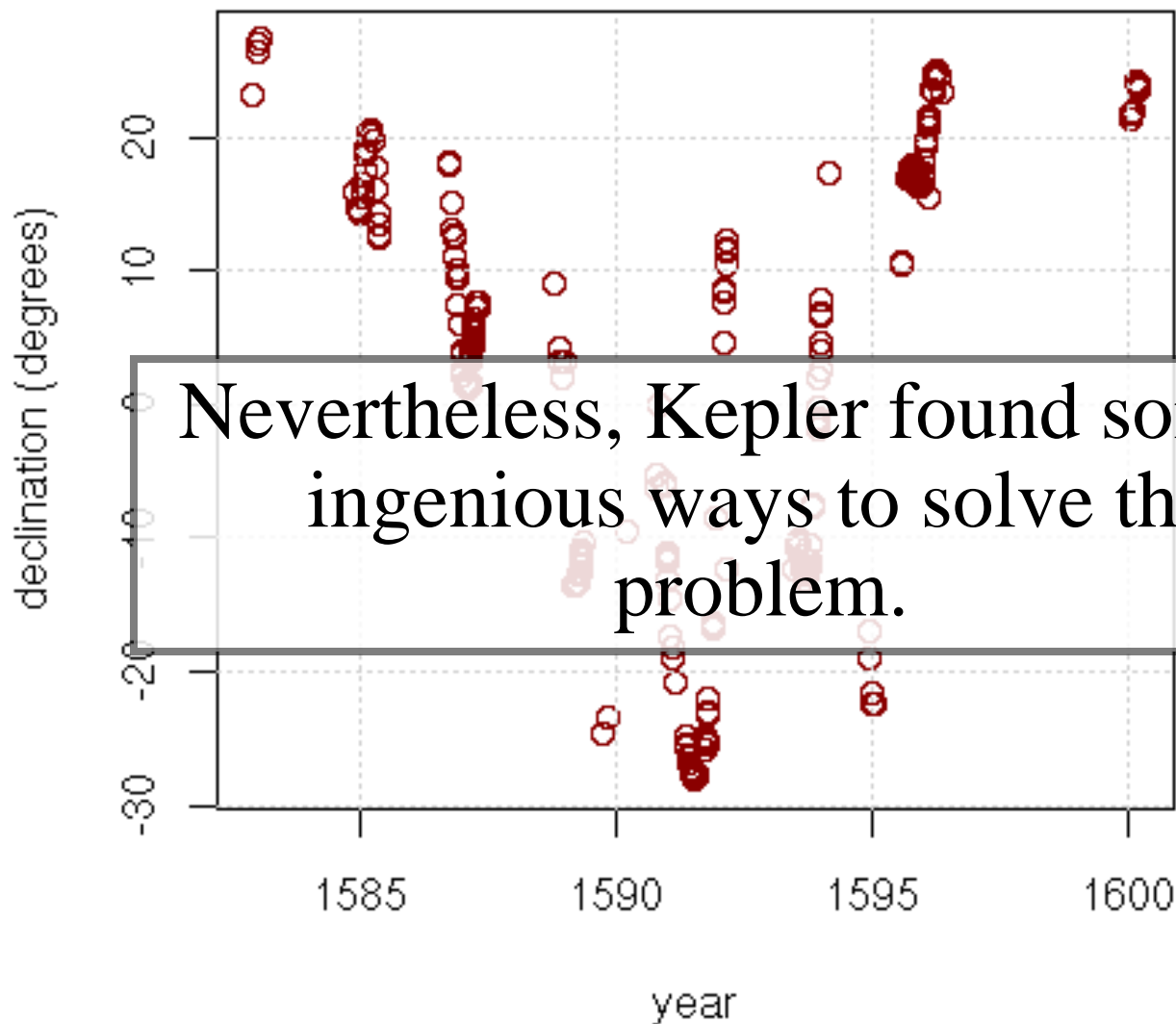
source: Tychonis Brahe Dani Opera Omnia

Tycho Brahe's Mars Observations



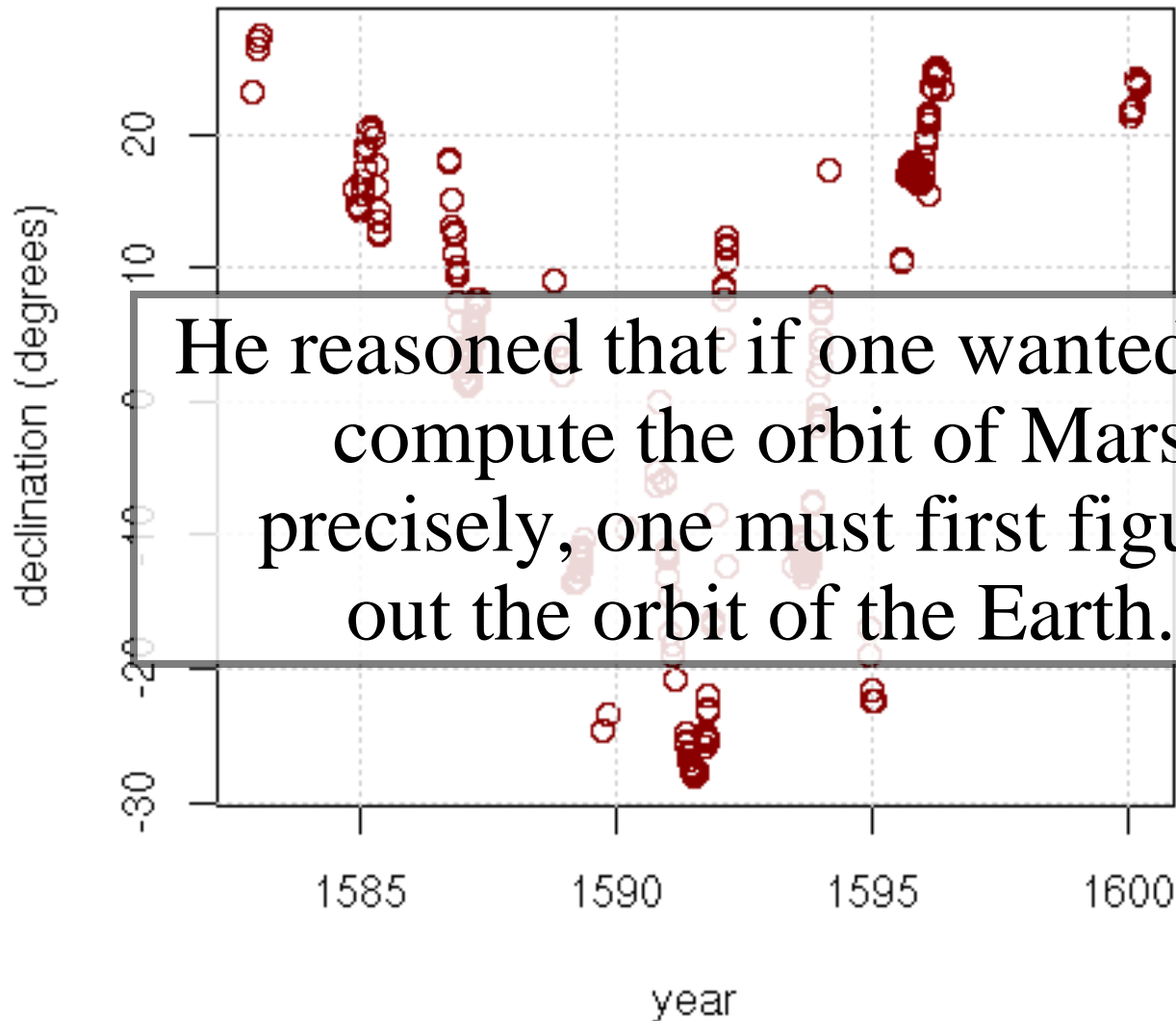
source: Tychonis Brahe Dani Opera Omnia

Tycho Brahe's Mars Observations



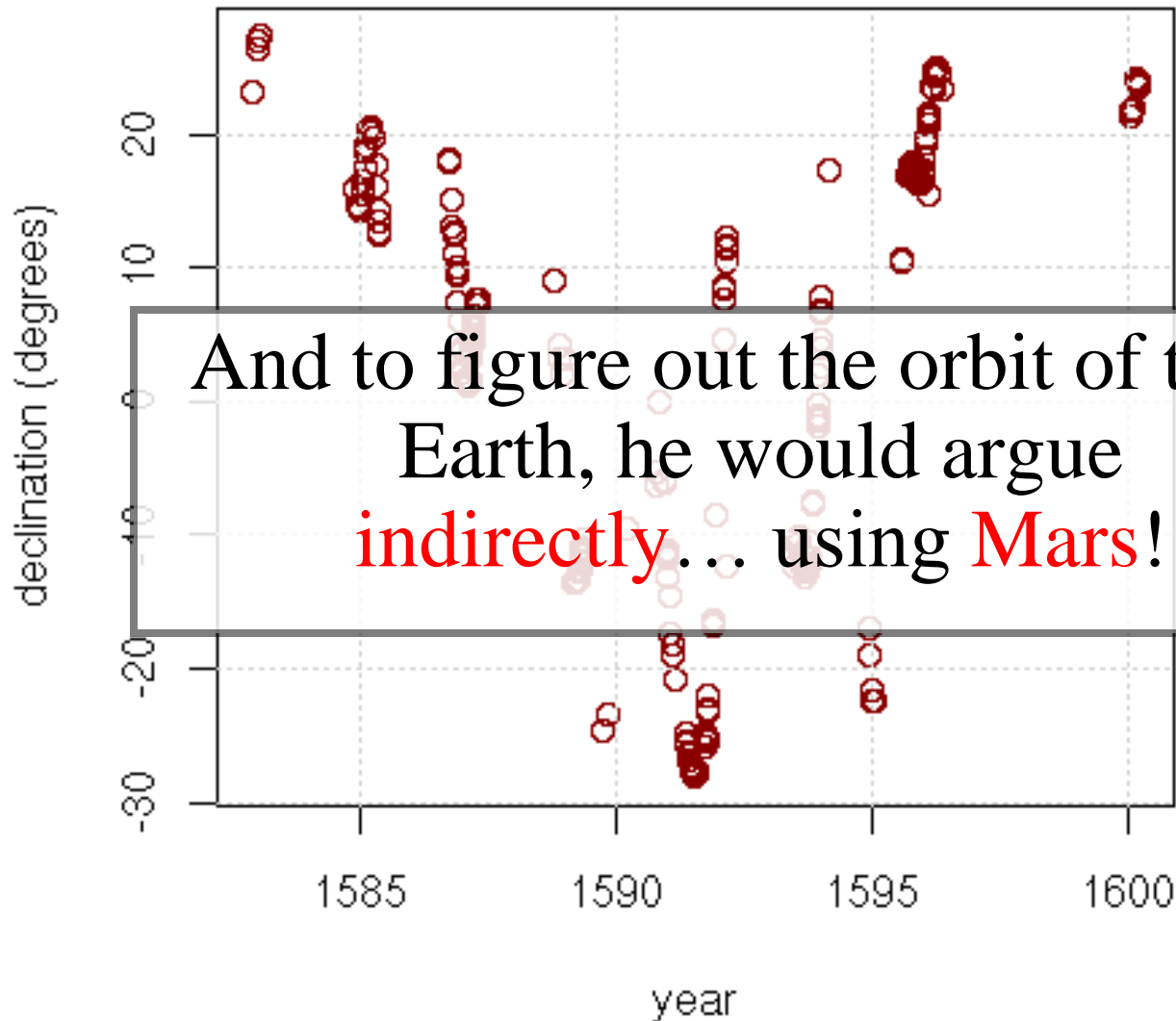
source: Tychonis Brahe Dani Opera Omnia

Tycho Brahe's Mars Observations

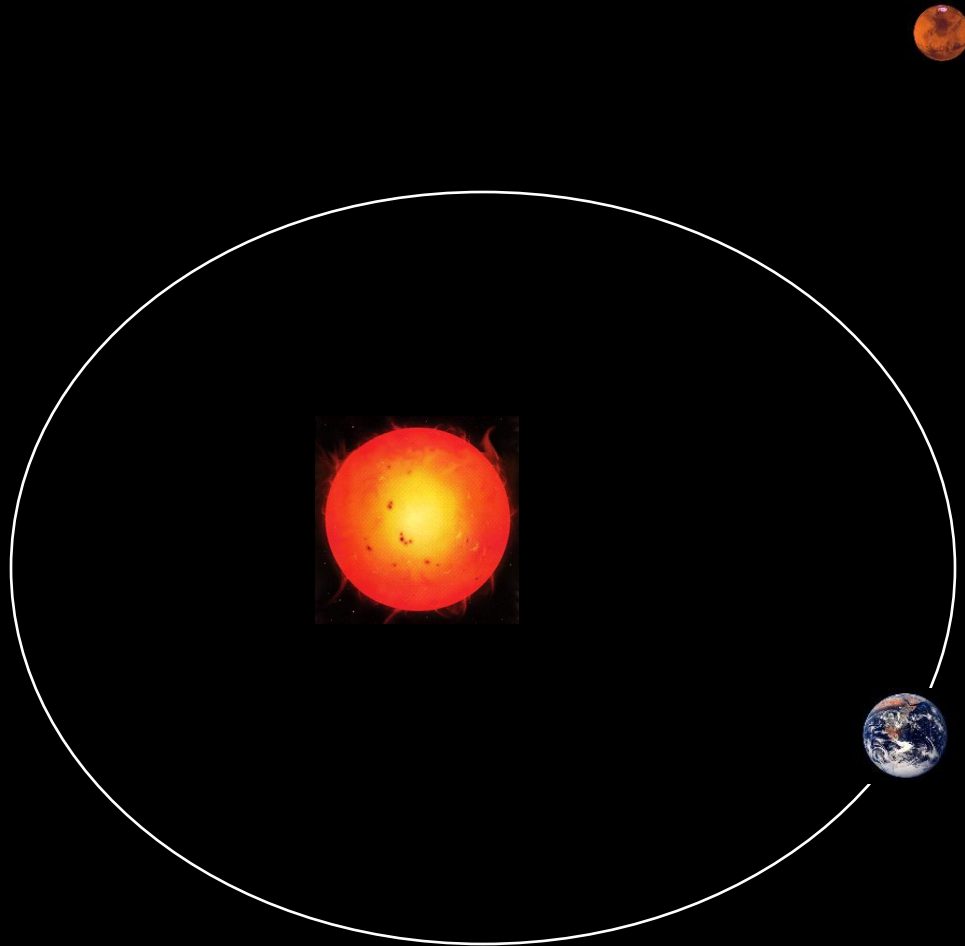


source: Tychonis Brahe Dani Opera Omnia

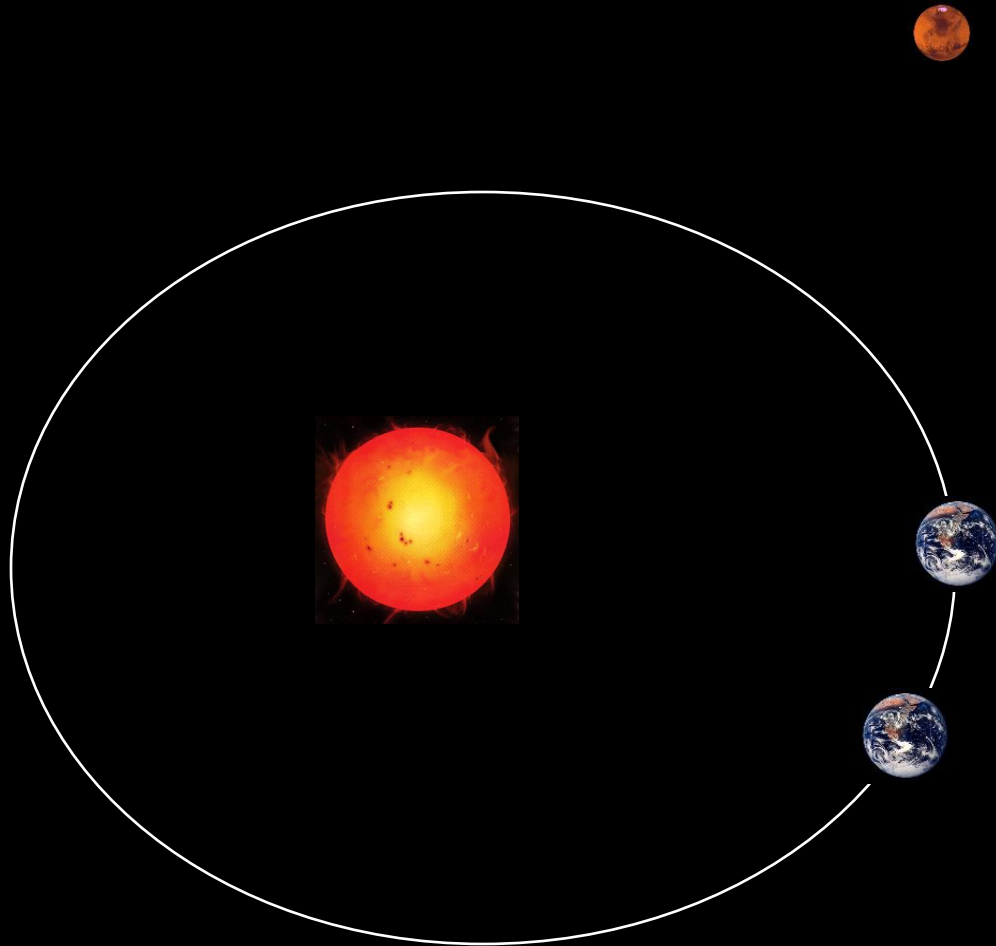
Tycho Brahe's Mars Observations



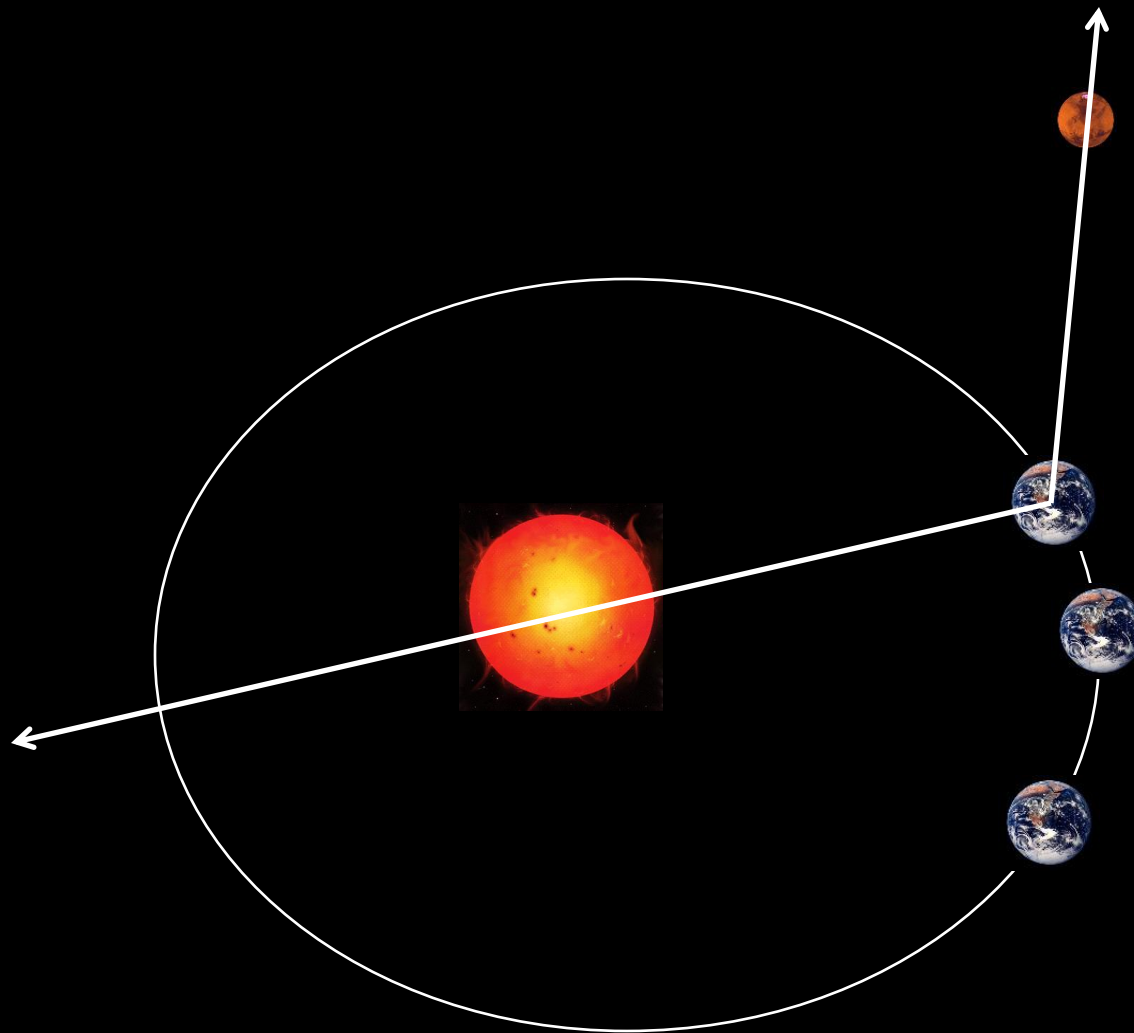
source: Tychonis Brahe Dani Opera Omnia



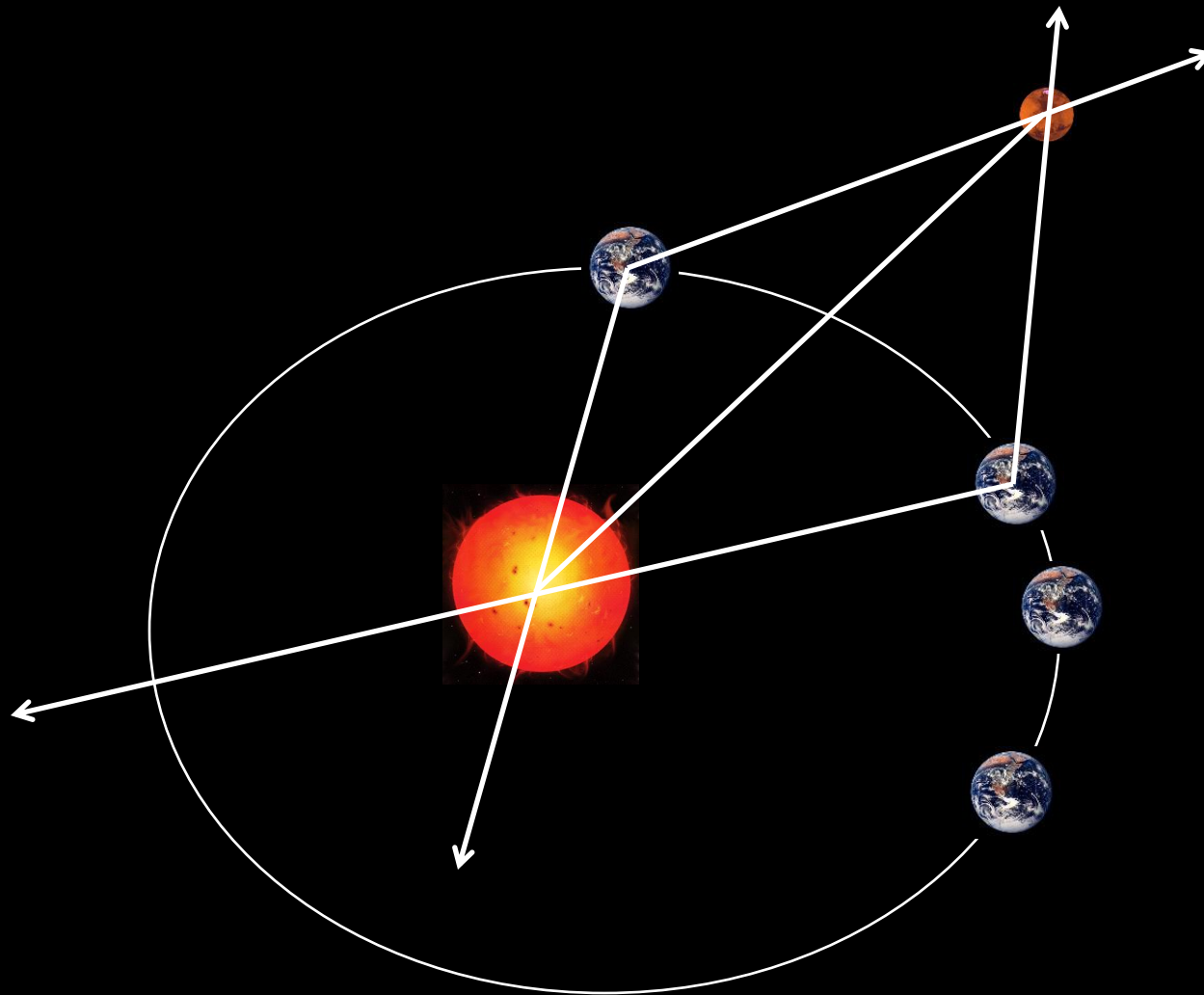
To explain how this works, let's first suppose that Mars is fixed, rather than orbiting the Sun.



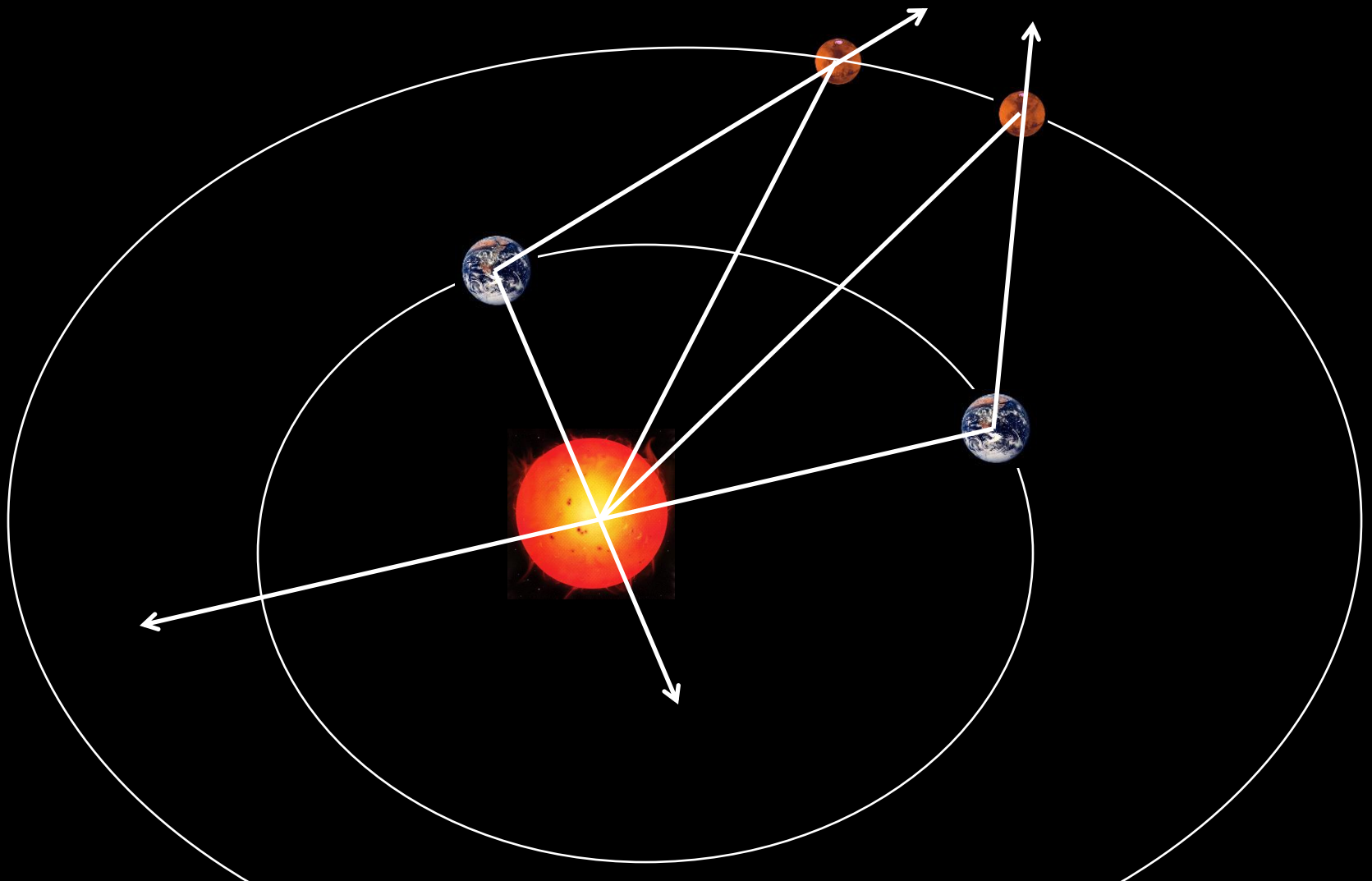
But the Earth is moving in an
unknown orbit.



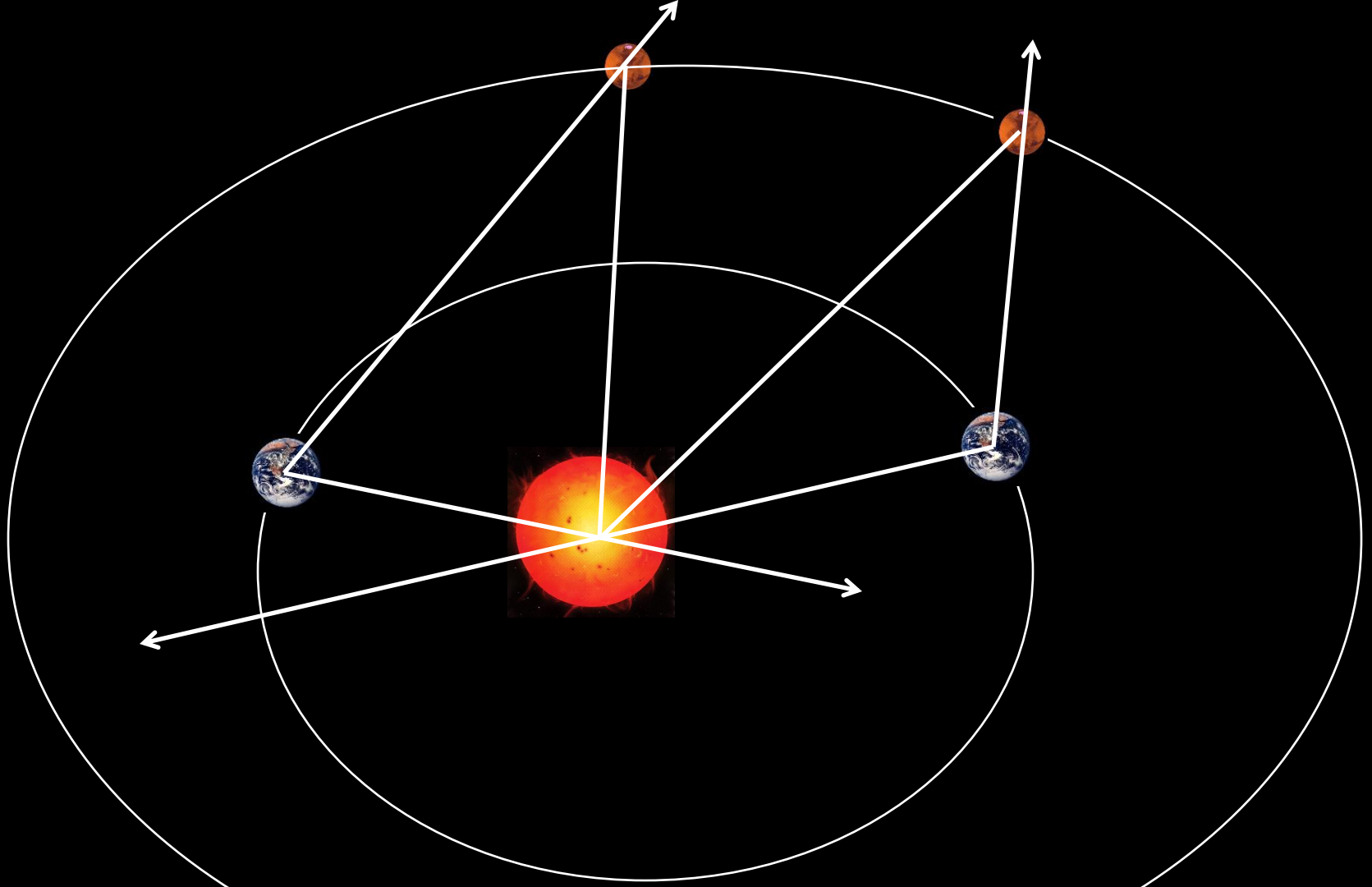
At any given time, one can measure the position of the Sun and Mars from Earth, with respect to the fixed stars (the Zodiac).



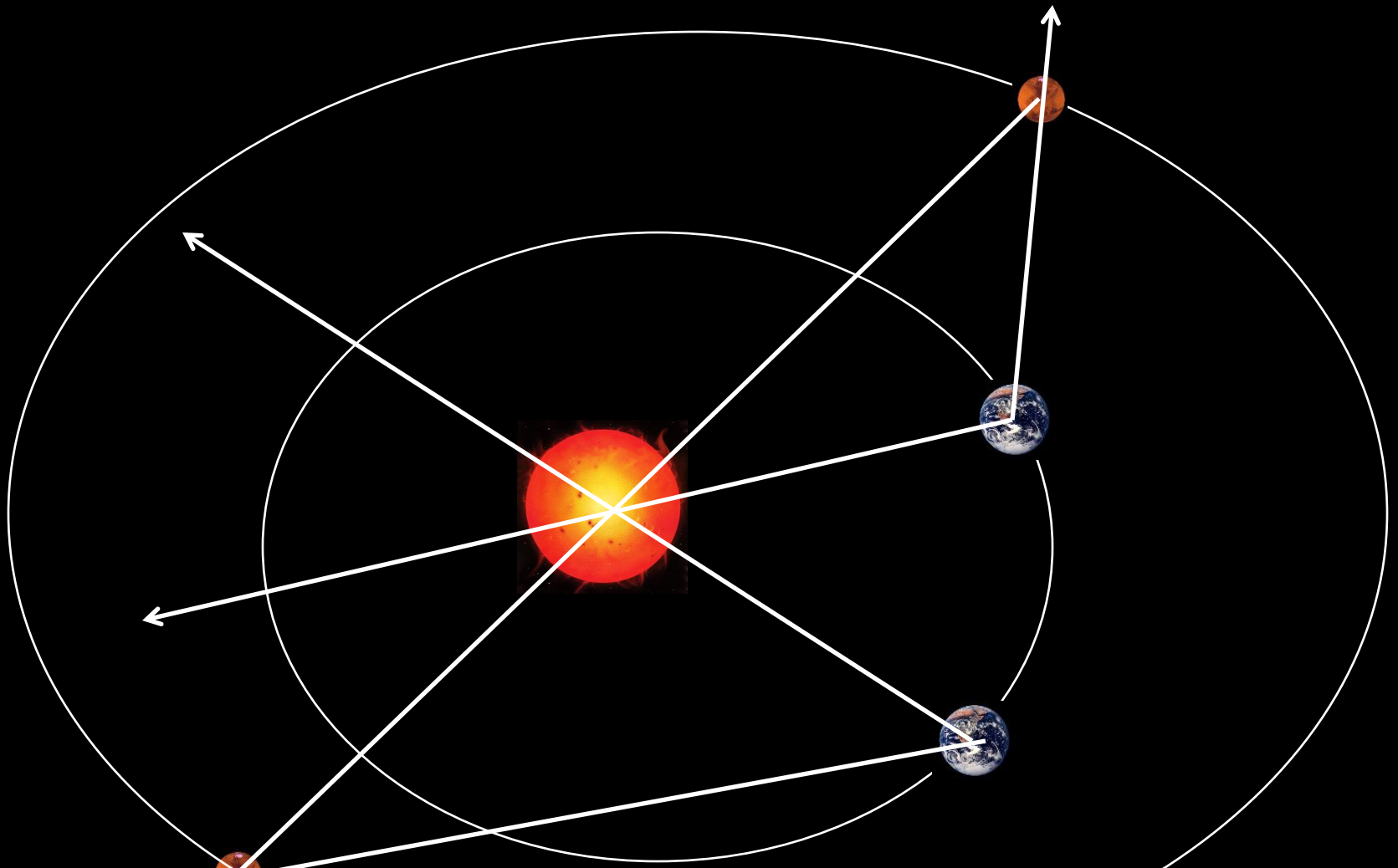
Assuming that the Sun and Mars are fixed, one can then **triangulate** to determine the position of the Earth relative to the Sun and Mars.



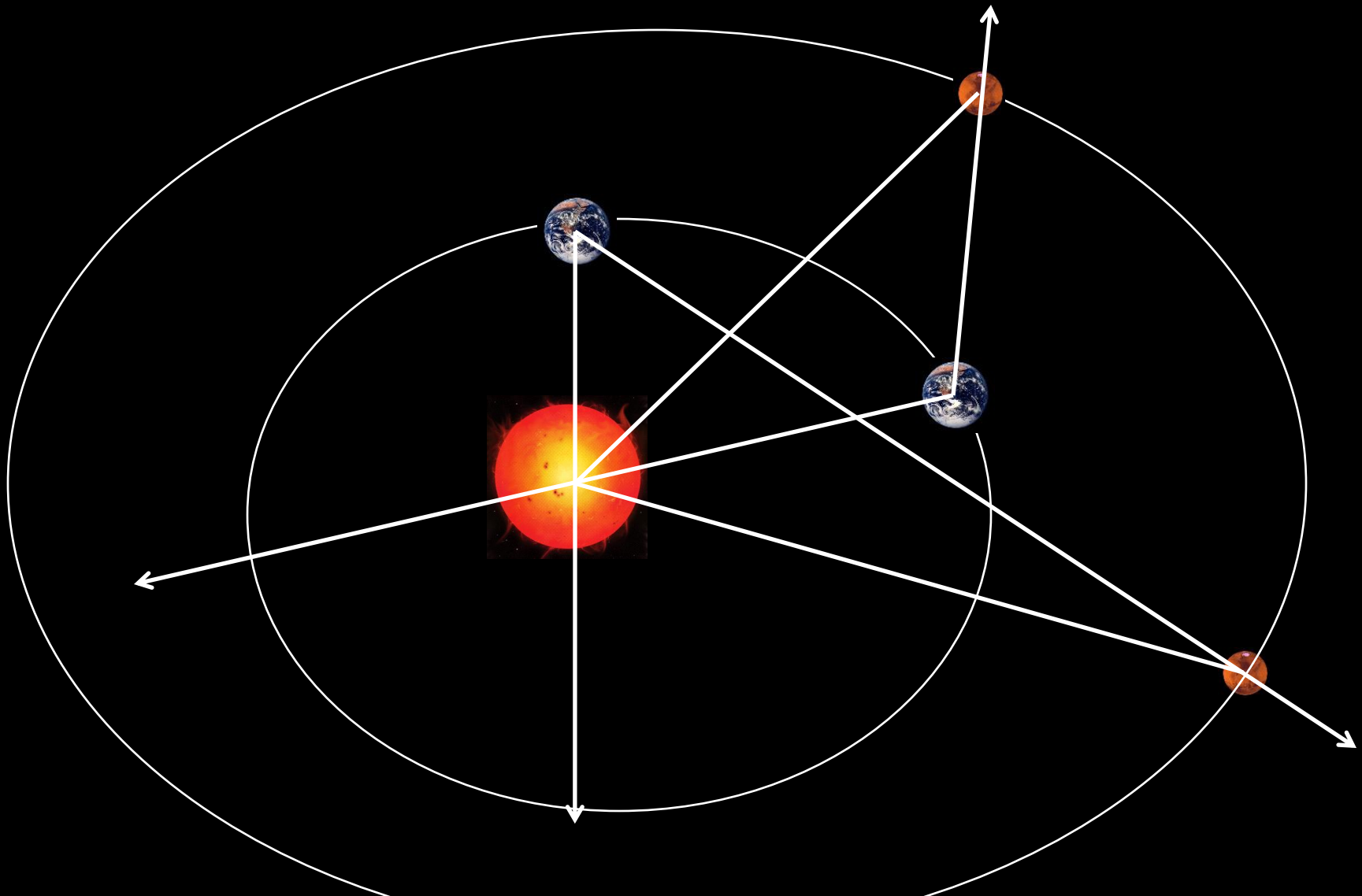
Unfortunately, Mars is not fixed;
it also moves, and along an
unknown orbit.



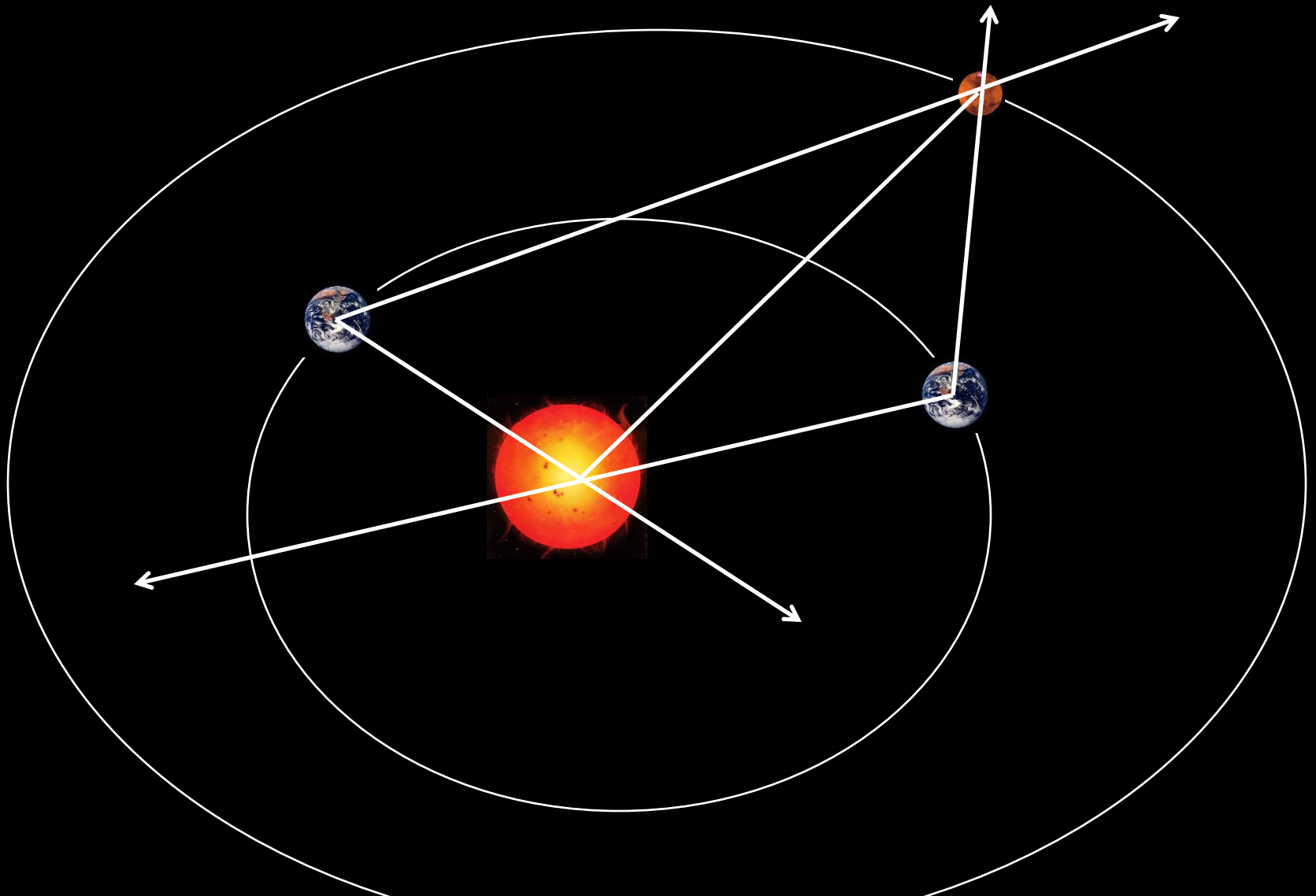
So it appears that
triangulation does not
work.



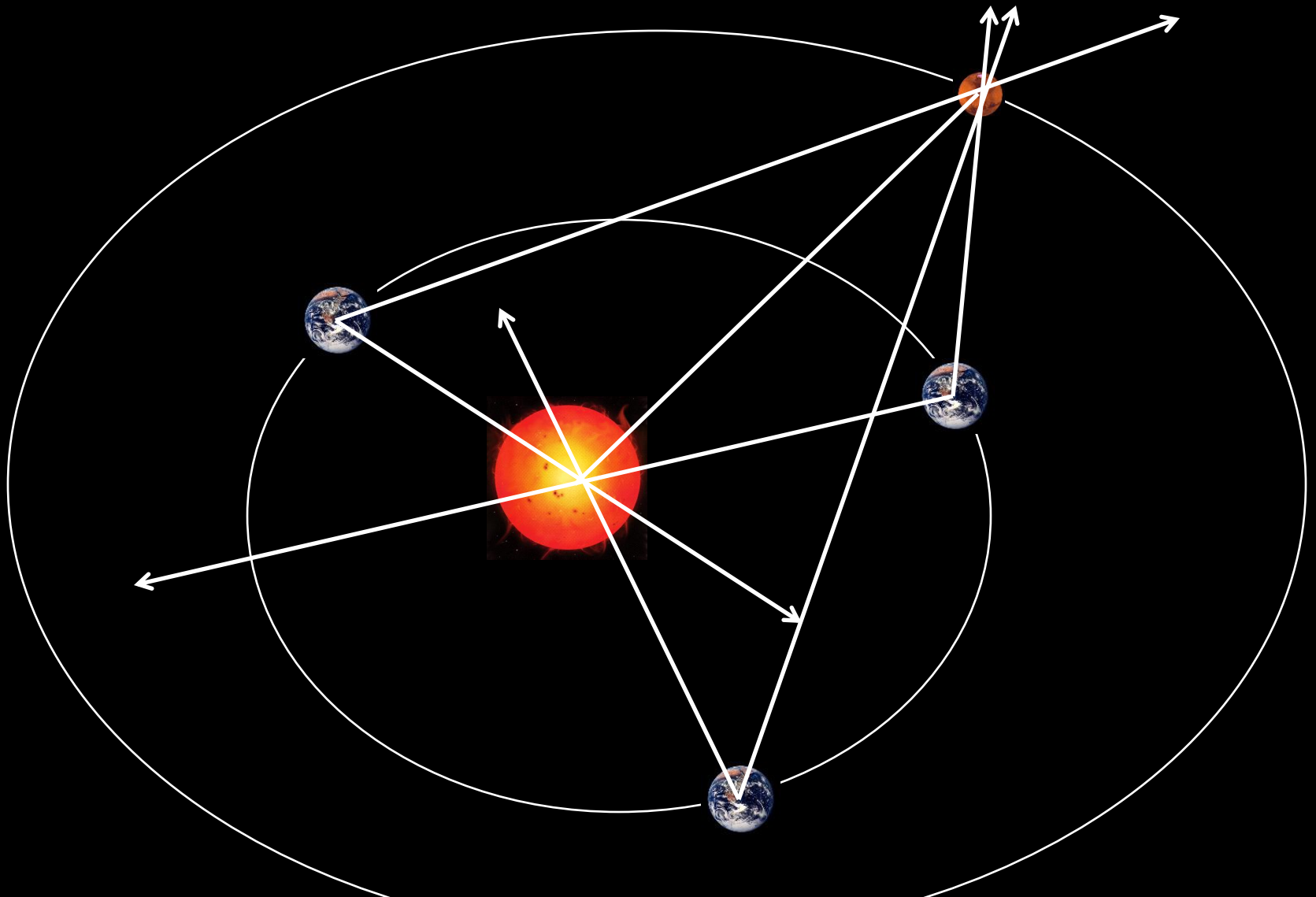
But Kepler had one additional piece of information:



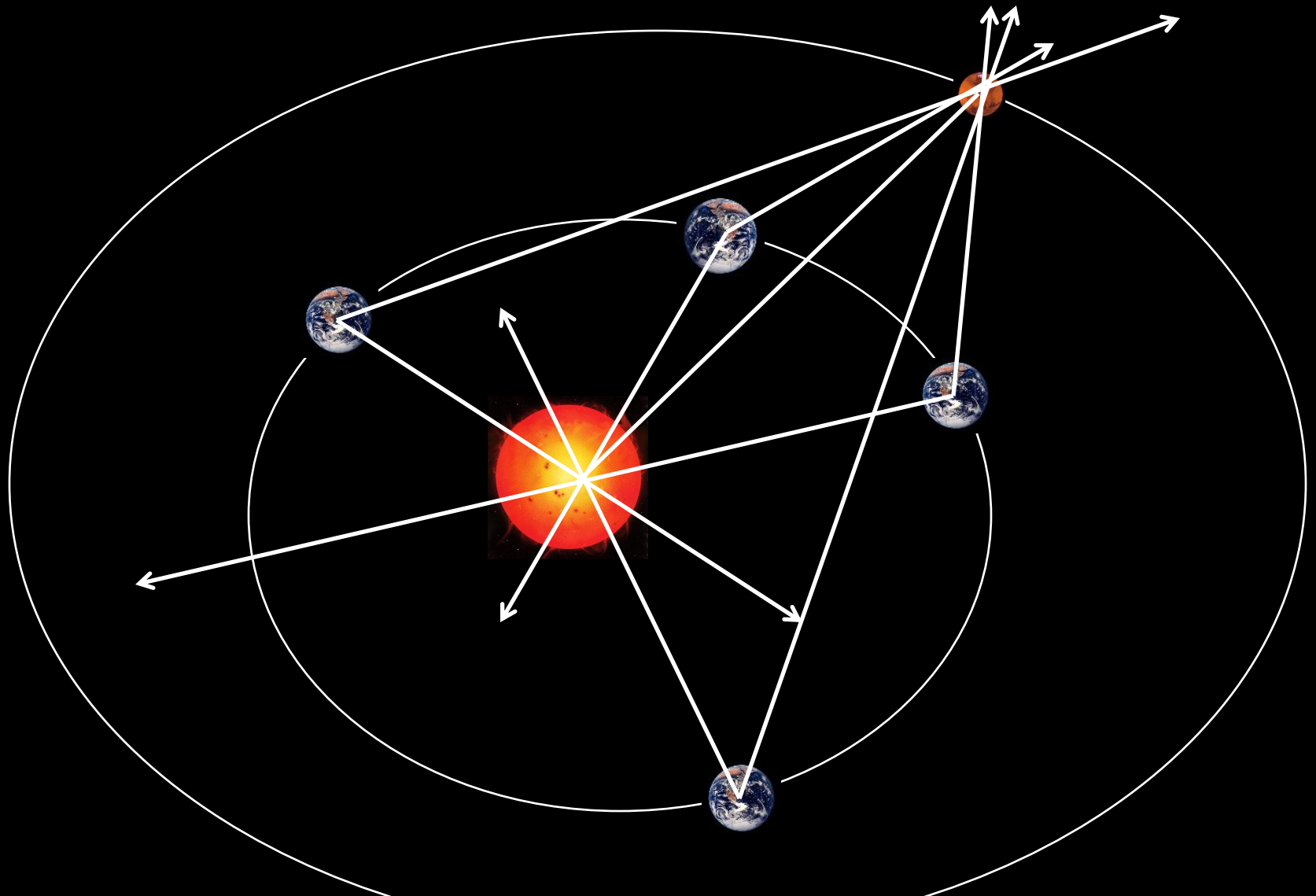
he knew that after every
687 days...



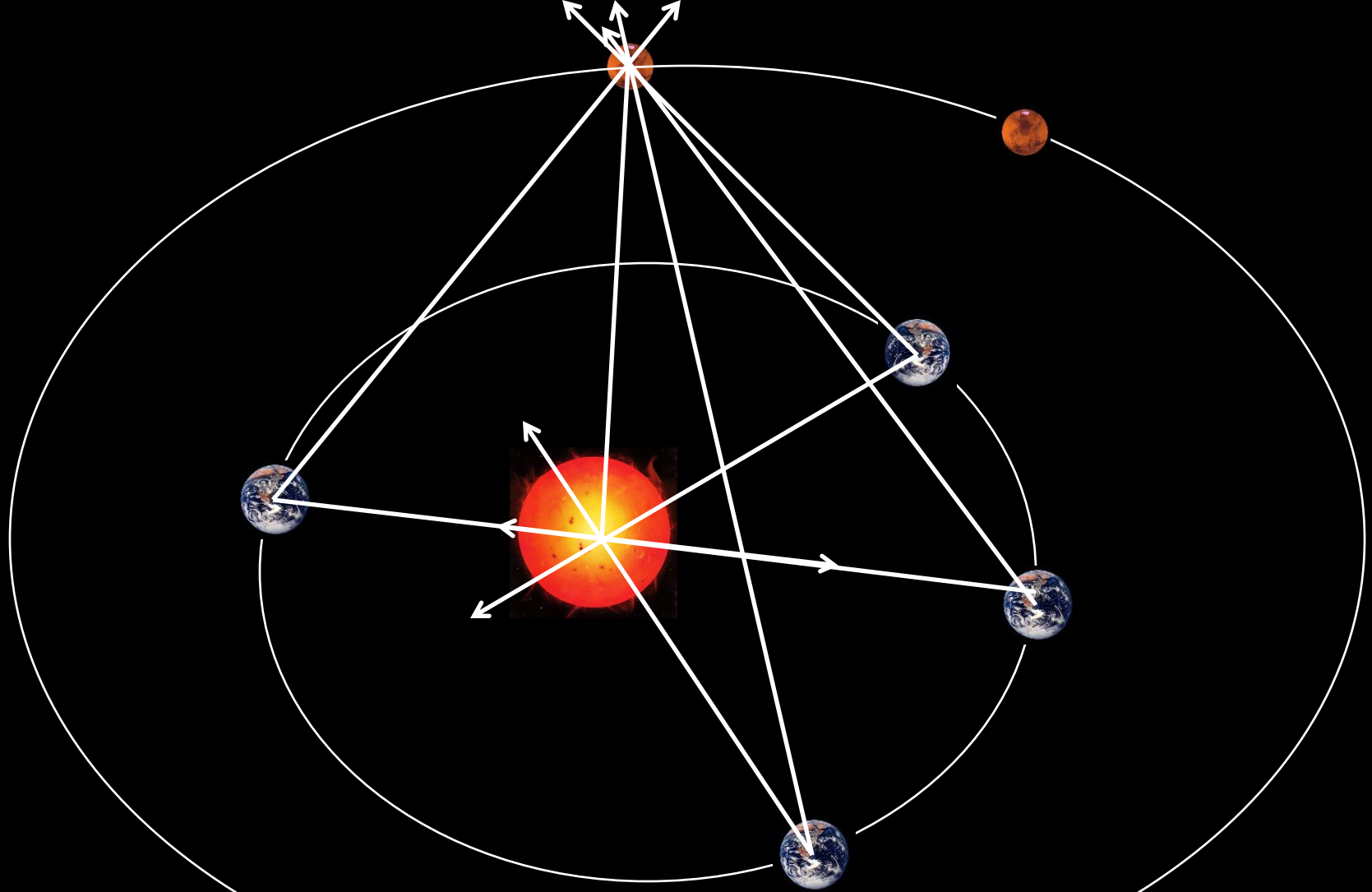
Mars returned to its original position.



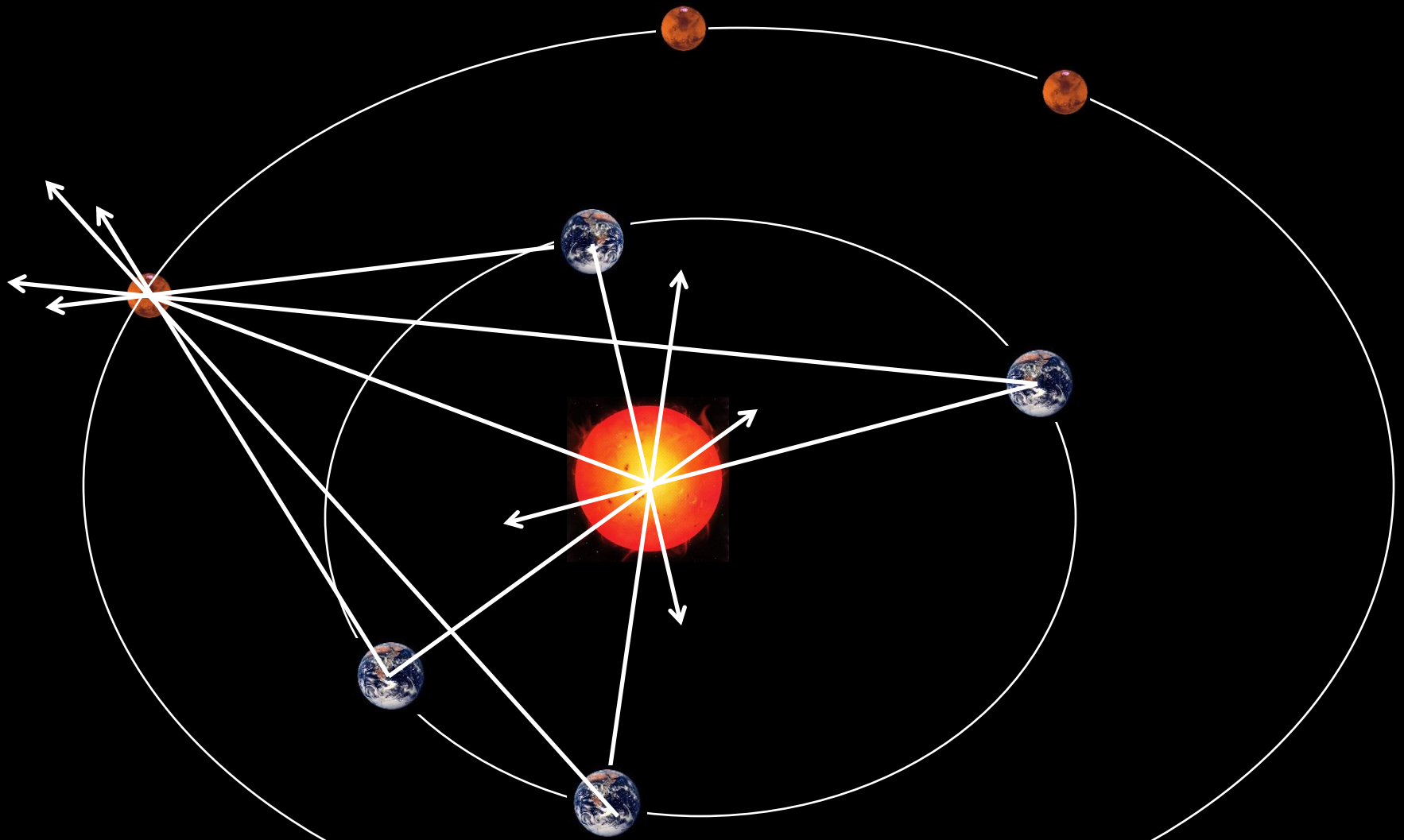
So by taking Brahe's data at intervals of 687 days...



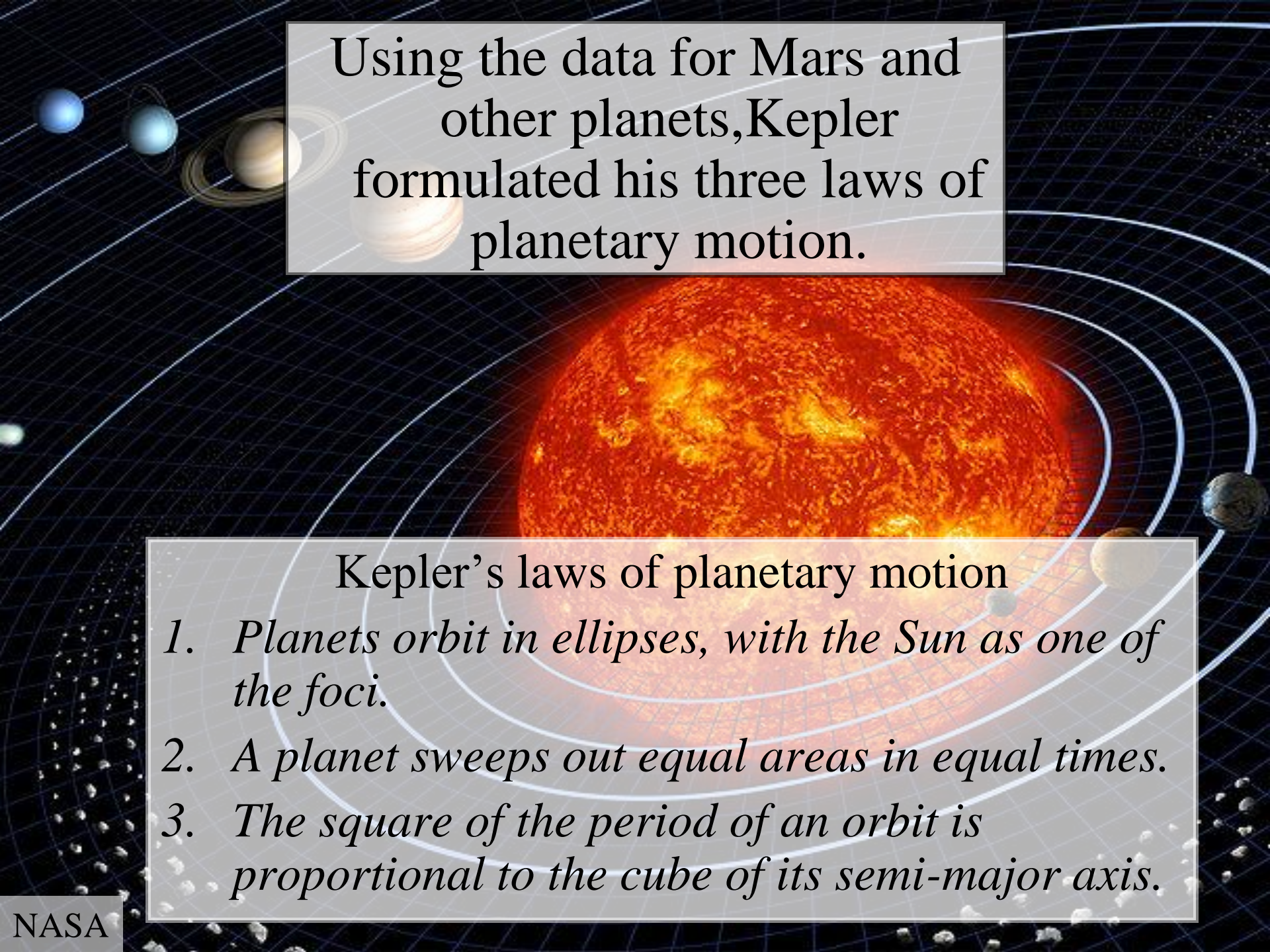
... Kepler could triangulate and compute Earth's orbit relative to any position of Mars.



Once Earth's orbit was known, it could be used to compute more positions of Mars by taking other sequences of data separated by 687 days...



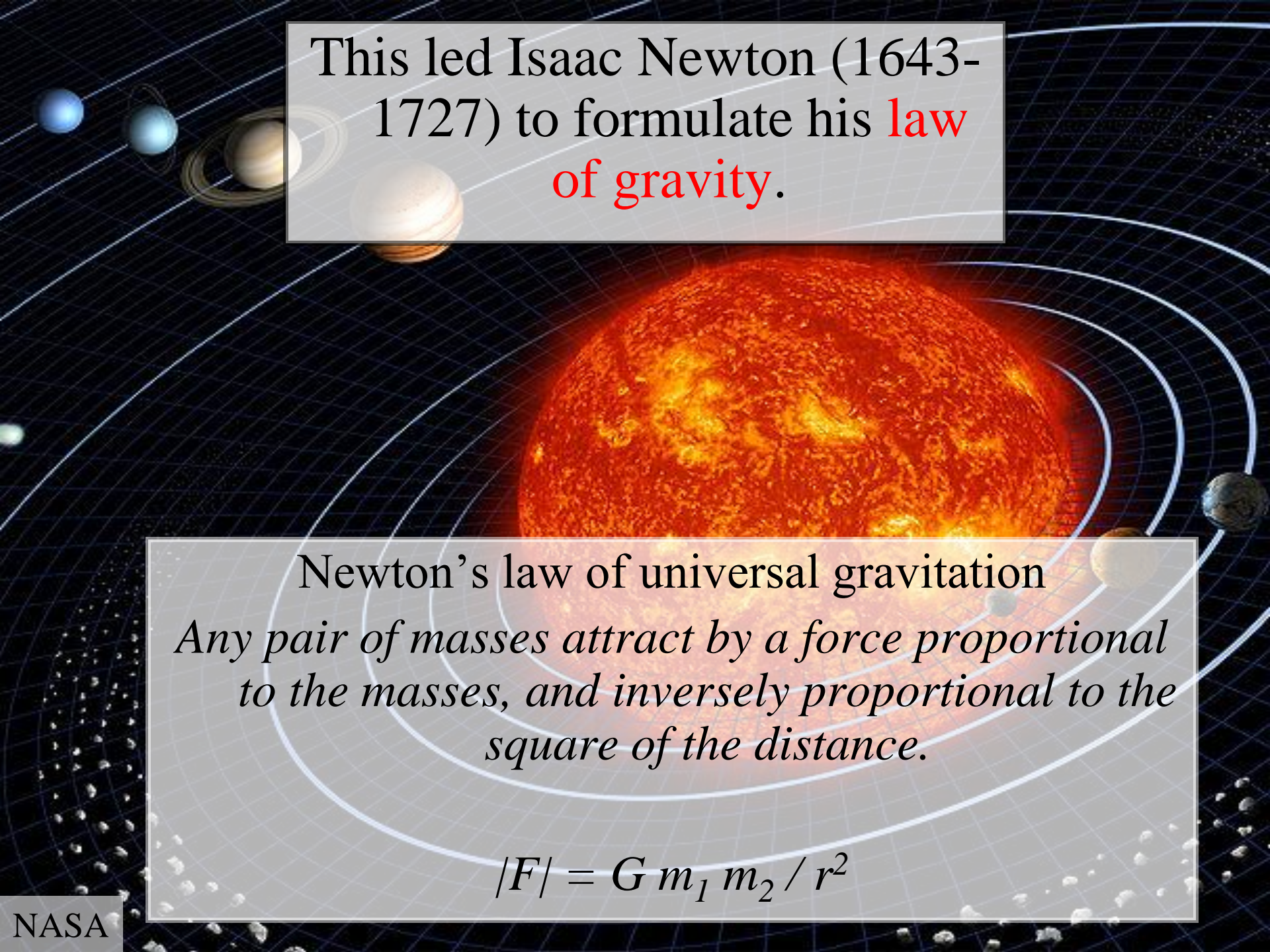
... which allows one to
compute the orbit of Mars.



Using the data for Mars and other planets, Kepler formulated his three laws of planetary motion.

Kepler's laws of planetary motion

1. *Planets orbit in ellipses, with the Sun as one of the foci.*
2. *A planet sweeps out equal areas in equal times.*
3. *The square of the period of an orbit is proportional to the cube of its semi-major axis.*

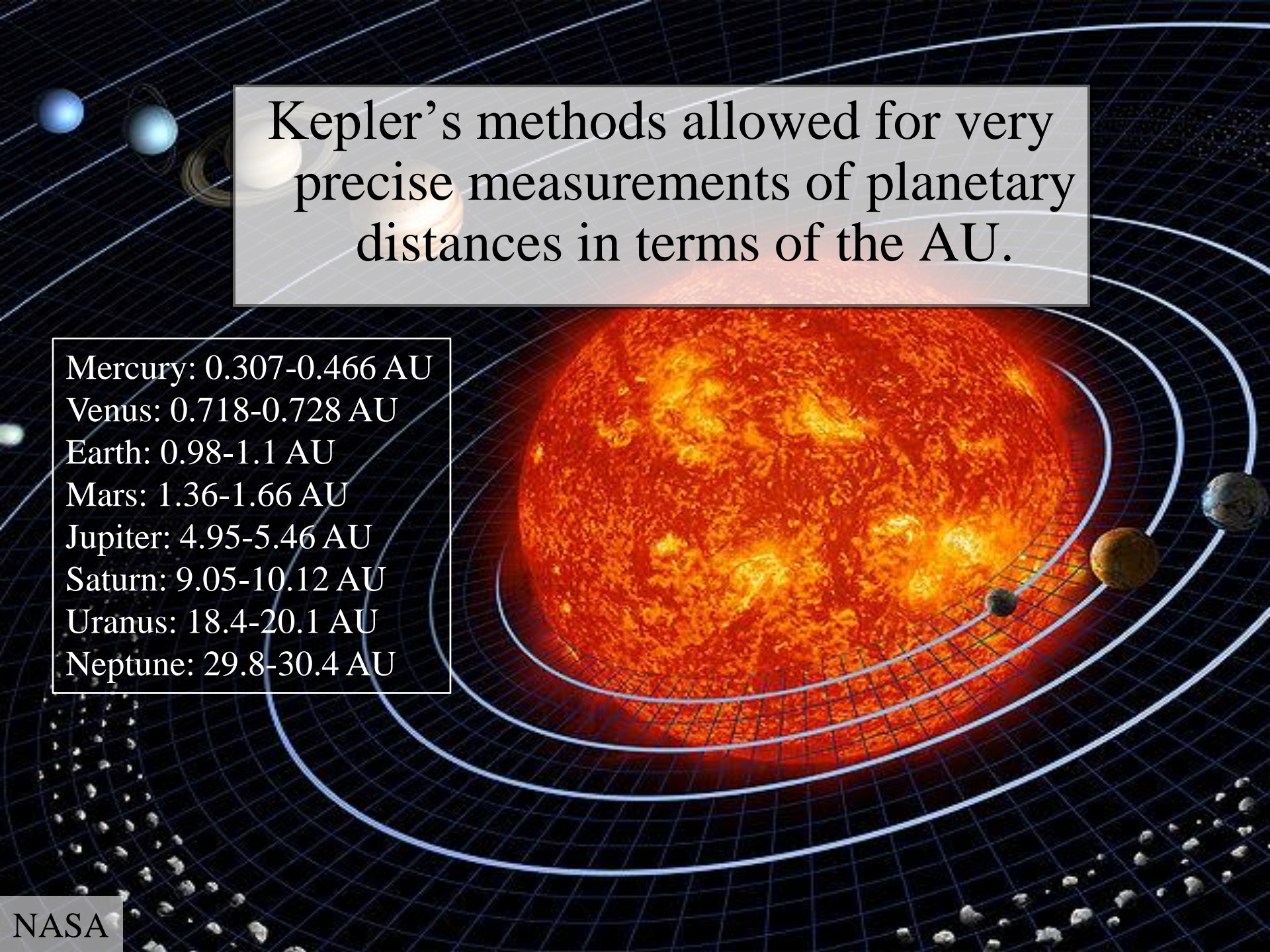


This led Isaac Newton (1643-1727) to formulate his **law of gravity**.

Newton's law of universal gravitation

Any pair of masses attract by a force proportional to the masses, and inversely proportional to the square of the distance.

$$|F| = G m_1 m_2 / r^2$$



Kepler's methods allowed for very precise measurements of planetary distances in terms of the AU.

Mercury: 0.307-0.466 AU

Venus: 0.718-0.728 AU

Earth: 0.98-1.1 AU

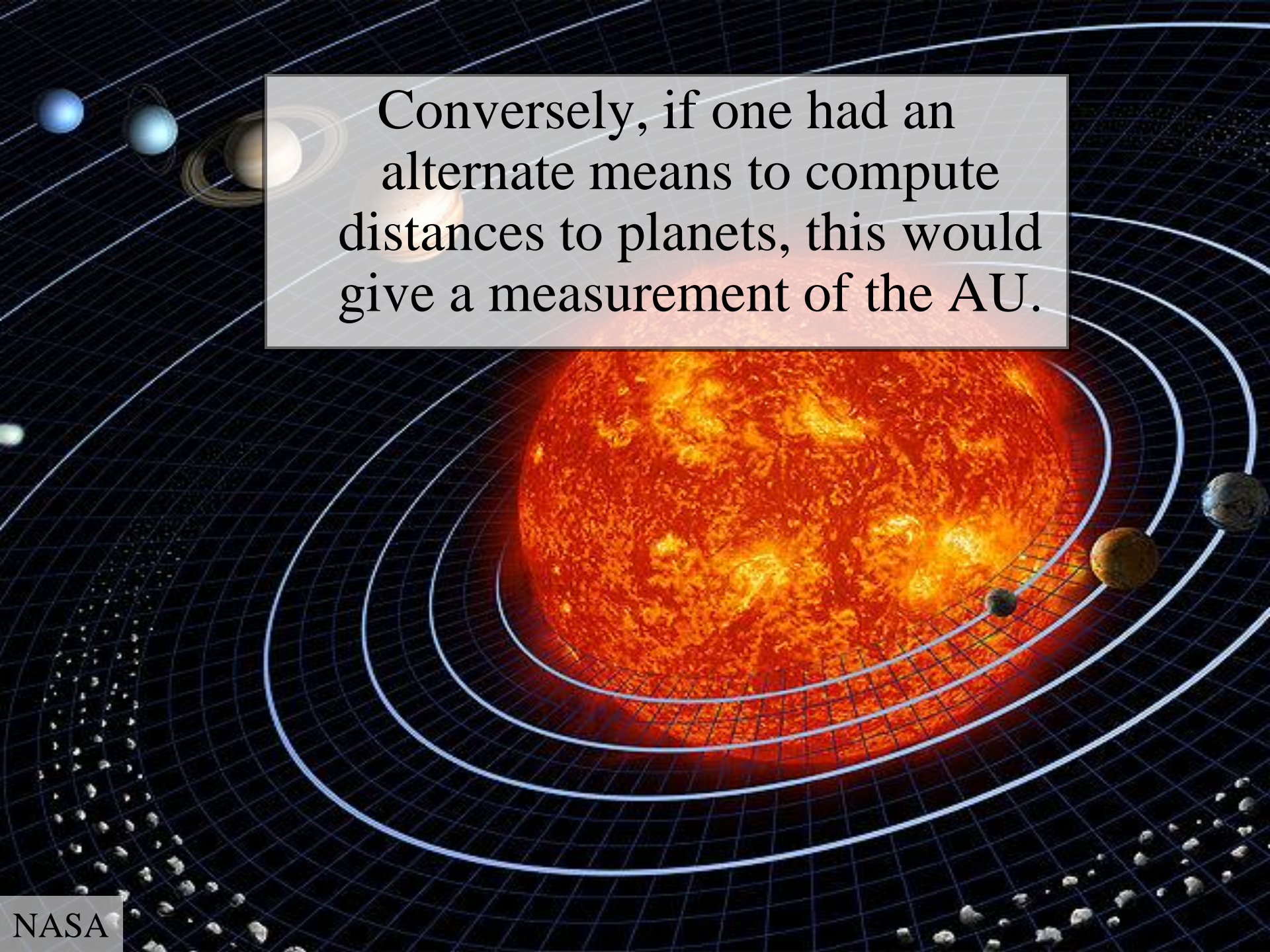
Mars: 1.36-1.66 AU

Jupiter: 4.95-5.46 AU

Saturn: 9.05-10.12 AU

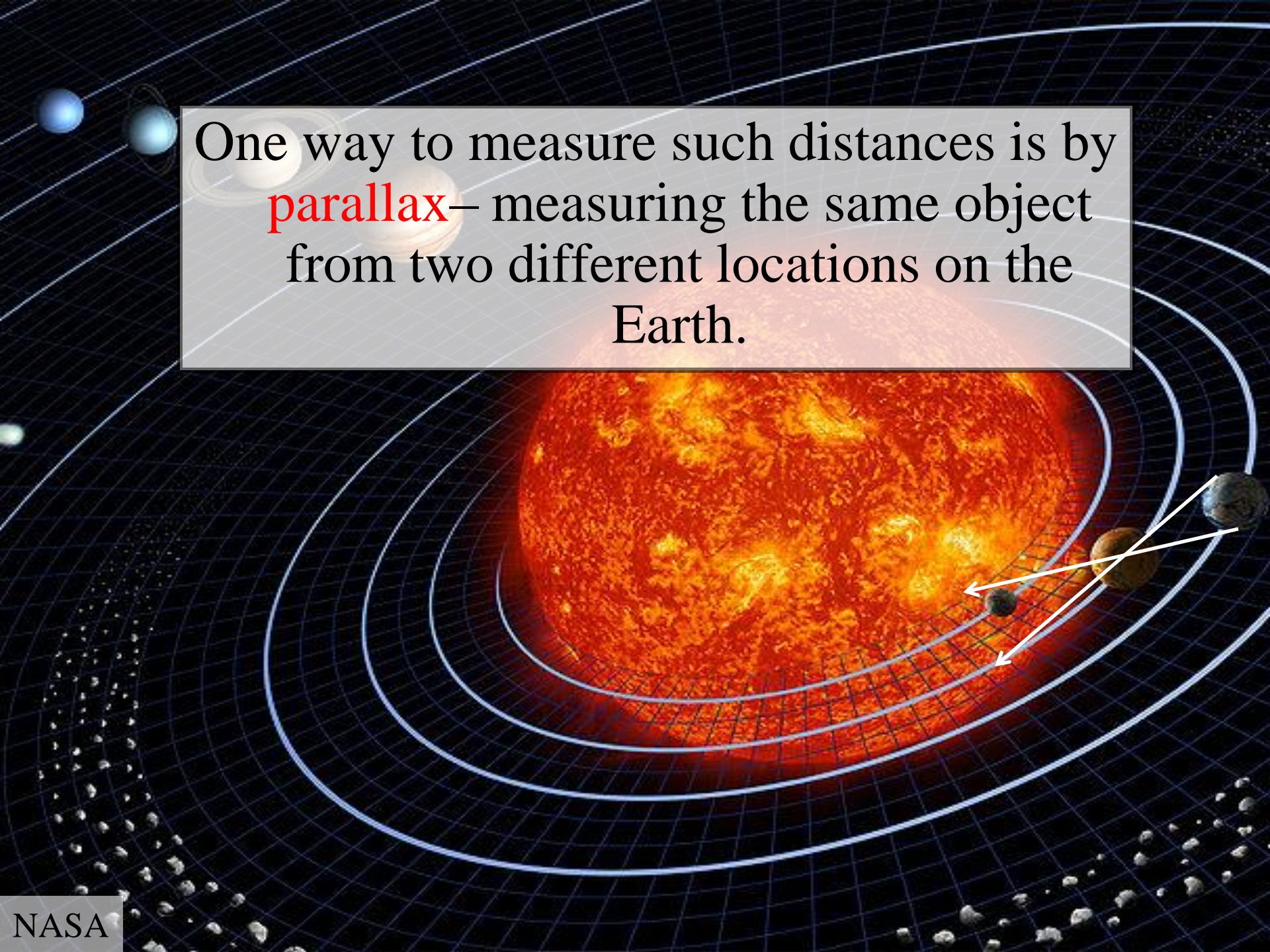
Uranus: 18.4-20.1 AU

Neptune: 29.8-30.4 AU

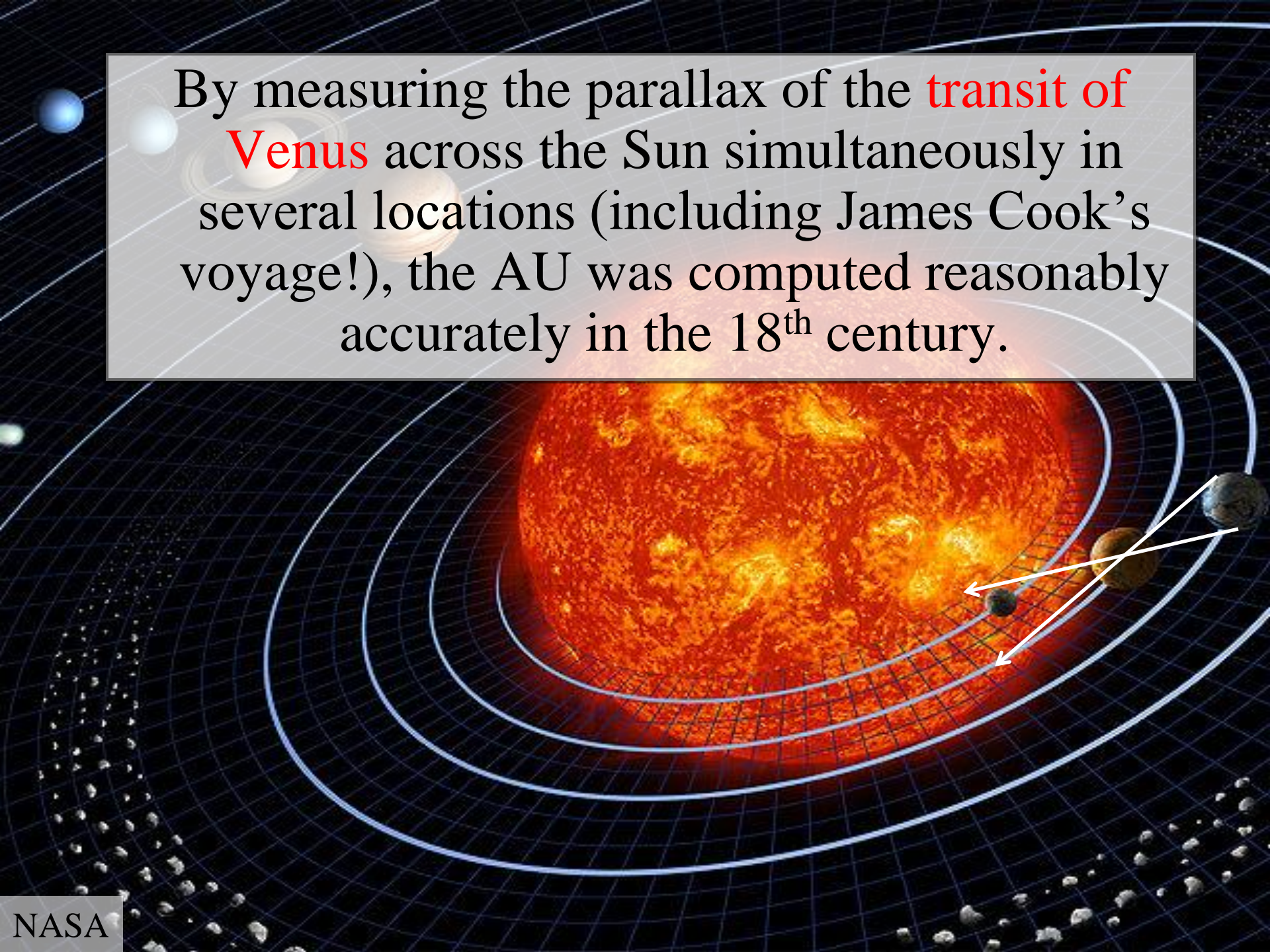


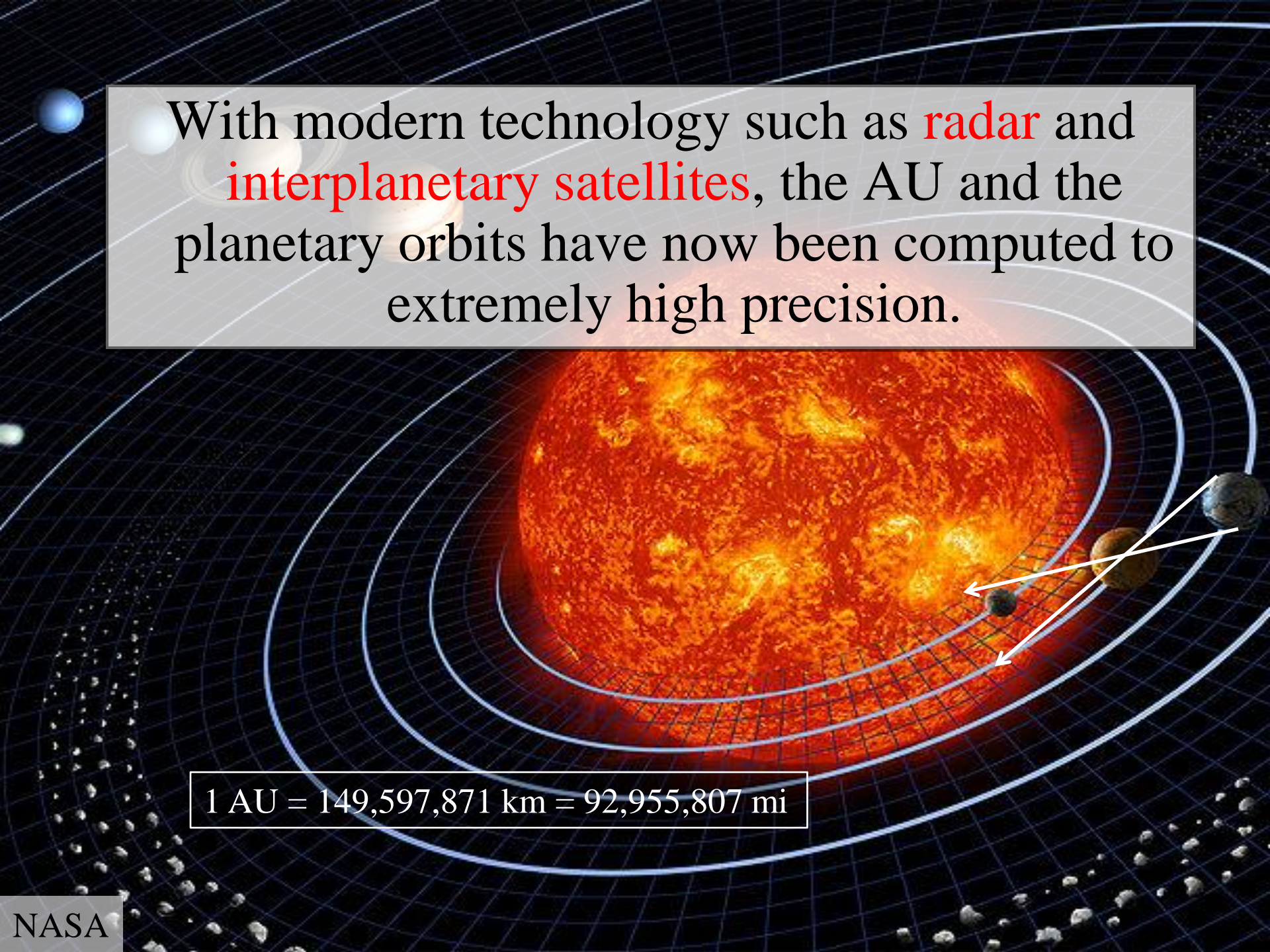
Conversely, if one had an alternate means to compute distances to planets, this would give a measurement of the AU.

One way to measure such distances is by **parallax**— measuring the same object from two different locations on the Earth.



By measuring the parallax of the **transit of Venus** across the Sun simultaneously in several locations (including James Cook's voyage!), the AU was computed reasonably accurately in the 18th century.

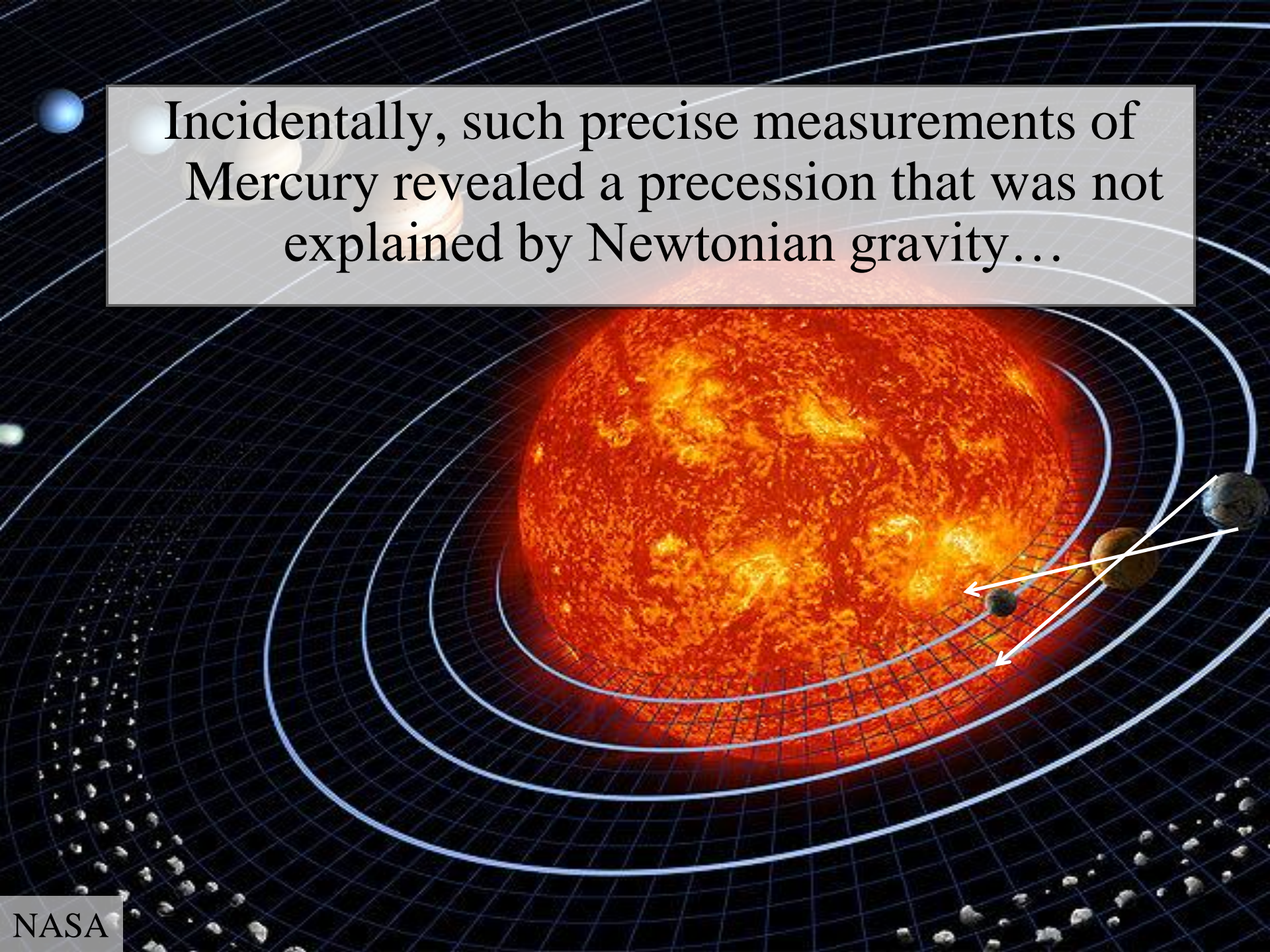


A diagram of the solar system with the Sun at the center, surrounded by concentric elliptical orbits. Planets are shown at various points on these orbits. A grid of latitude and longitude lines is overlaid on the orbits. Two white arrows point from the Earth's orbit towards the Sun, indicating the distance of 1 AU.

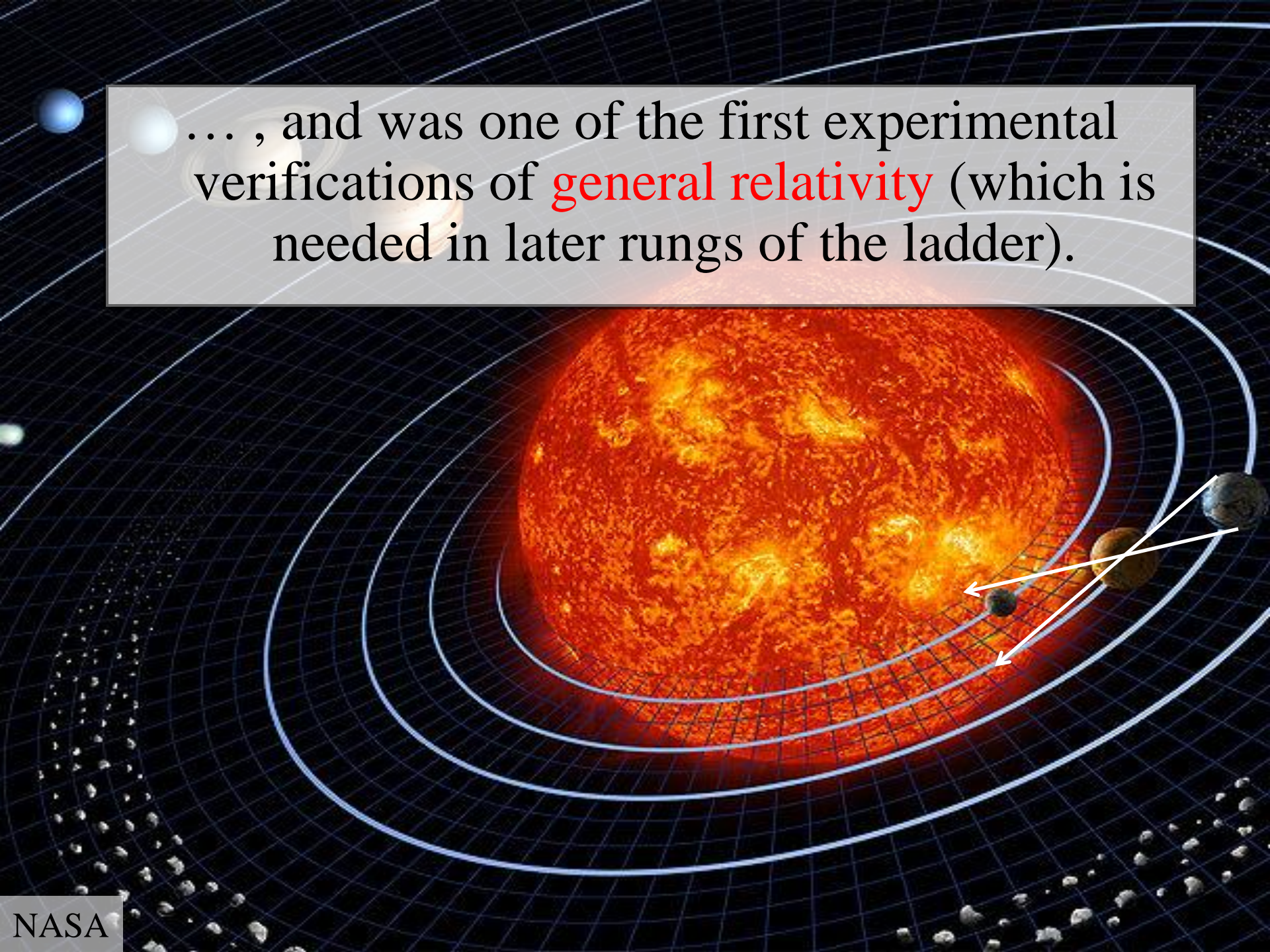
With modern technology such as **radar** and **interplanetary satellites**, the AU and the planetary orbits have now been computed to extremely high precision.

1 AU = 149,597,871 km = 92,955,807 mi

Incidentally, such precise measurements of Mercury revealed a precession that was not explained by Newtonian gravity...




... , and was one of the first experimental verifications of **general relativity** (which is needed in later rungs of the ladder).

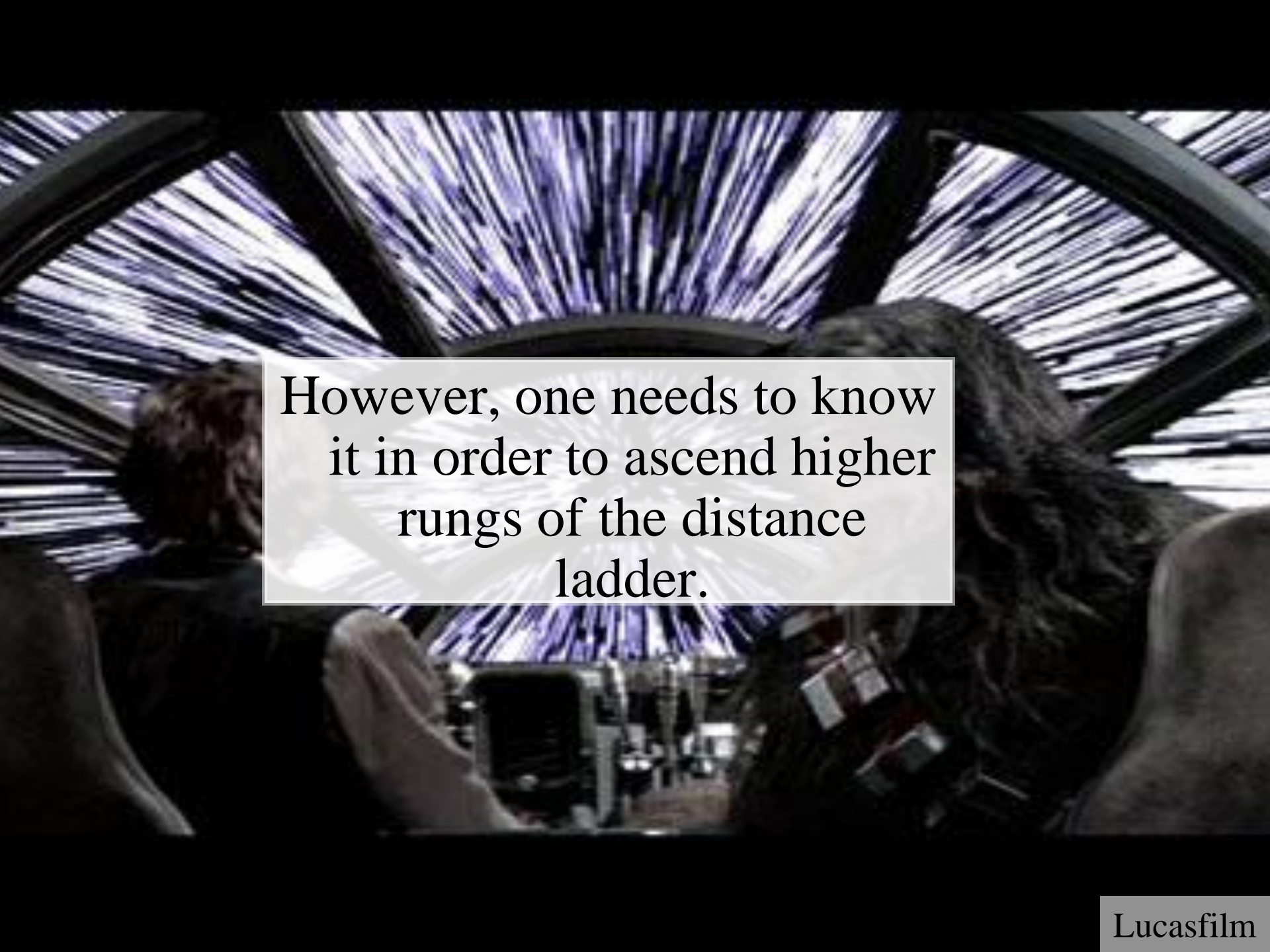




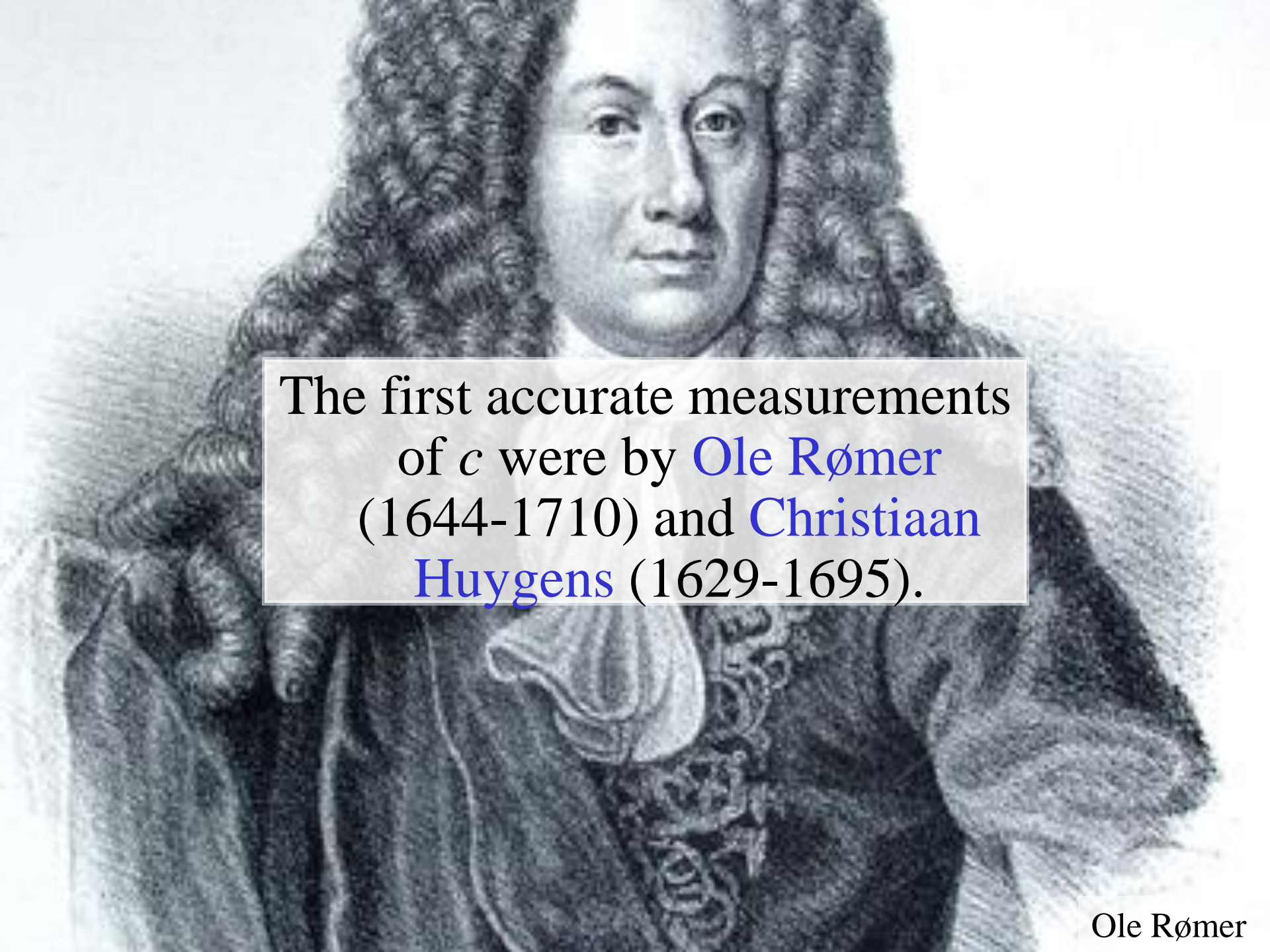
**5th rung: the speed
of light**

A cinematic shot from Star Wars showing the interior of a spaceship cockpit. The view is from the perspective of the pilot, looking out through a large, curved window. The window shows a dense field of blue and white streaks, representing the speed of light. In the foreground, the backs of two characters are visible: a man with short brown hair on the left and a woman with long, dark, curly hair on the right. A semi-transparent white text box is centered over the image, containing the text: "Technically, the speed of light, c, is not a distance." The text is in a serif font, with "speed of light" in red and "c" in black.

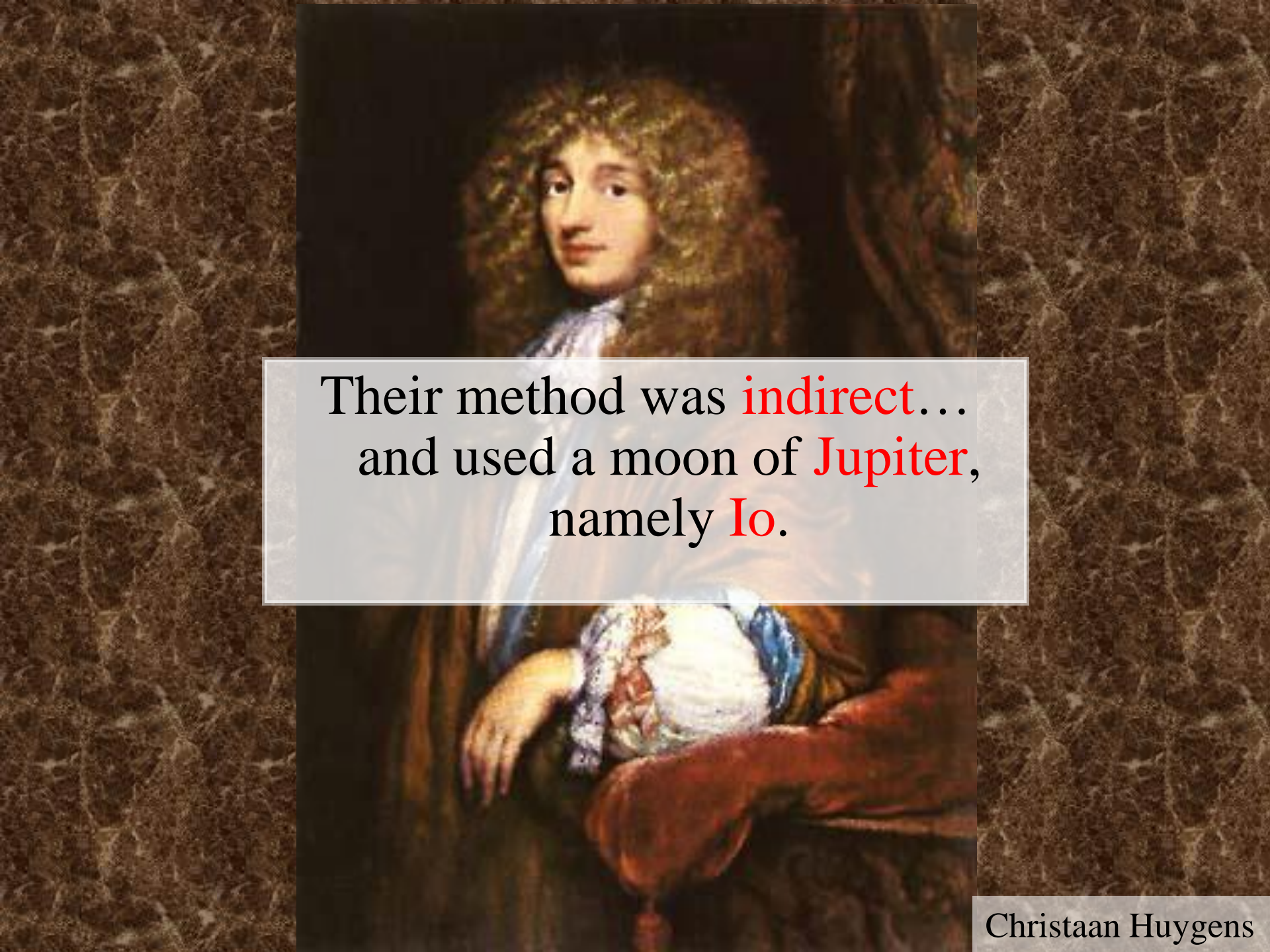
Technically, the **speed of light**, c , is not a distance.



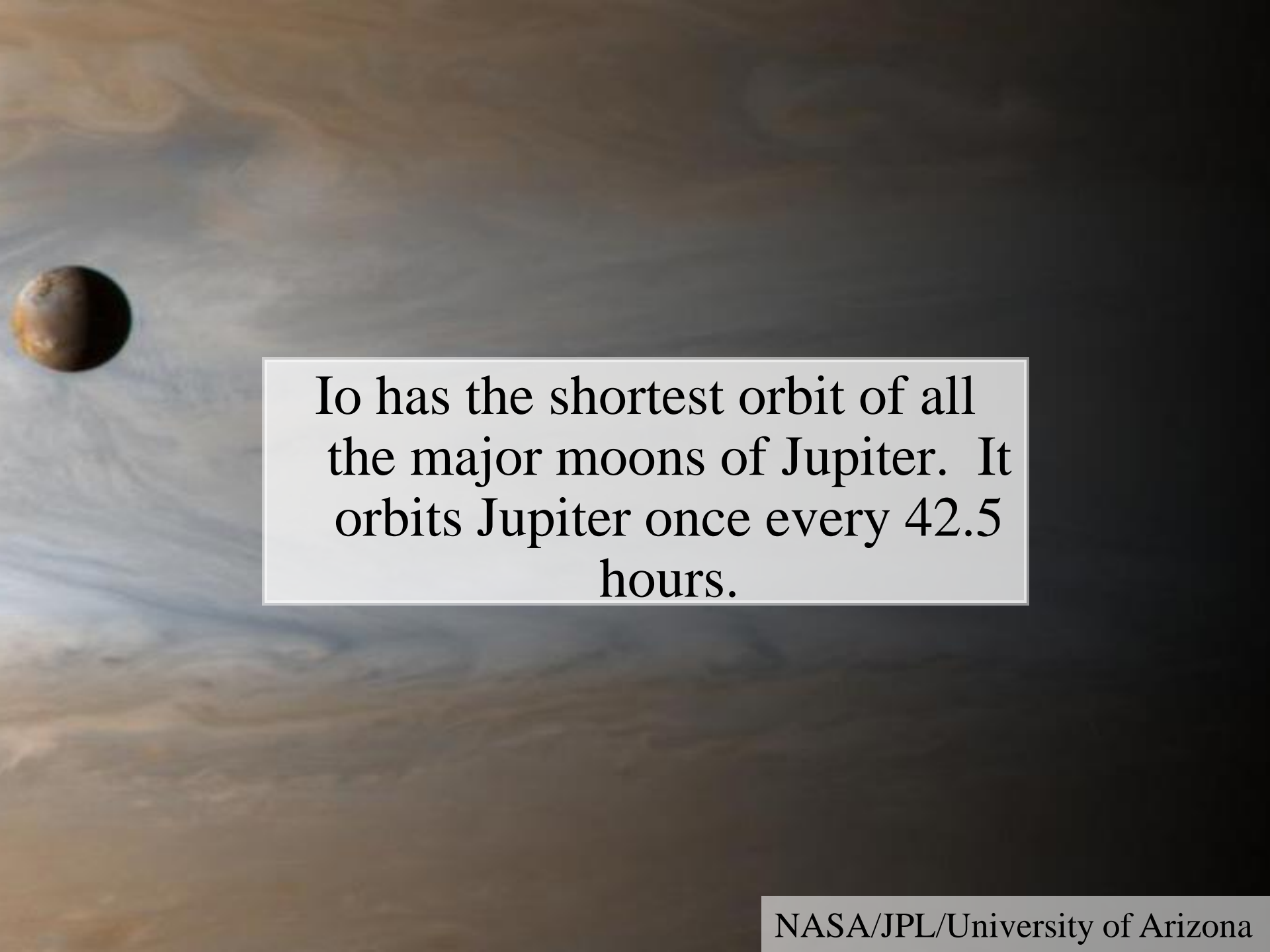
However, one needs to know
it in order to ascend higher
rungs of the distance
ladder.

A black and white engraving of Ole Rømer, a Danish astronomer. He is depicted from the chest up, wearing a large, curly wig and a dark, patterned coat with a white ruffled collar. The background is a light, textured grey.

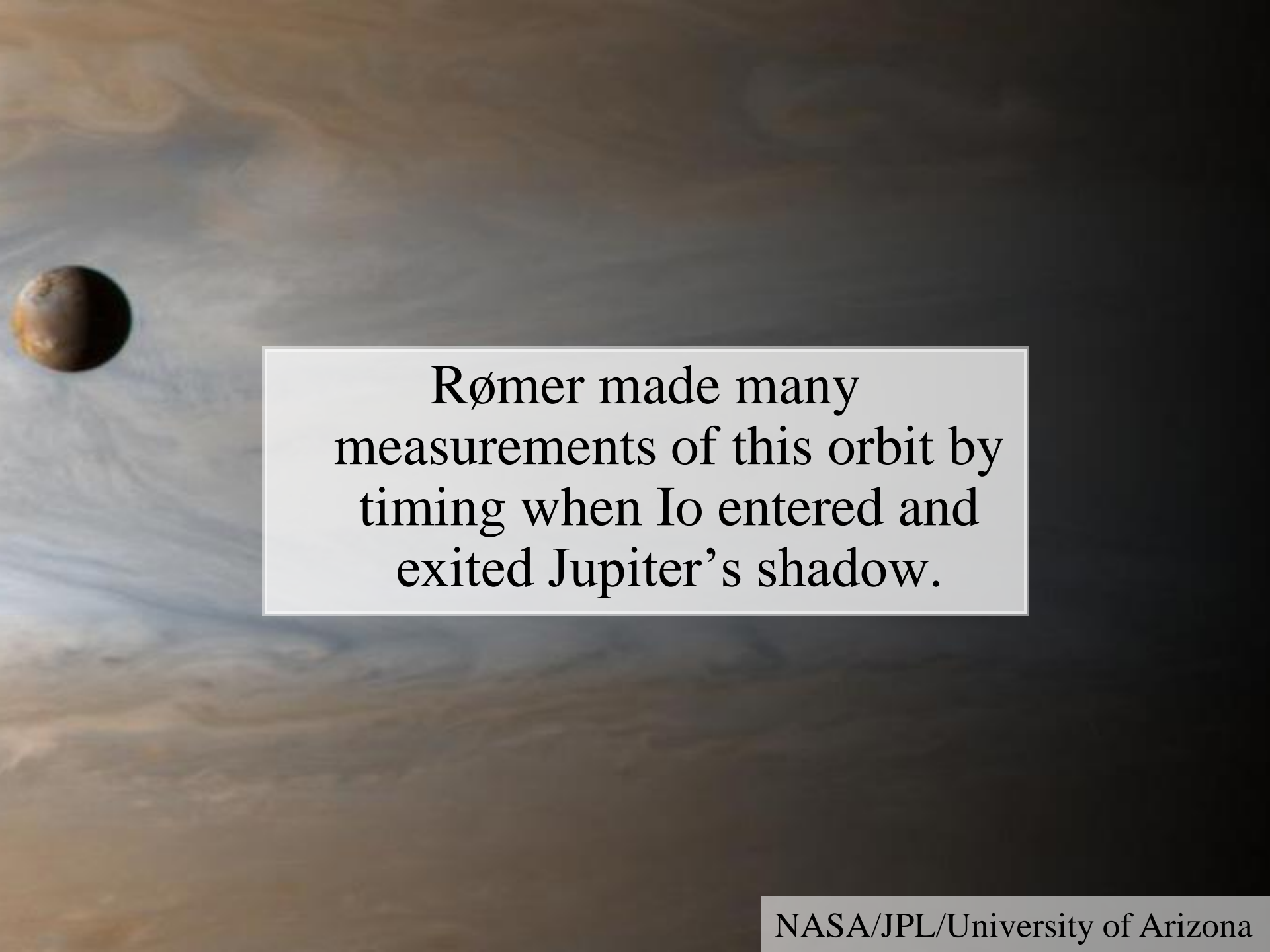
The first accurate measurements
of c were by **Ole Rømer**
(1644-1710) and **Christiaan**
Huygens (1629-1695).

A portrait of Christiaan Huygens, a Dutch astronomer, mathematician, and physicist. He is depicted from the chest up, wearing a large, curly wig and a blue and white striped shirt. He is seated and looking slightly to the left. The background is dark and textured.

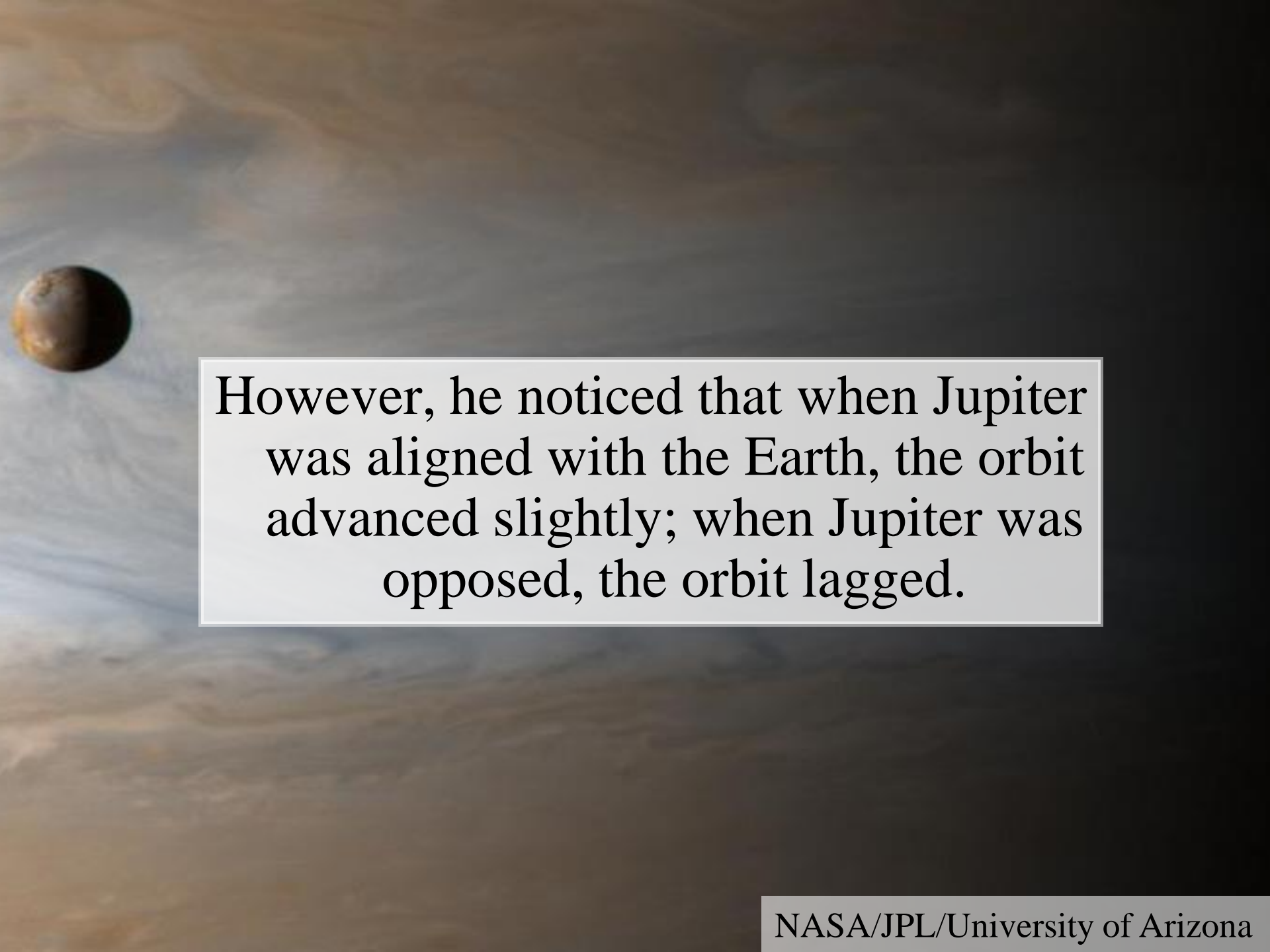
Their method was **indirect**...
and used a moon of **Jupiter**,
namely **Io**.

A photograph of the planet Jupiter with its characteristic bands of orange, brown, and white. In the upper left corner, the moon Io is visible as a small, dark, spherical object. A white rectangular box with a thin black border is centered on the right side of the image, containing text.

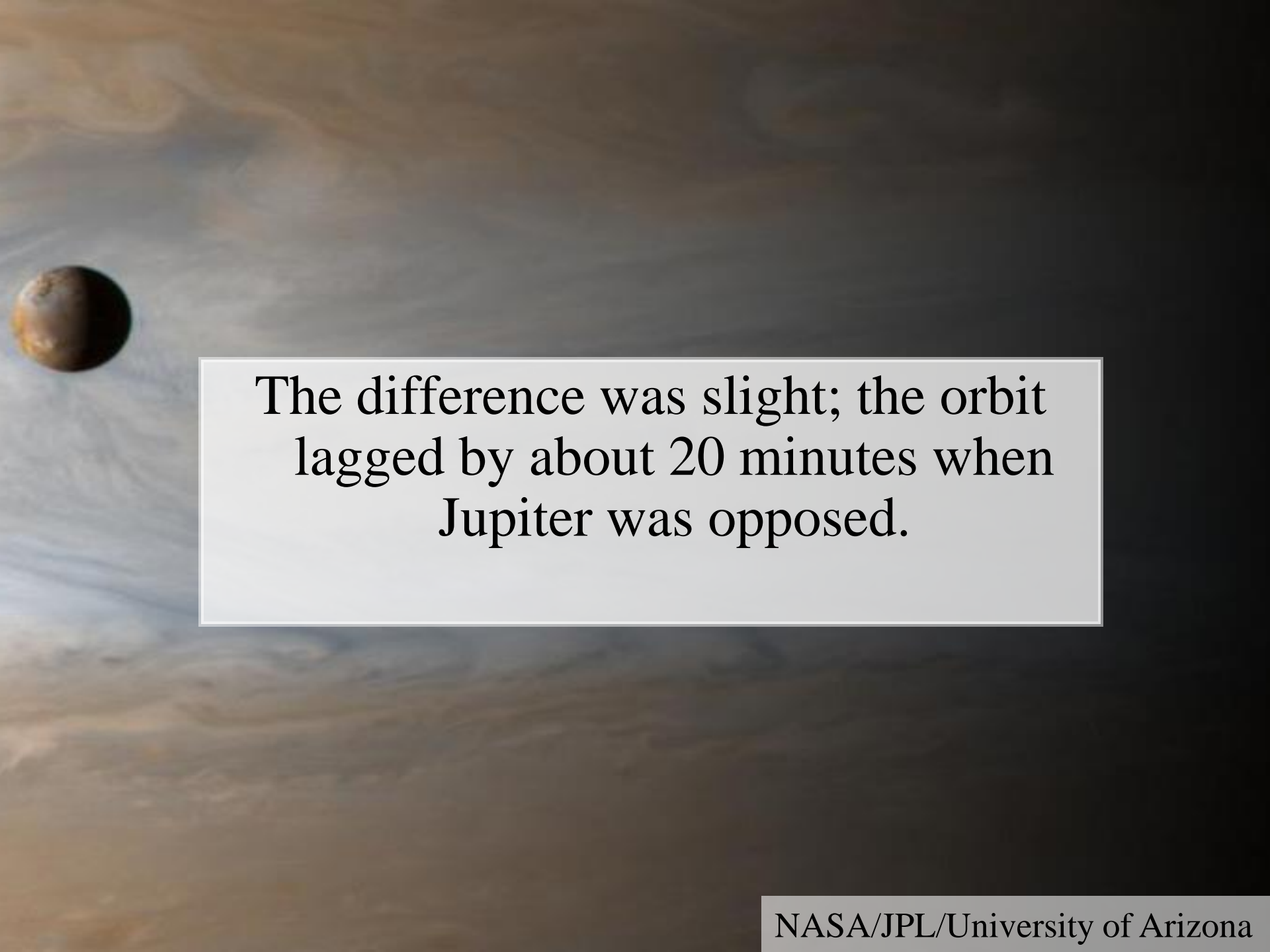
Io has the shortest orbit of all the major moons of Jupiter. It orbits Jupiter once every 42.5 hours.

A photograph of the moon Io in Jupiter's shadow. The moon is a small, reddish-brown sphere on the left side of the frame. The background is a dark, cloudy sky with a gradient from light brown to dark grey. A white rectangular box with a thin black border is centered in the lower half of the image, containing text.

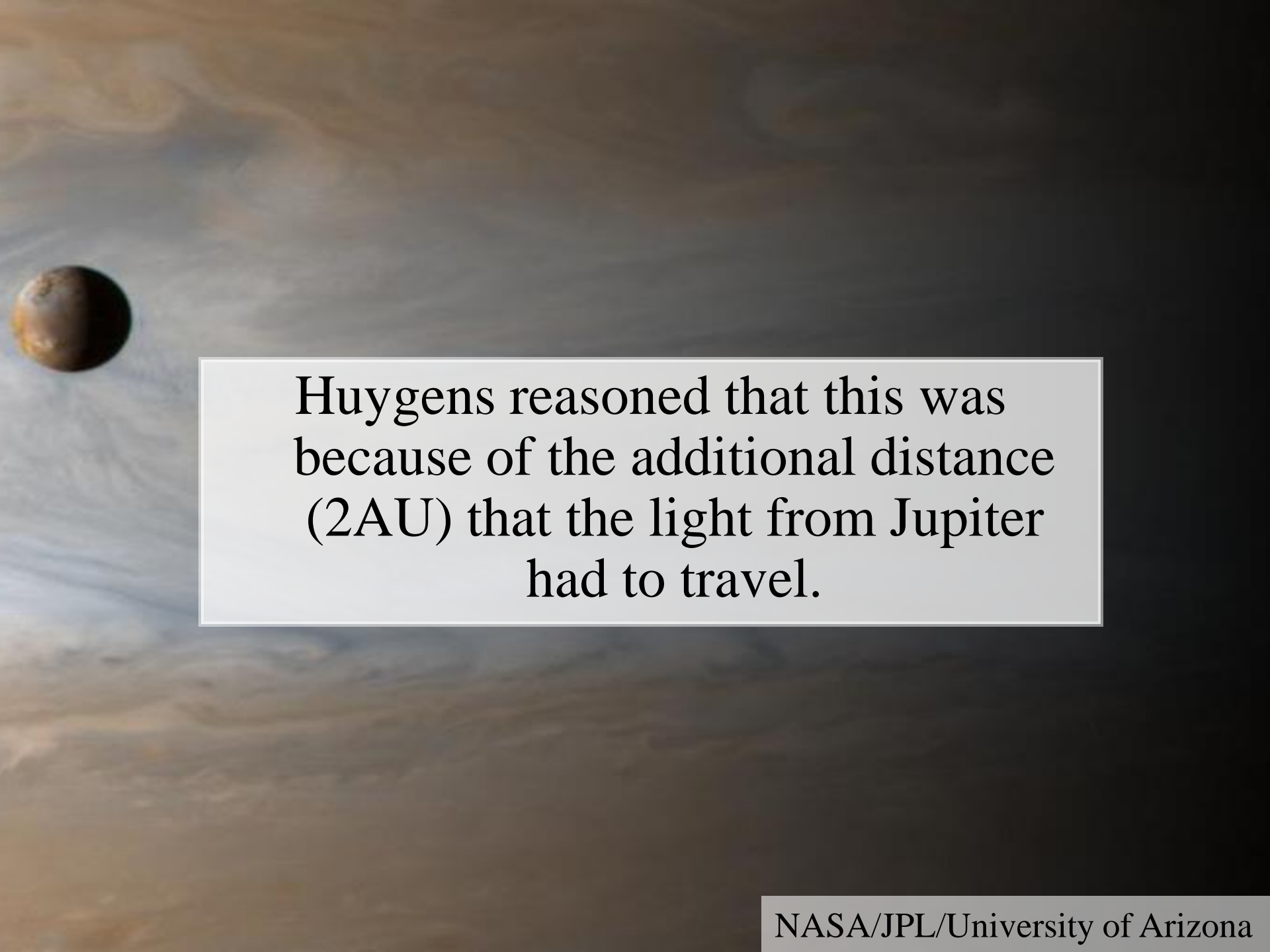
Rømer made many measurements of this orbit by timing when Io entered and exited Jupiter's shadow.



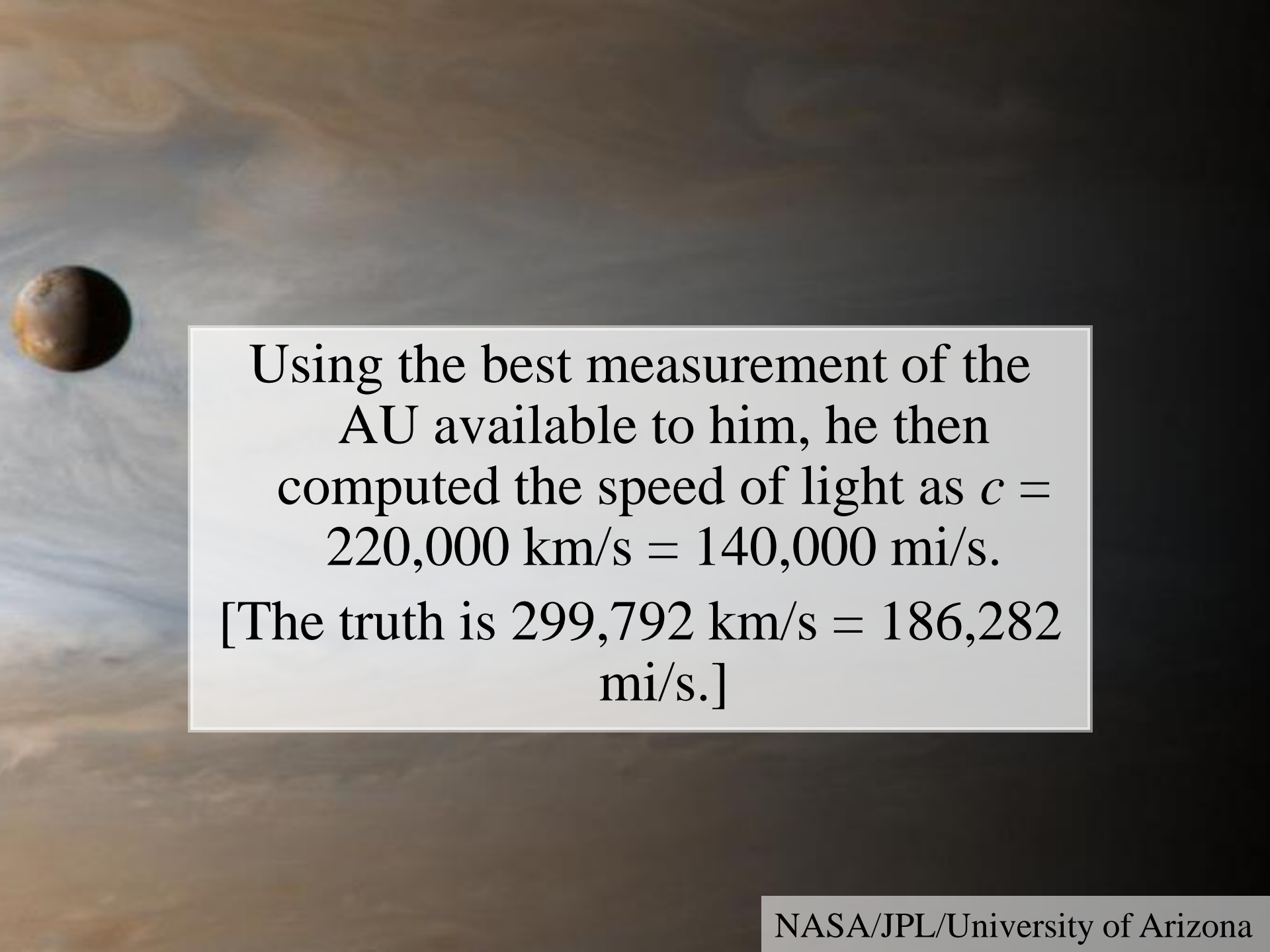
However, he noticed that when Jupiter was aligned with the Earth, the orbit advanced slightly; when Jupiter was opposed, the orbit lagged.



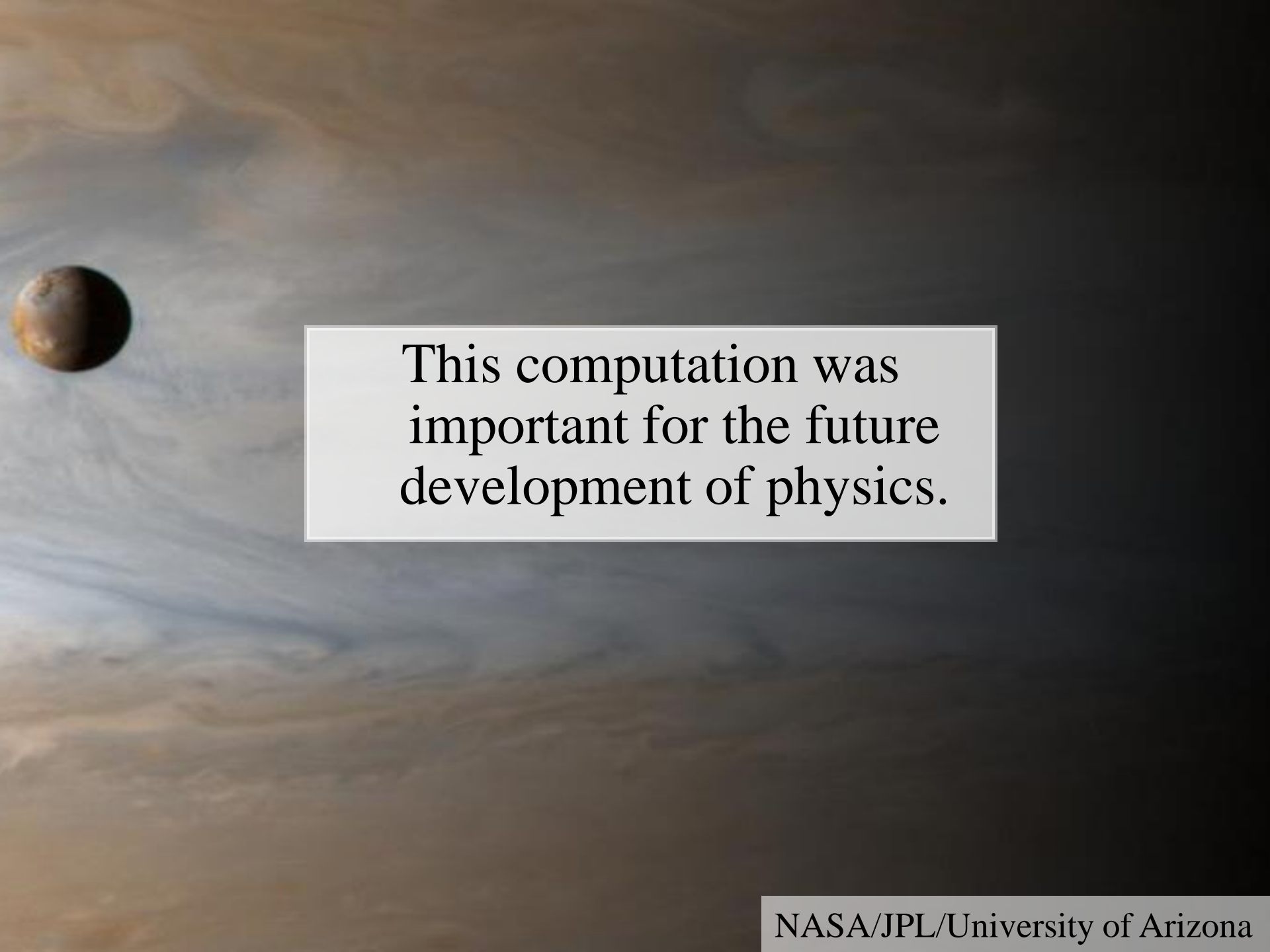
The difference was slight; the orbit lagged by about 20 minutes when Jupiter was opposed.



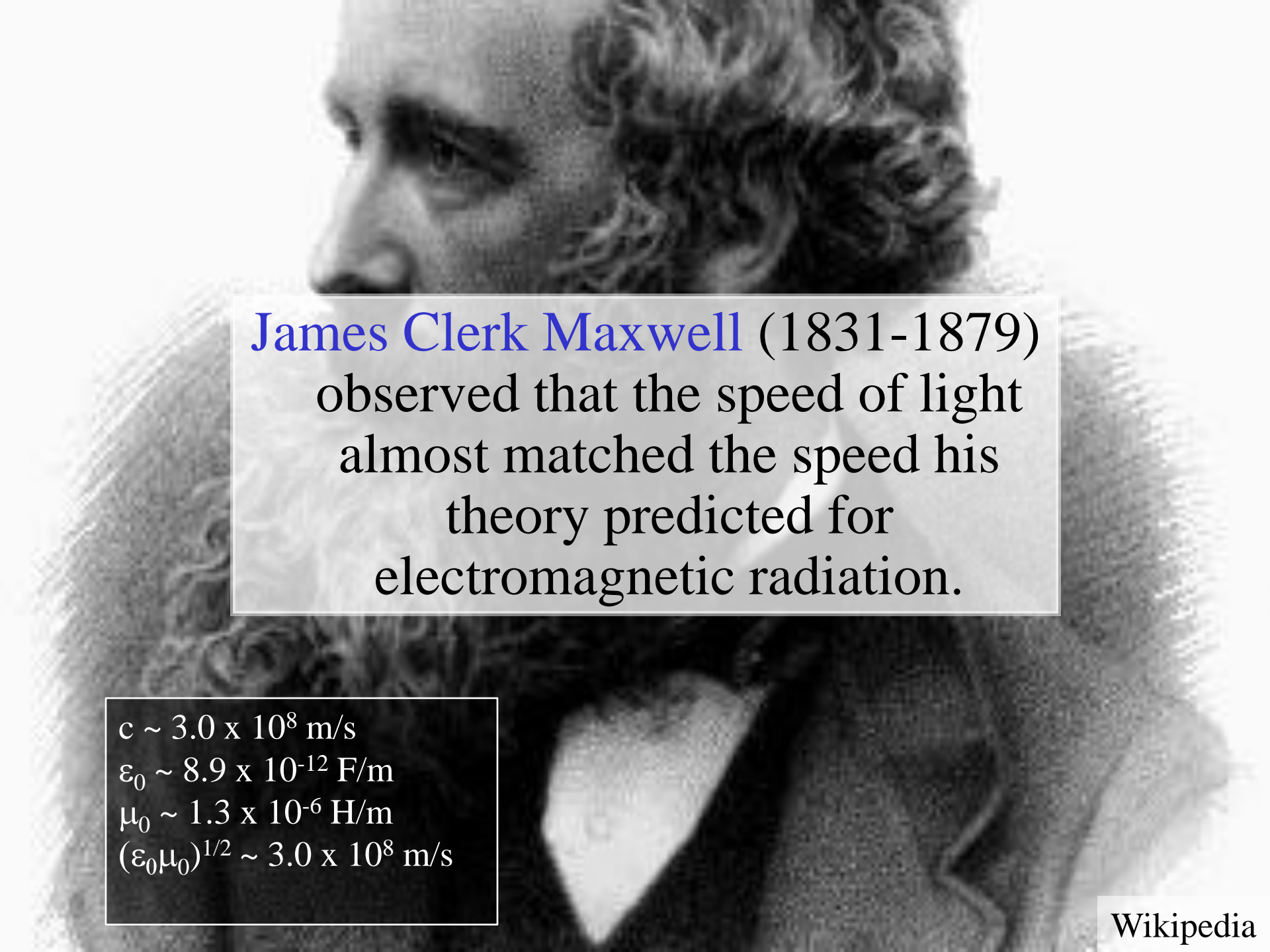
Huygens reasoned that this was because of the additional distance (2AU) that the light from Jupiter had to travel.



Using the best measurement of the
AU available to him, he then
computed the speed of light as $c =$
 $220,000 \text{ km/s} = 140,000 \text{ mi/s}.$
[The truth is $299,792 \text{ km/s} = 186,282$
 $\text{mi/s}.$]

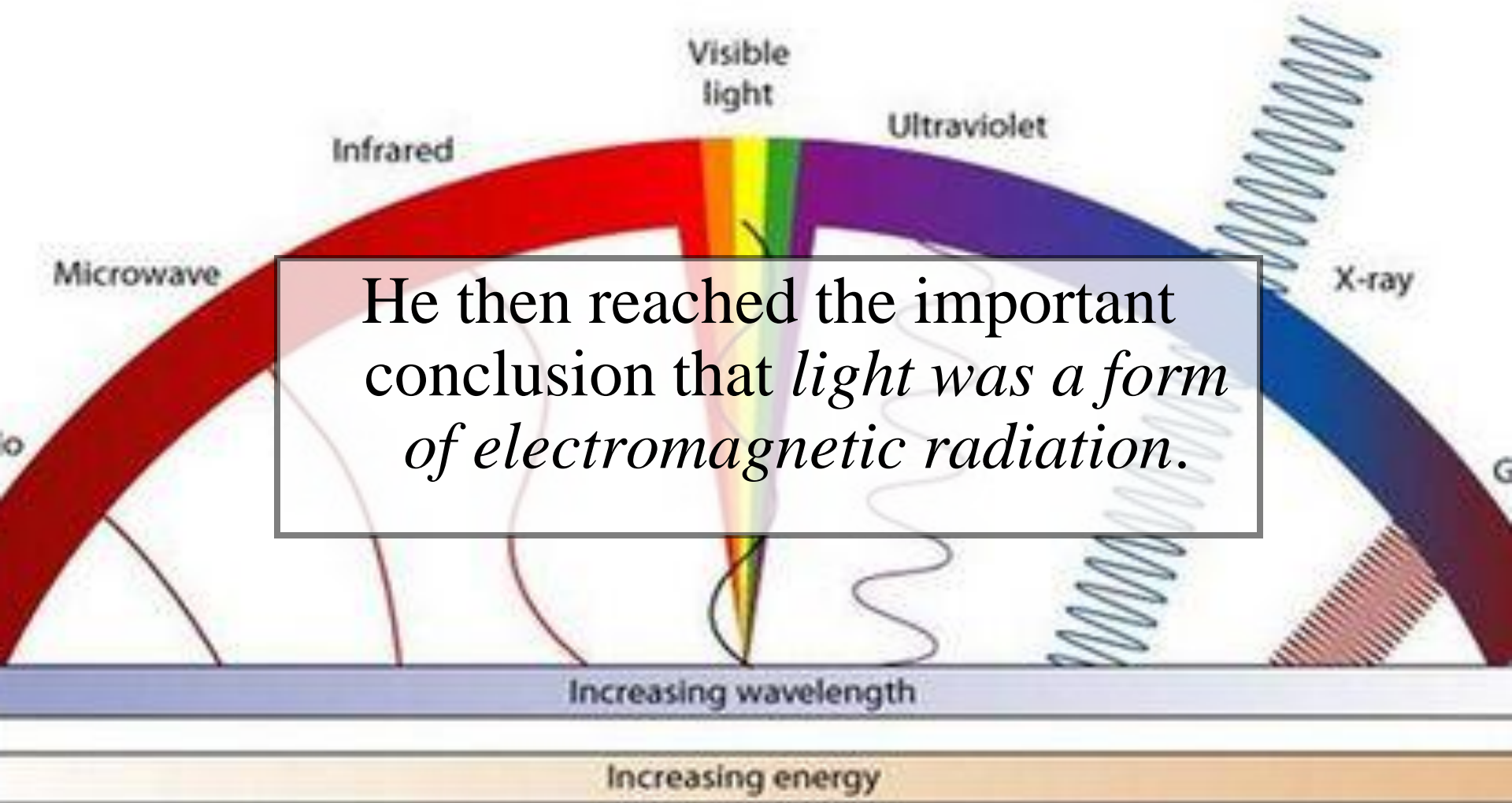


This computation was
important for the future
development of physics.

A black and white portrait of James Clerk Maxwell, showing his head and shoulders in profile, facing left. He has thick, curly hair and is wearing a dark suit jacket over a white shirt and a dark cravat.

James Clerk Maxwell (1831-1879)
observed that the speed of light
almost matched the speed his
theory predicted for
electromagnetic radiation.

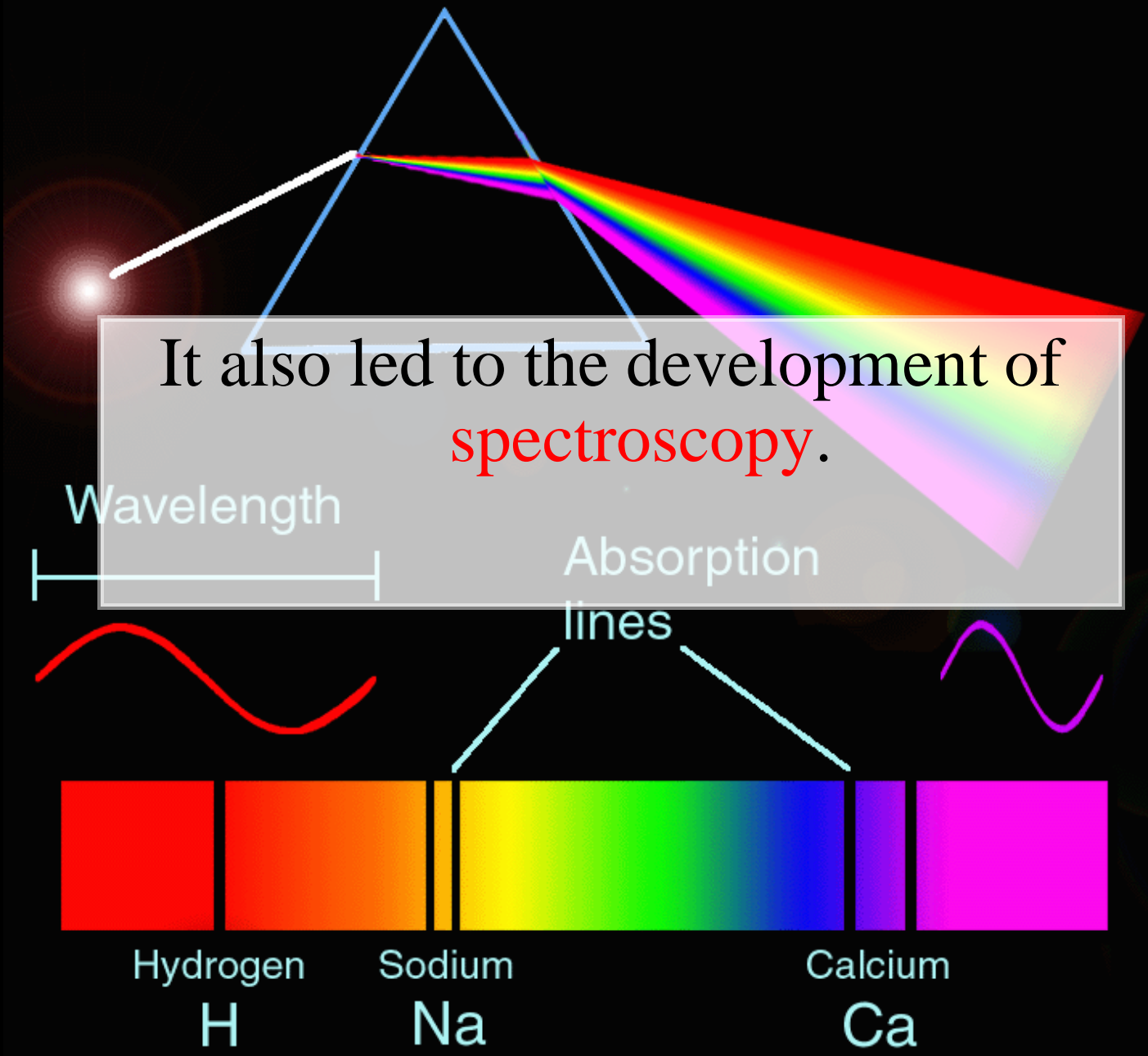
$$c \sim 3.0 \times 10^{10} \text{ m/s}$$
$$\epsilon_0 \sim 8.9 \times 10^{-12} \text{ F/m}$$
$$\mu_0 \sim 1.3 \times 10^{-6} \text{ H/m}$$
$$(\epsilon_0 \mu_0)^{1/2} \sim 3.0 \times 10^{10} \text{ m/s}$$



This observation was instrumental
in leading to **Einstein's theory of
special relativity.**

$$\begin{aligned}x &= vt \leftrightarrow x' = 0 \\x &= ct \leftrightarrow x' = ct' \\x &= -ct \leftrightarrow x' = -ct'\end{aligned}$$

$$\begin{aligned}x' &= (x-vt)/(1-v^2/c^2)^{1/2} \\t' &= (t-vx/c^2)/(1-v^2/c^2)^{1/2}\end{aligned}$$



It also led to the development of spectroscopy.

Wavelength

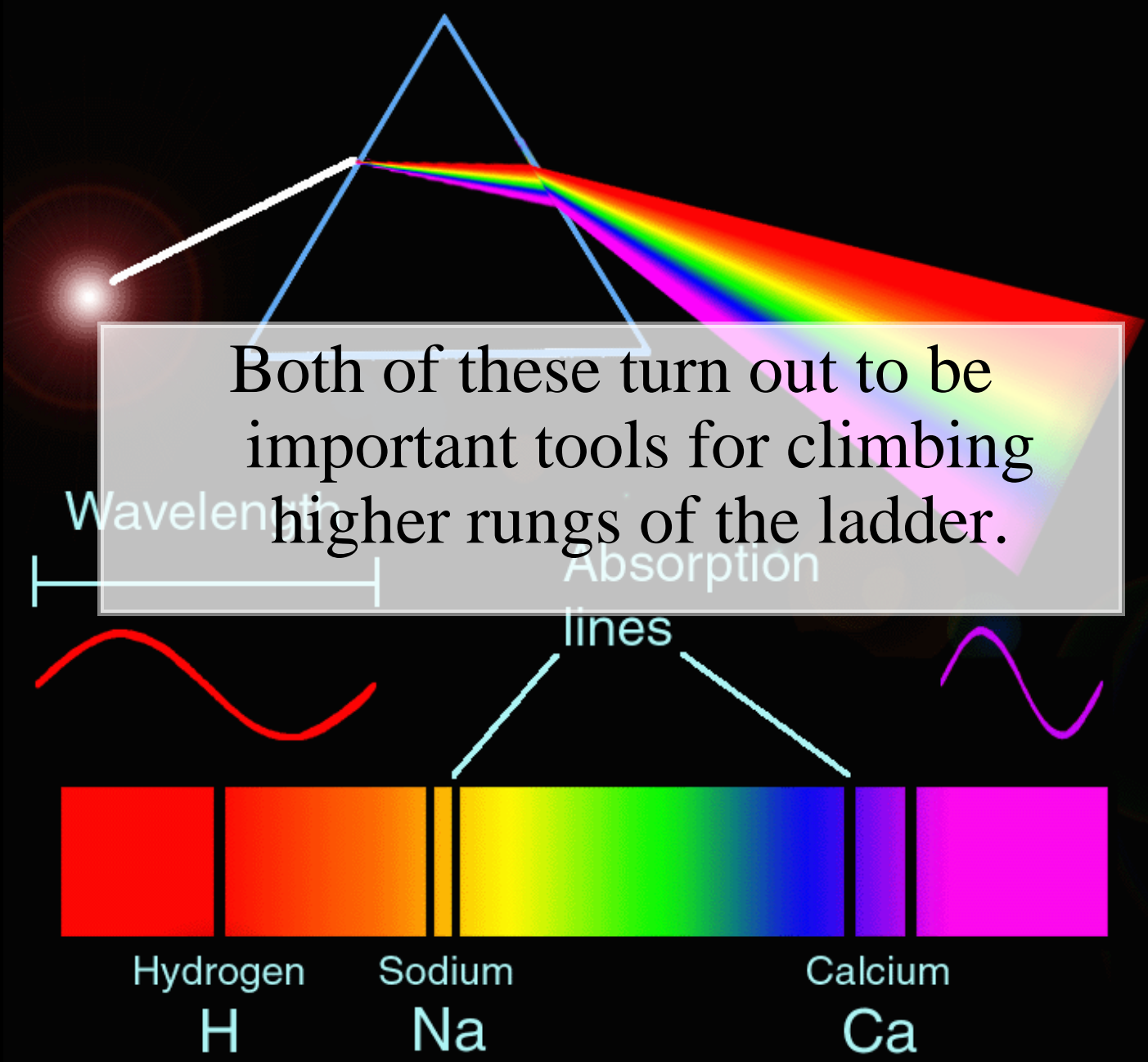
Absorption lines



Hydrogen
H

Sodium
Na

Calcium
Ca



Both of these turn out to be important tools for climbing higher rungs of the ladder.

Wavelength

Absorption

lines

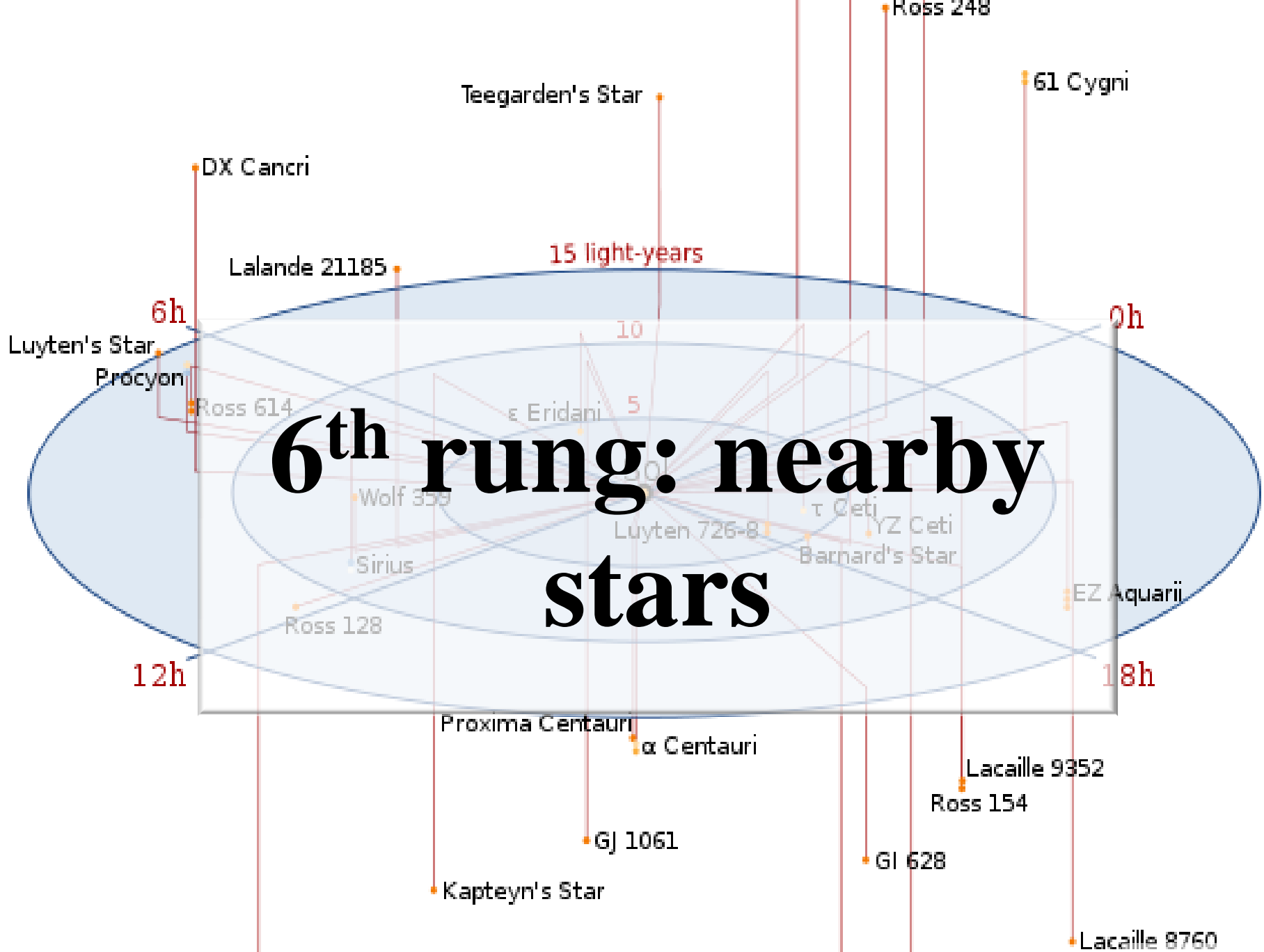


Hydrogen
H

Sodium
Na

Calcium
Ca

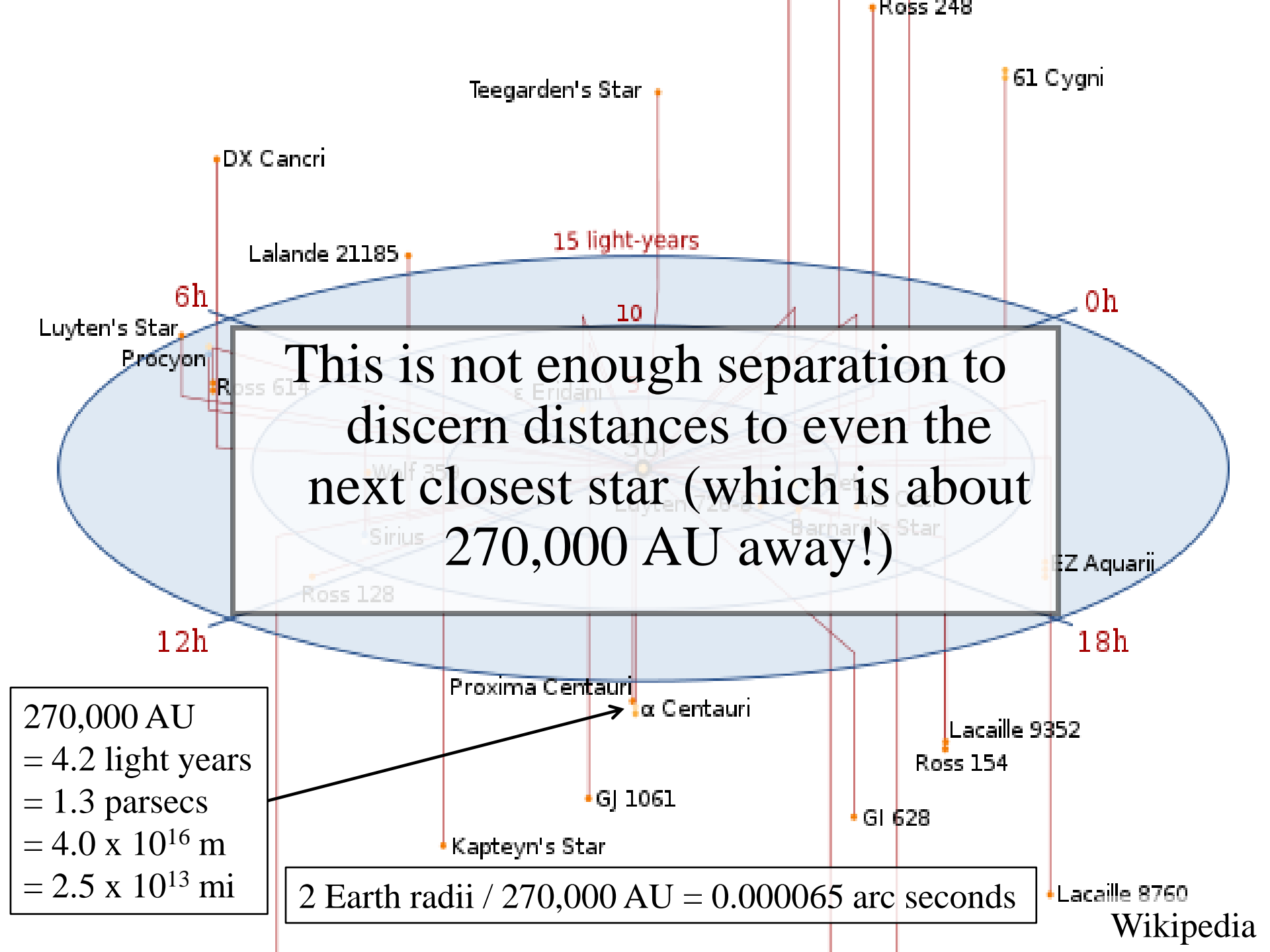
6th rung: nearby stars



This is not enough separation to discern distances to even the next closest star (which is about 270,000 AU away!)

270,000 AU
 = 4.2 light years
 = 1.3 parsecs
 = 4.0×10^{16} m
 = 2.5×10^{13} mi

$2 \text{ Earth radii} / 270,000 \text{ AU} = 0.000065 \text{ arc seconds}$



2 Earth radii = 12,700 km
2 AU = 300,000,000 km

Every January,
we see this:



Every July,
we see this:

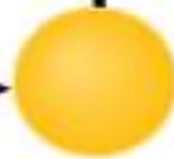


nearby star

However, if one takes
measurements six months apart,
one gets a distance separation of
2AU...



1 AU



(not to scale)



July

January

From "The Essential Cosmic Perspective", Bennett et al.

1 light year = 9.5×10^{15} m
1 parsec = 3.1×10^{16} m

distant stars

Every January,
we see this:



Every July,
we see this:



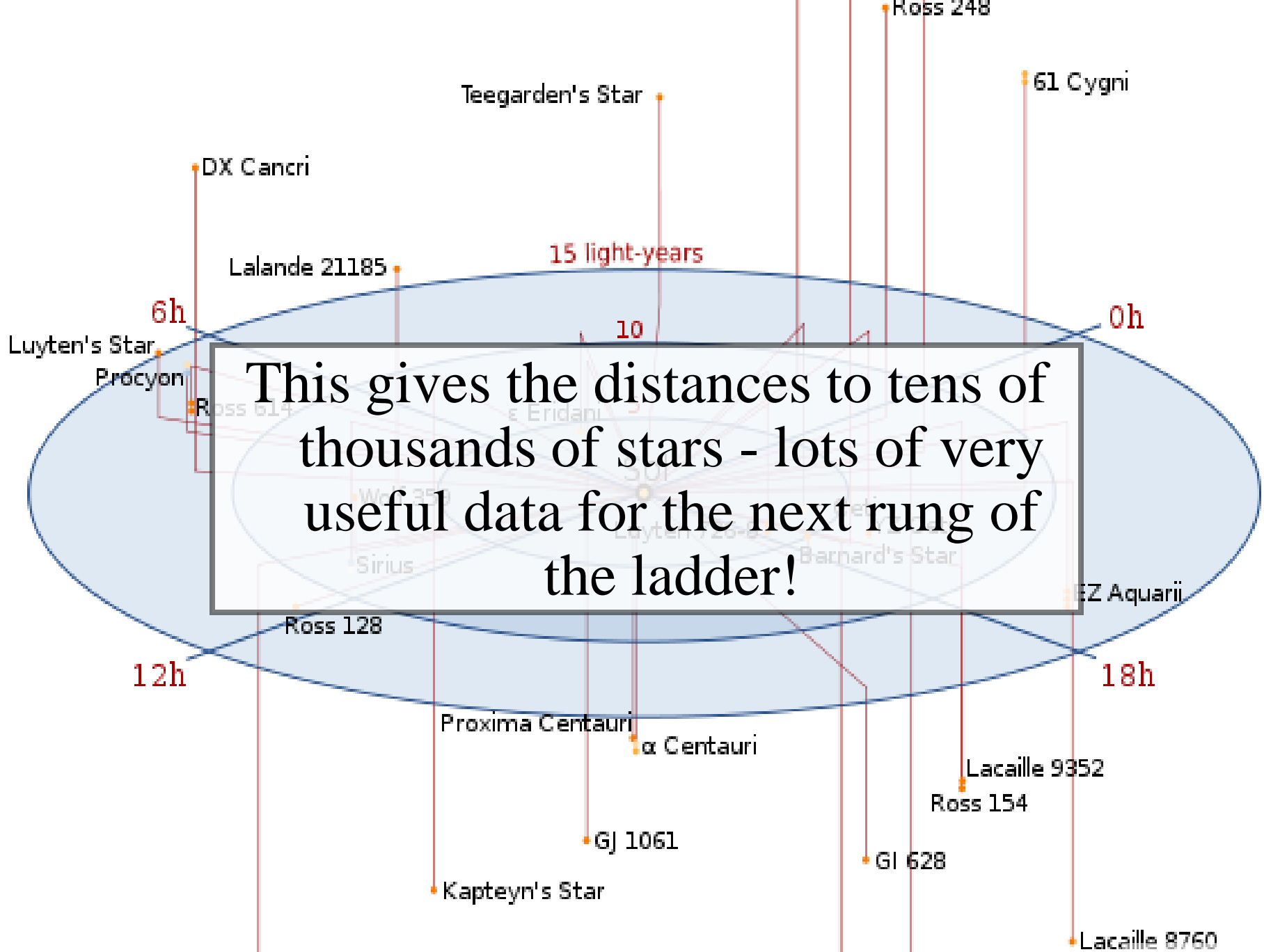
nearby star


... which gives enough parallax to
measure all stars within about
100 light years (30 parsecs).



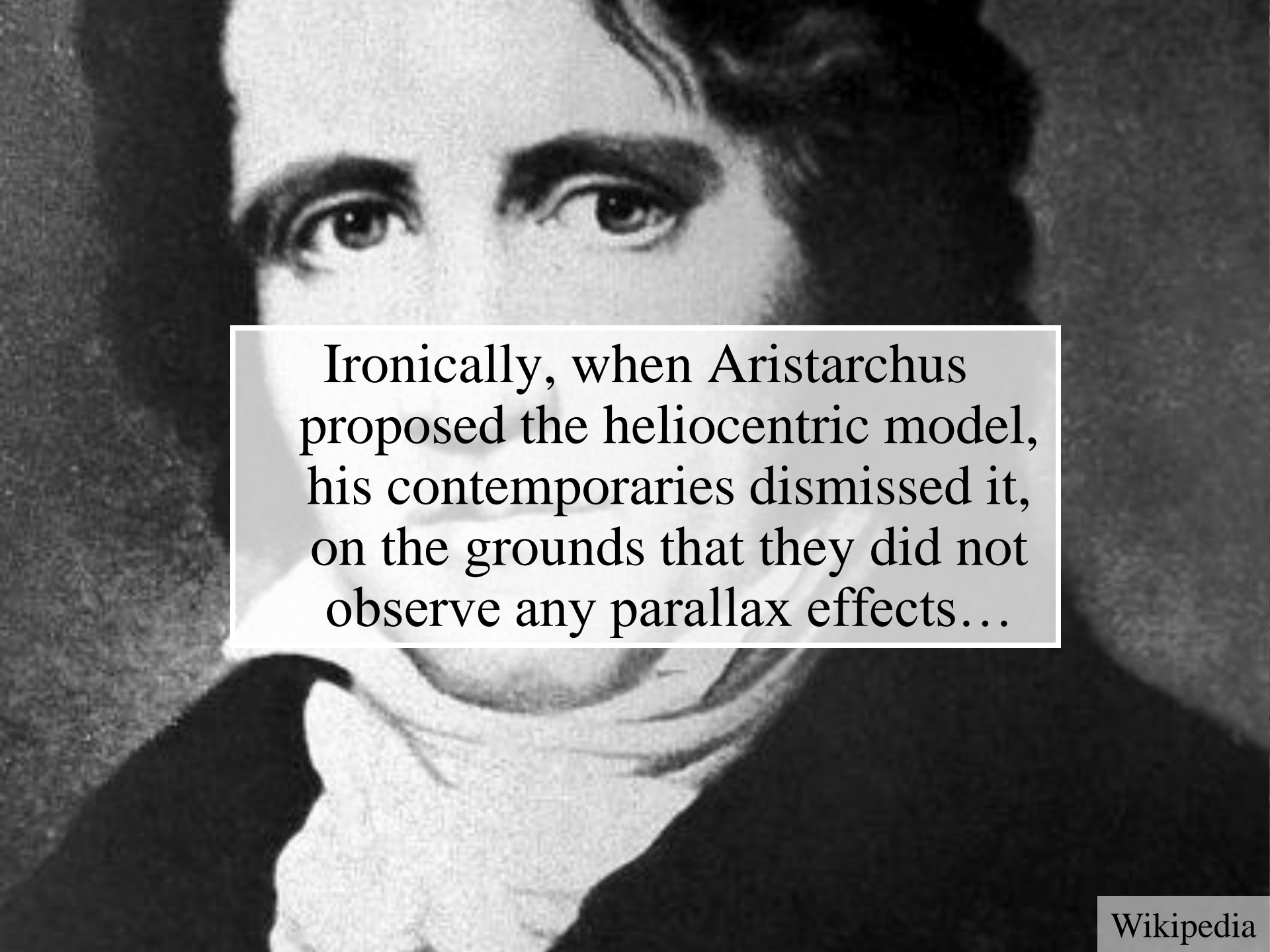
From "The Essential Cosmic Perspective", Bennett et al.

This gives the distances to tens of thousands of stars - lots of very useful data for the next rung of the ladder!

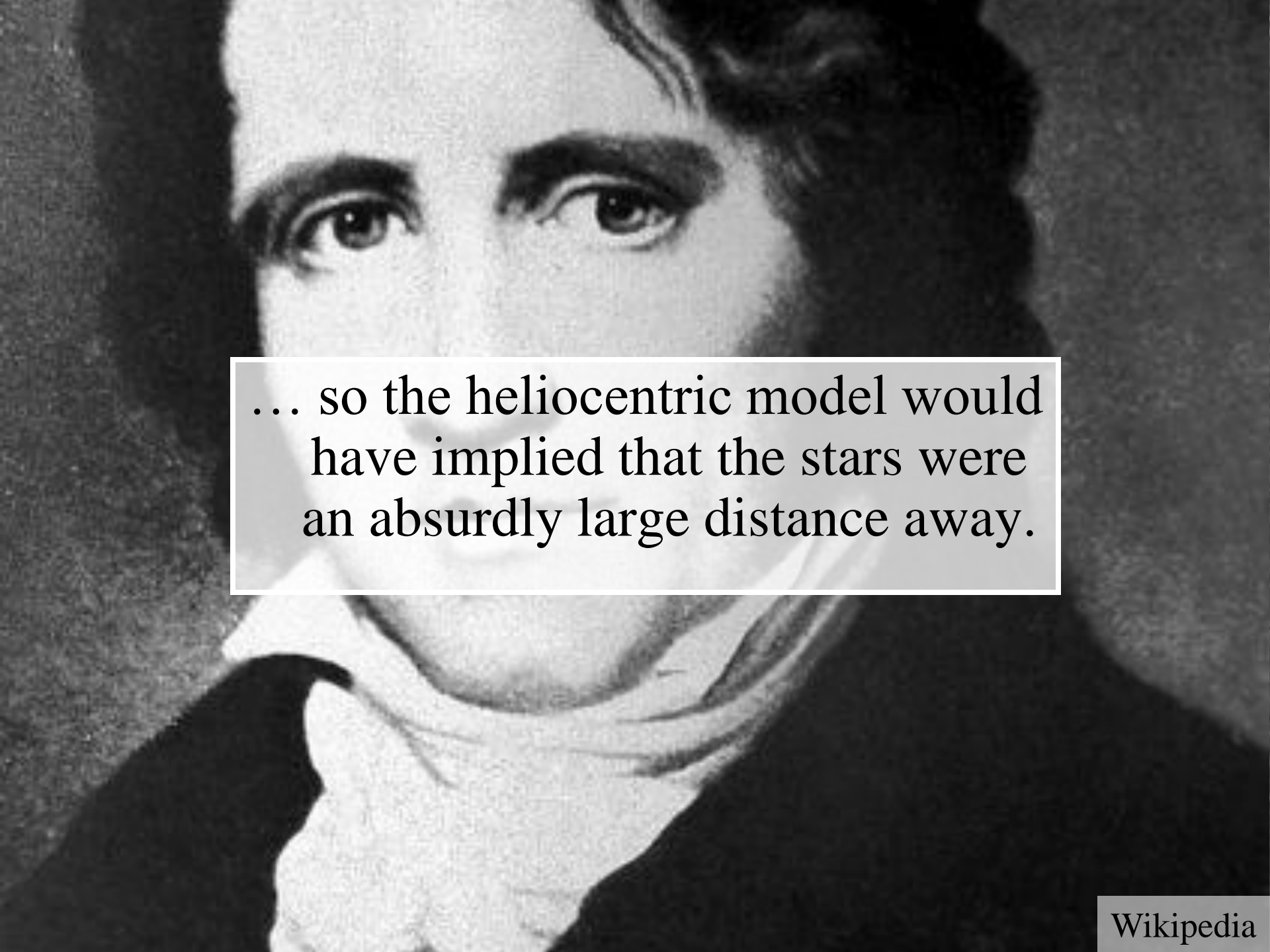


A black and white portrait of Friedrich Bessel, a German astronomer, mathematician, physicist, and engineer. He is shown from the chest up, wearing a dark coat and a white cravat. His hair is dark and wavy, and he has a serious expression.

These parallax computations,
which require accurate
telescropy, were first done by
Friedrich Bessel (1784-1846) in
1838.



Ironically, when Aristarchus proposed the heliocentric model, his contemporaries dismissed it, on the grounds that they did not observe any parallax effects...

A black and white portrait of a man, likely a historical figure, wearing a white cravat and a dark coat. The image is a close-up, focusing on the man's face and upper torso. The background is dark and out of focus.

... so the heliocentric model would have implied that the stars were an absurdly large distance away.

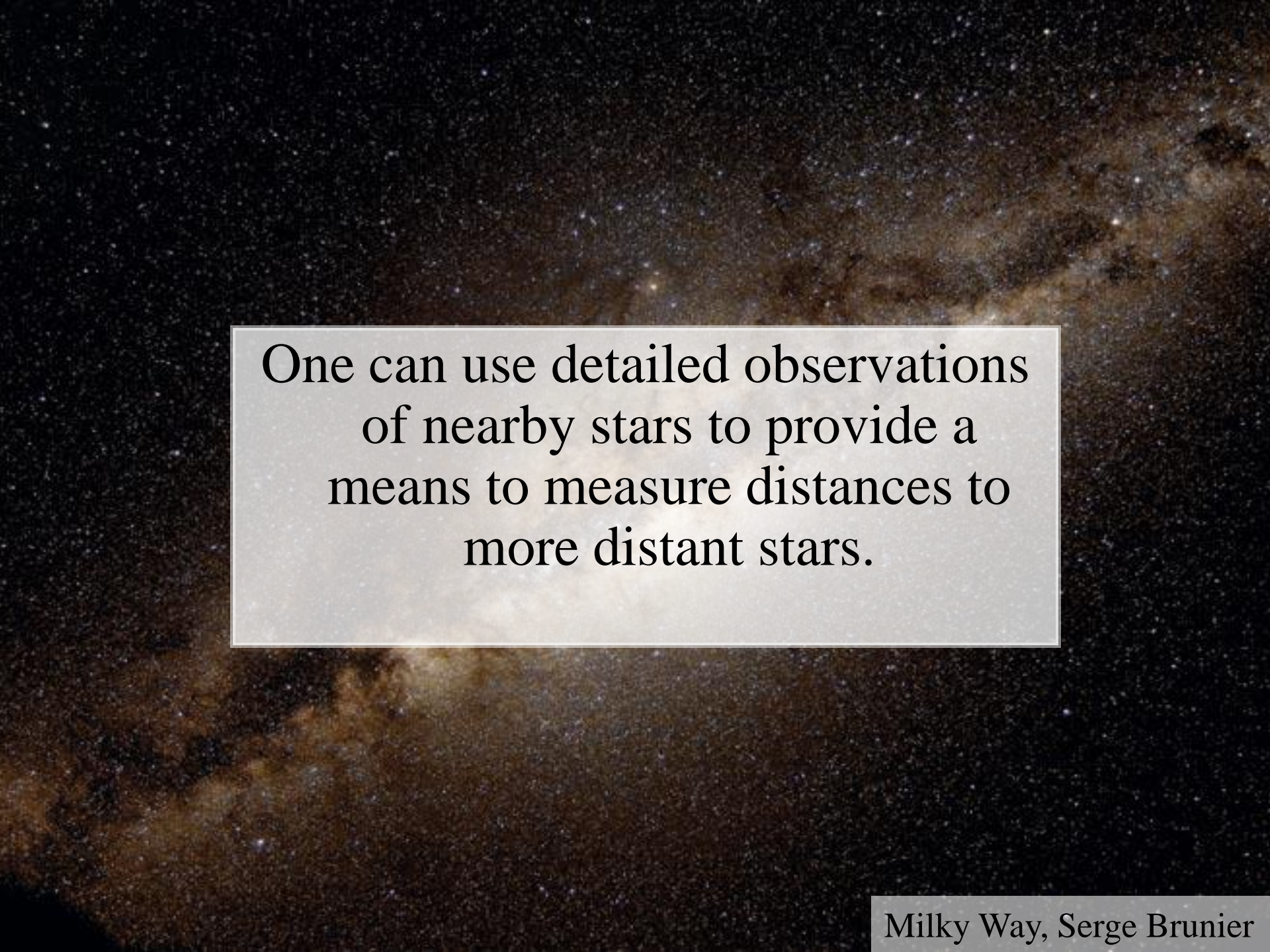


[Which, of course, they are.]

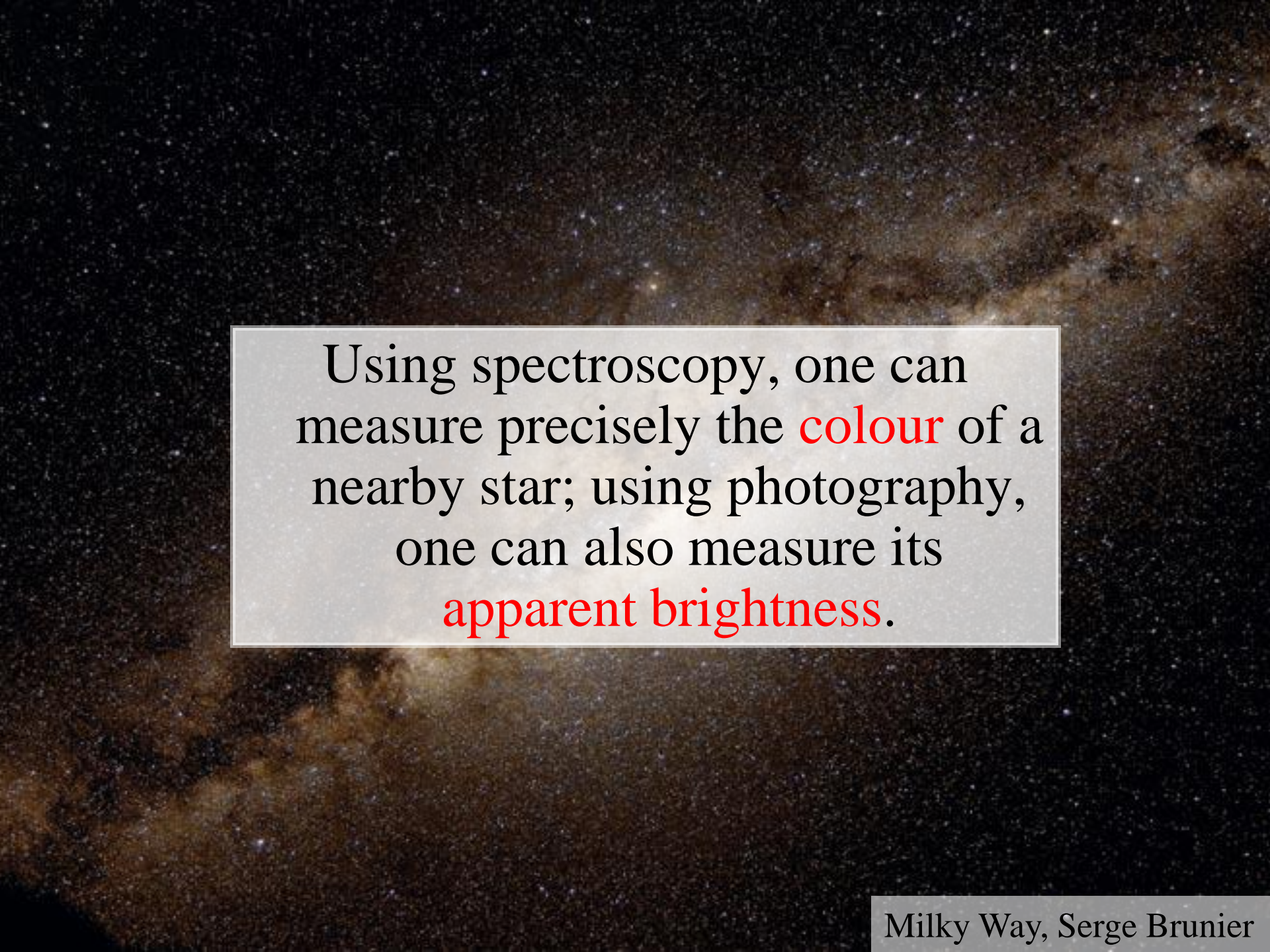
Distance to Proxima Centauri
= 40,000,000,000 km
= 25,000,000,000 mi




**7th rung: the
Milky Way**



One can use detailed observations of nearby stars to provide a means to measure distances to more distant stars.



Using spectroscopy, one can measure precisely the **colour** of a nearby star; using photography, one can also measure its **apparent brightness**.



Using the apparent brightness, the distance, and inverse square law, one can compute the **absolute brightness** of these stars.

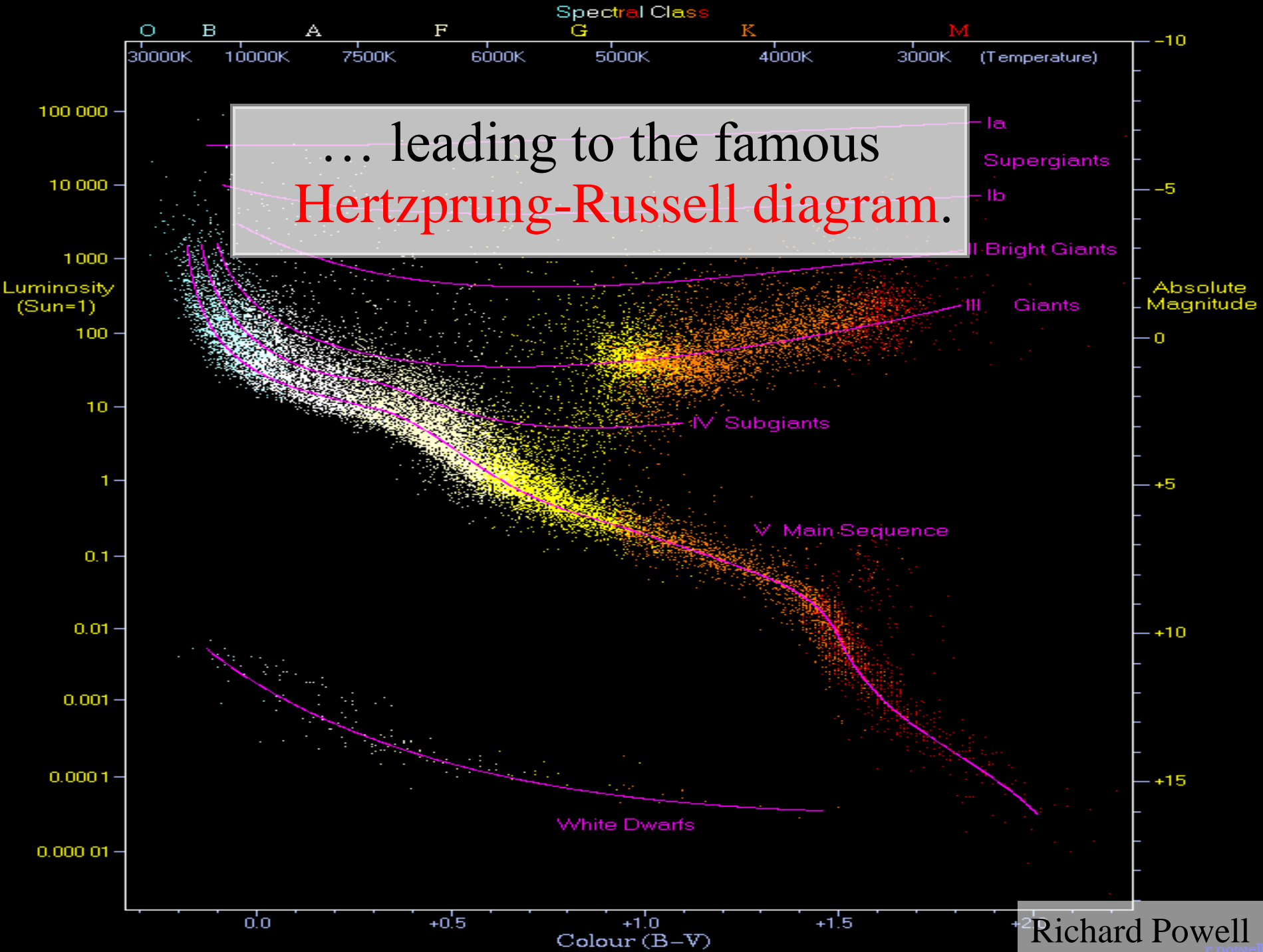
$$M = m - 5(\log_{10} D_L - 1)$$

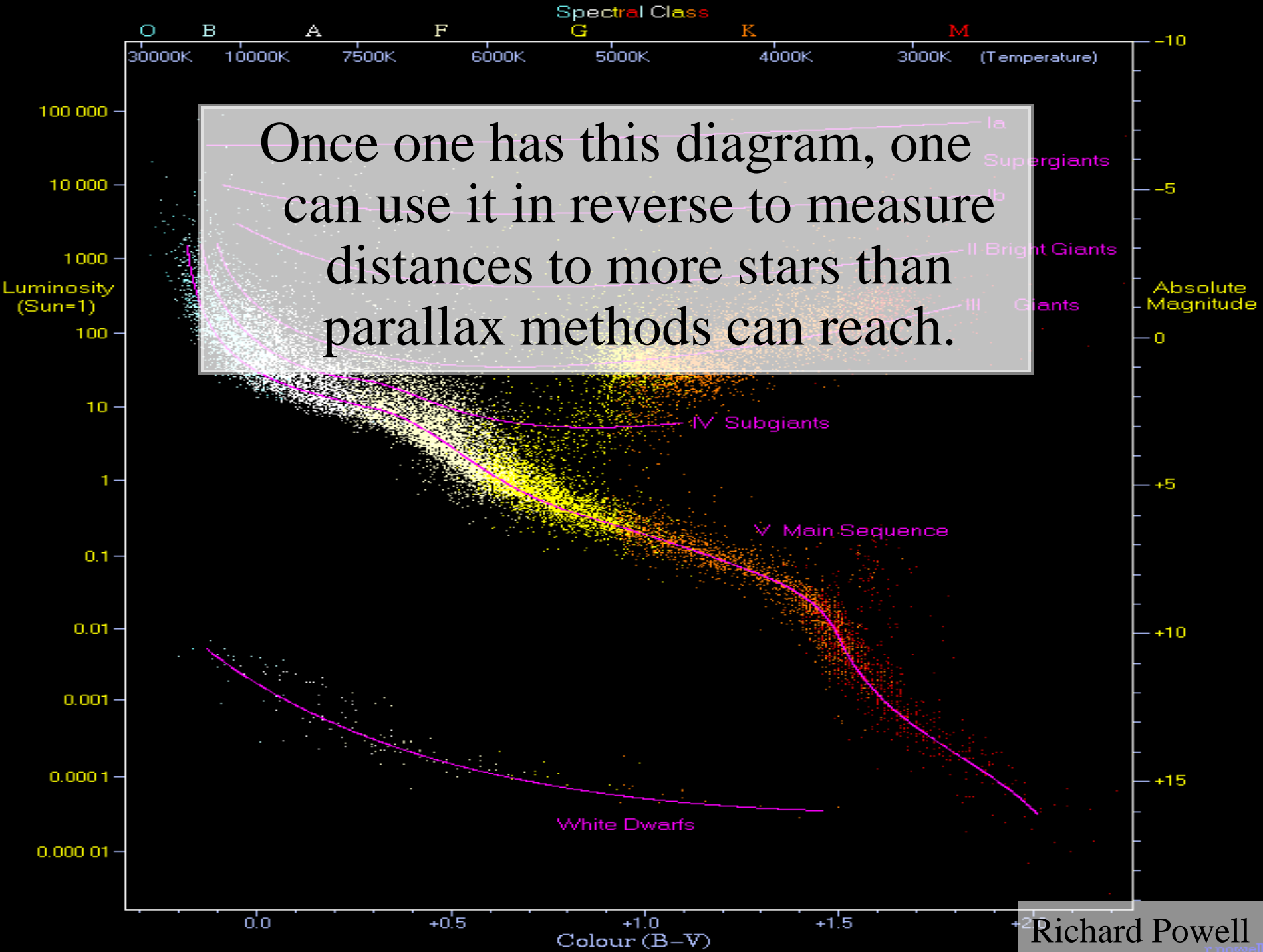


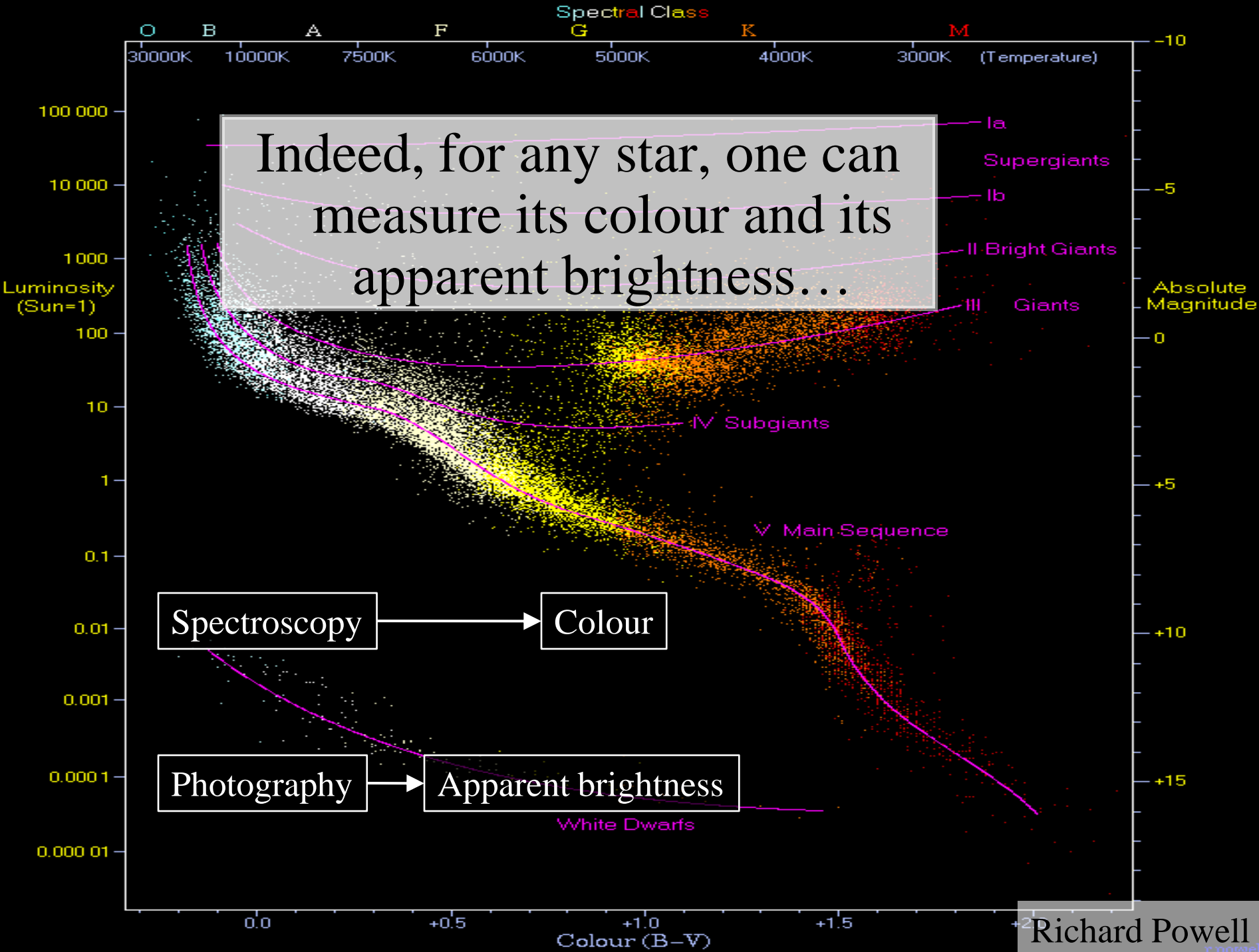
Ejnar Hertzsprung (1873-1967)
and **Henry Russell** (1877-1957)
plotted this absolute brightness
against color for thousands of
nearby stars in 1905-1915...

Leiden Observatory

University of Chicago/Yerkes Observatory

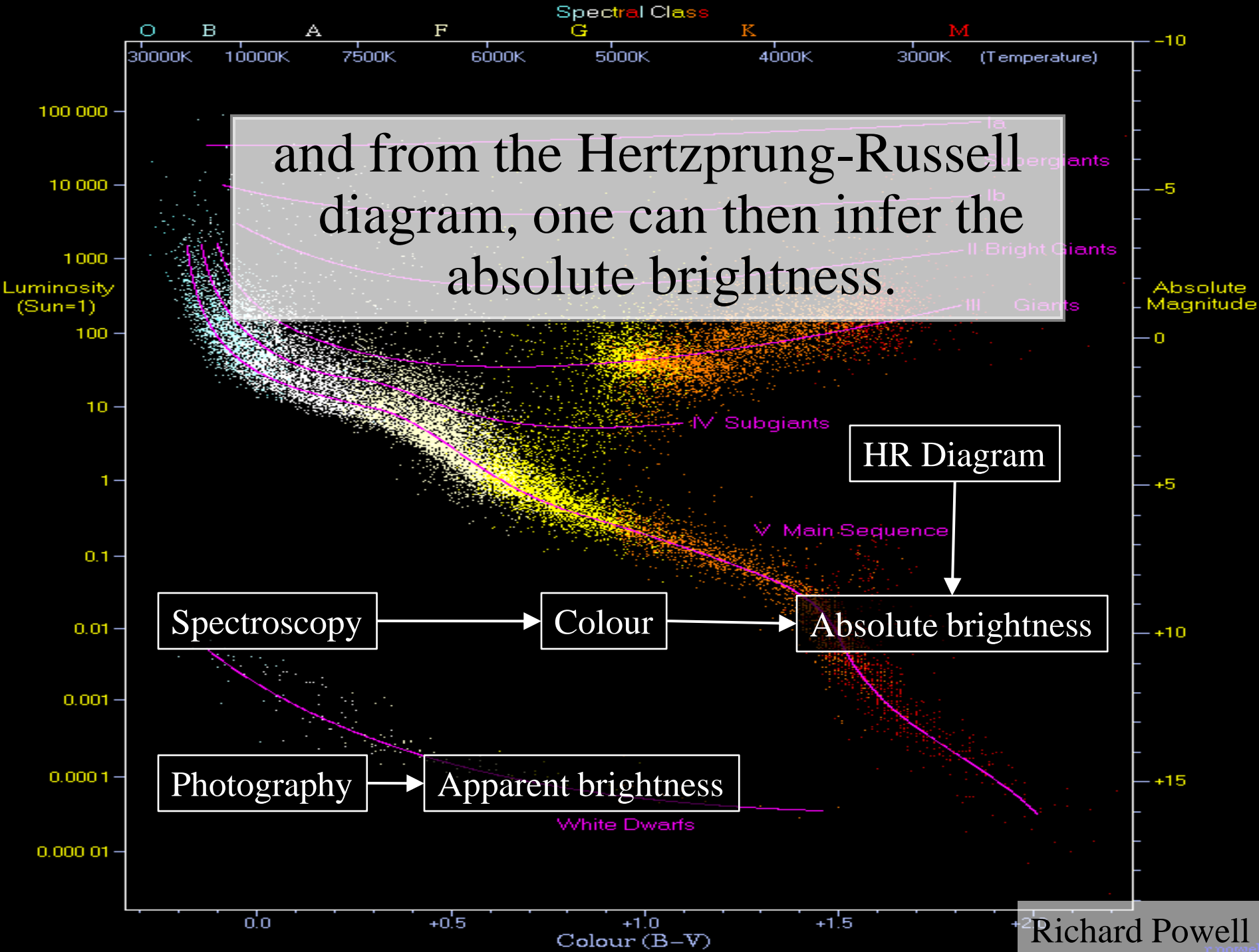


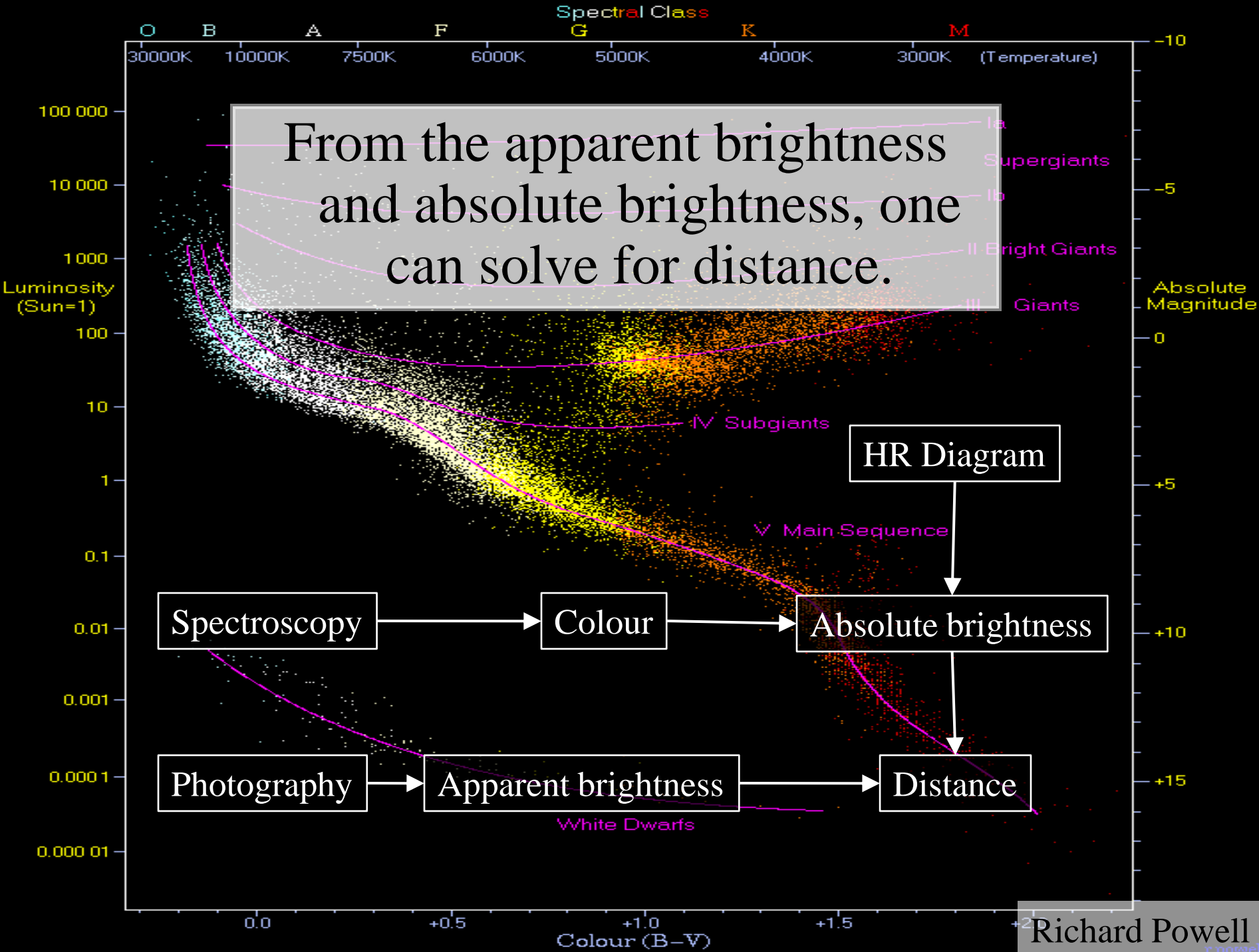


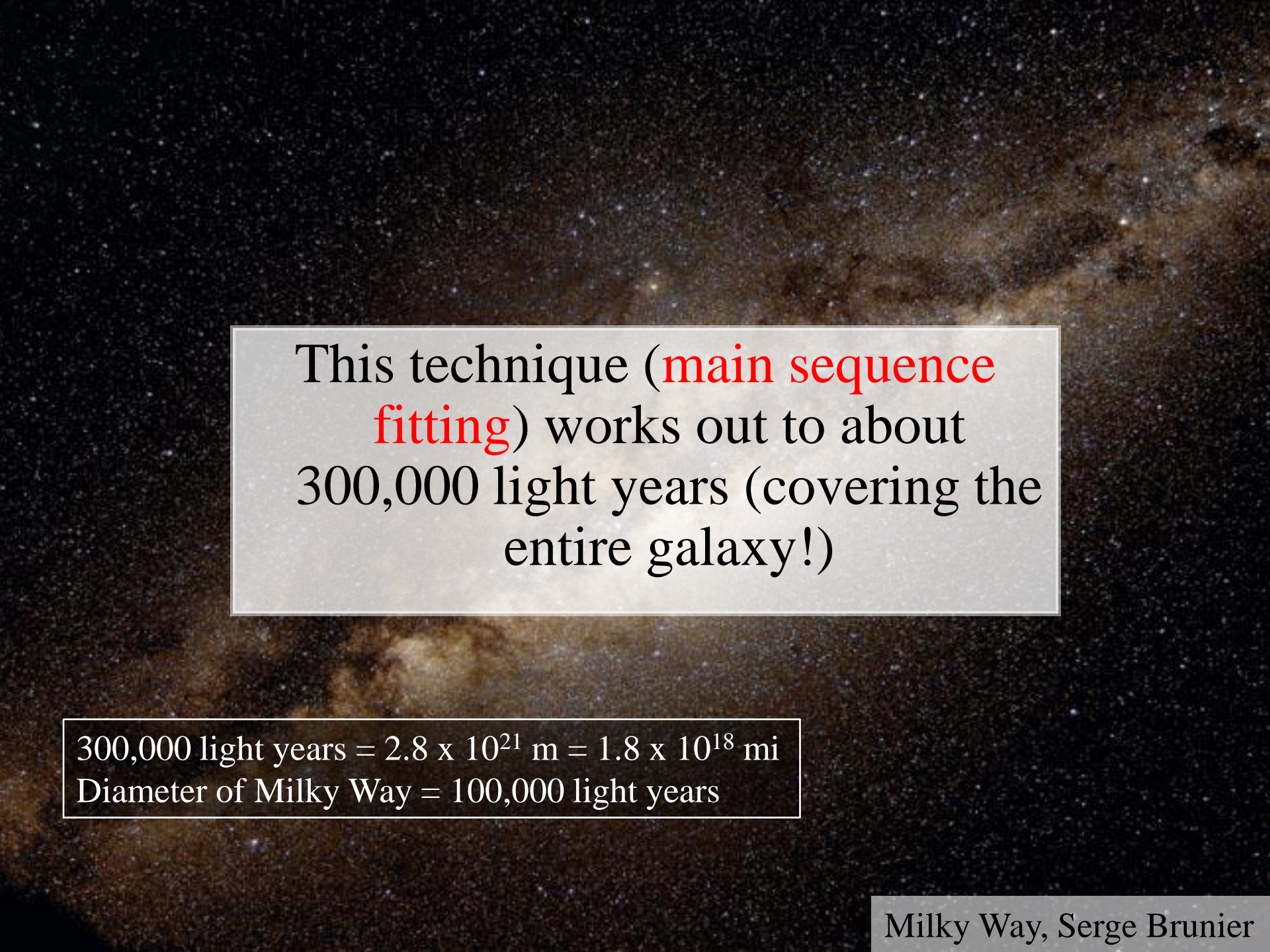


Indeed, for any star, one can measure its colour and its apparent brightness...



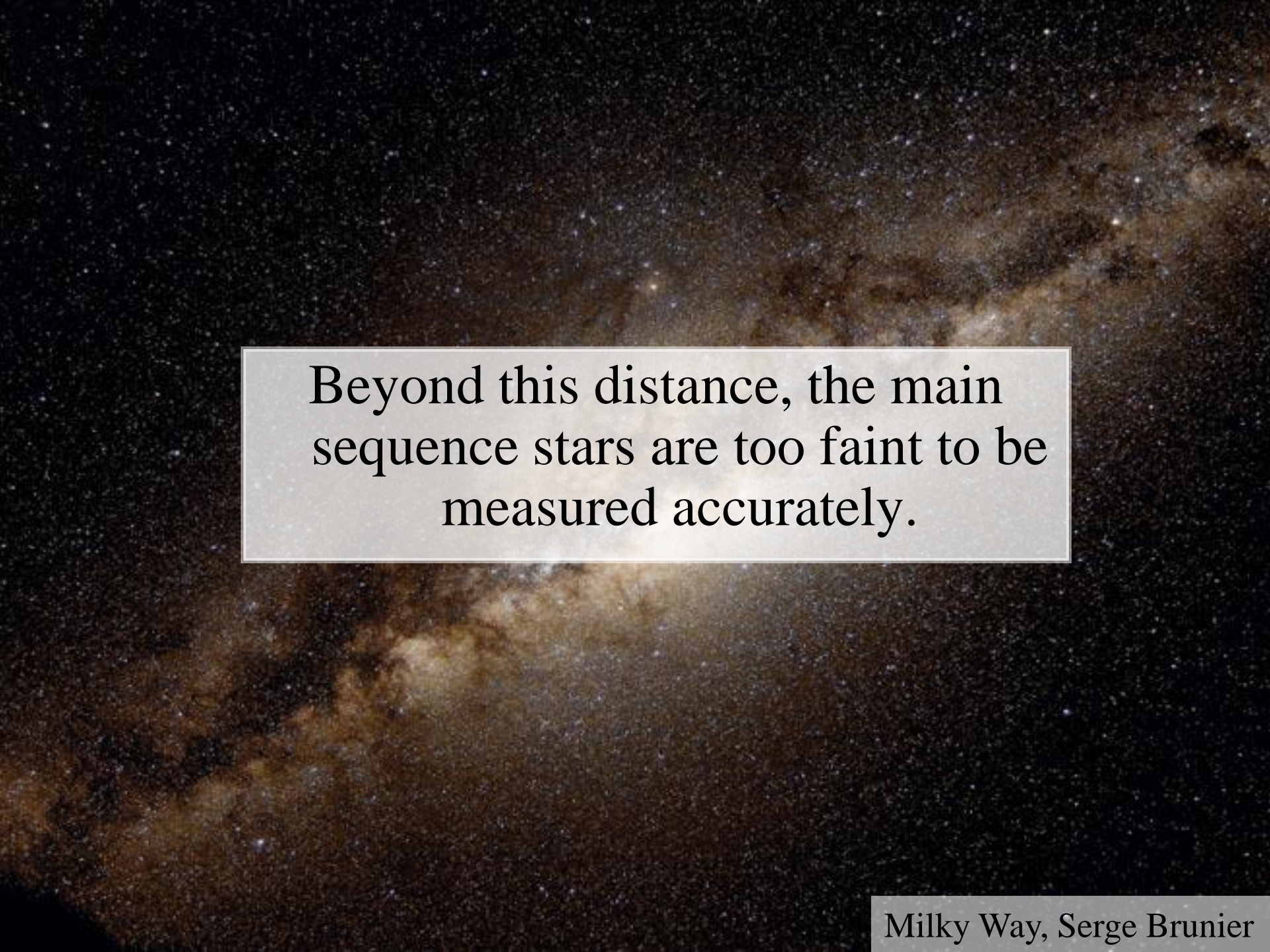






This technique (**main sequence fitting**) works out to about 300,000 light years (covering the entire galaxy!)

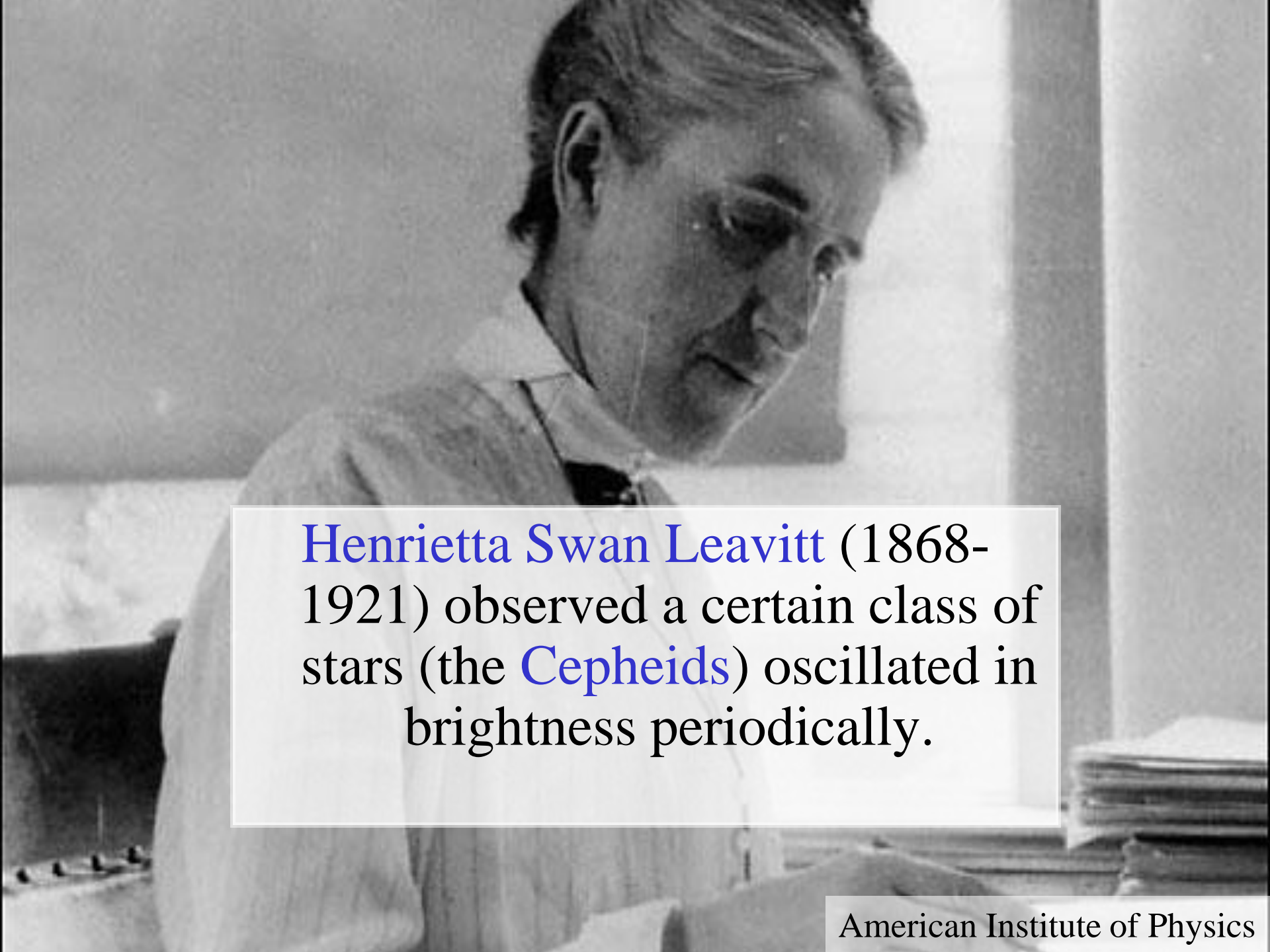
300,000 light years = 2.8×10^{21} m = 1.8×10^{18} mi
Diameter of Milky Way = 100,000 light years



Beyond this distance, the main sequence stars are too faint to be measured accurately.

A vast field of galaxies, including many small, distant ones, set against a dark background. The galaxies vary in color and shape, with some appearing as bright yellow or orange points and others as more complex, multi-colored structures.

**8th rung: Other
galaxies**



Henrietta Swan Leavitt (1868-1921) observed a certain class of stars (the **Cepheids**) oscillated in brightness periodically.

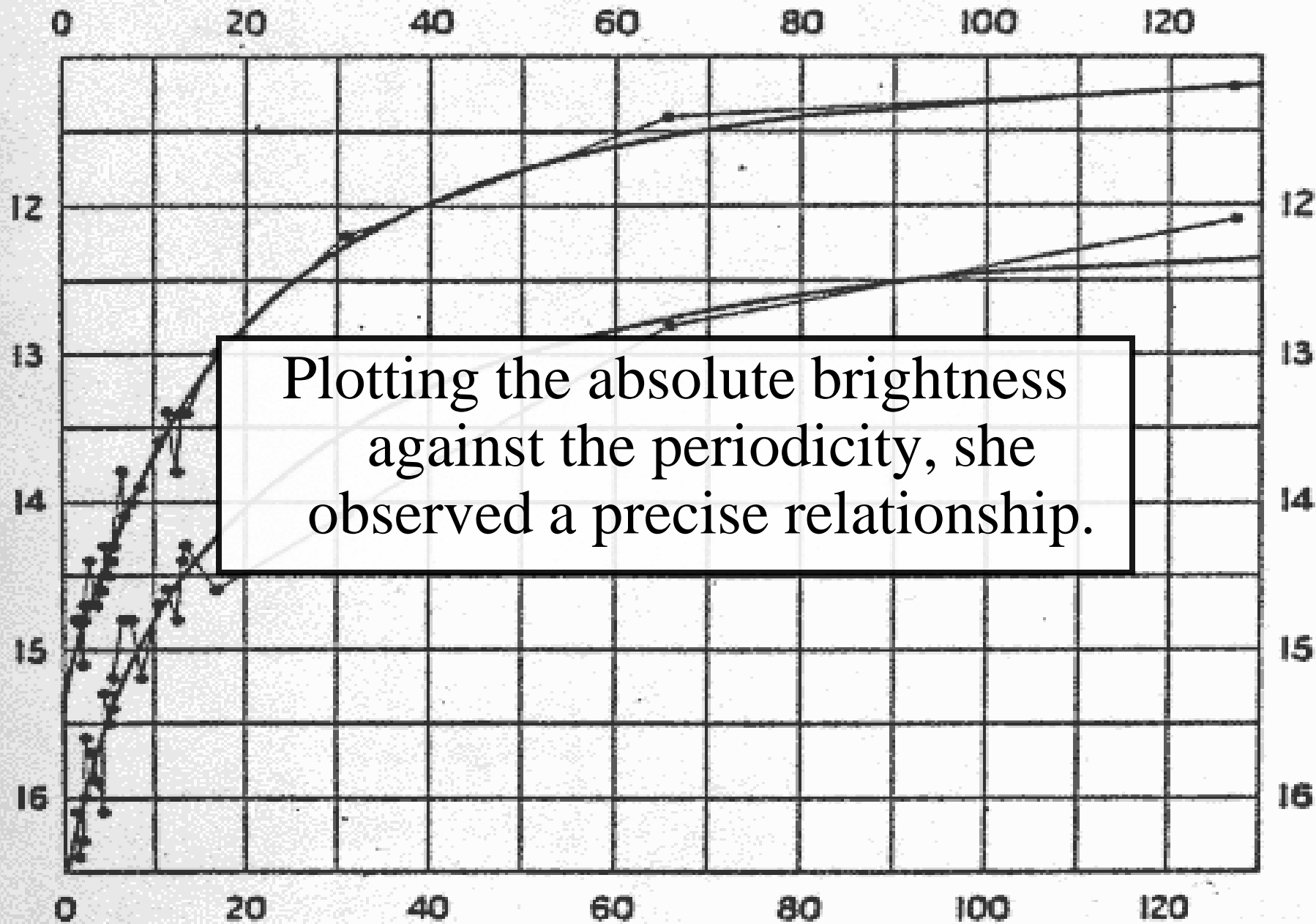


FIG. 1.

Henrietta Swan Leavitt, 1912

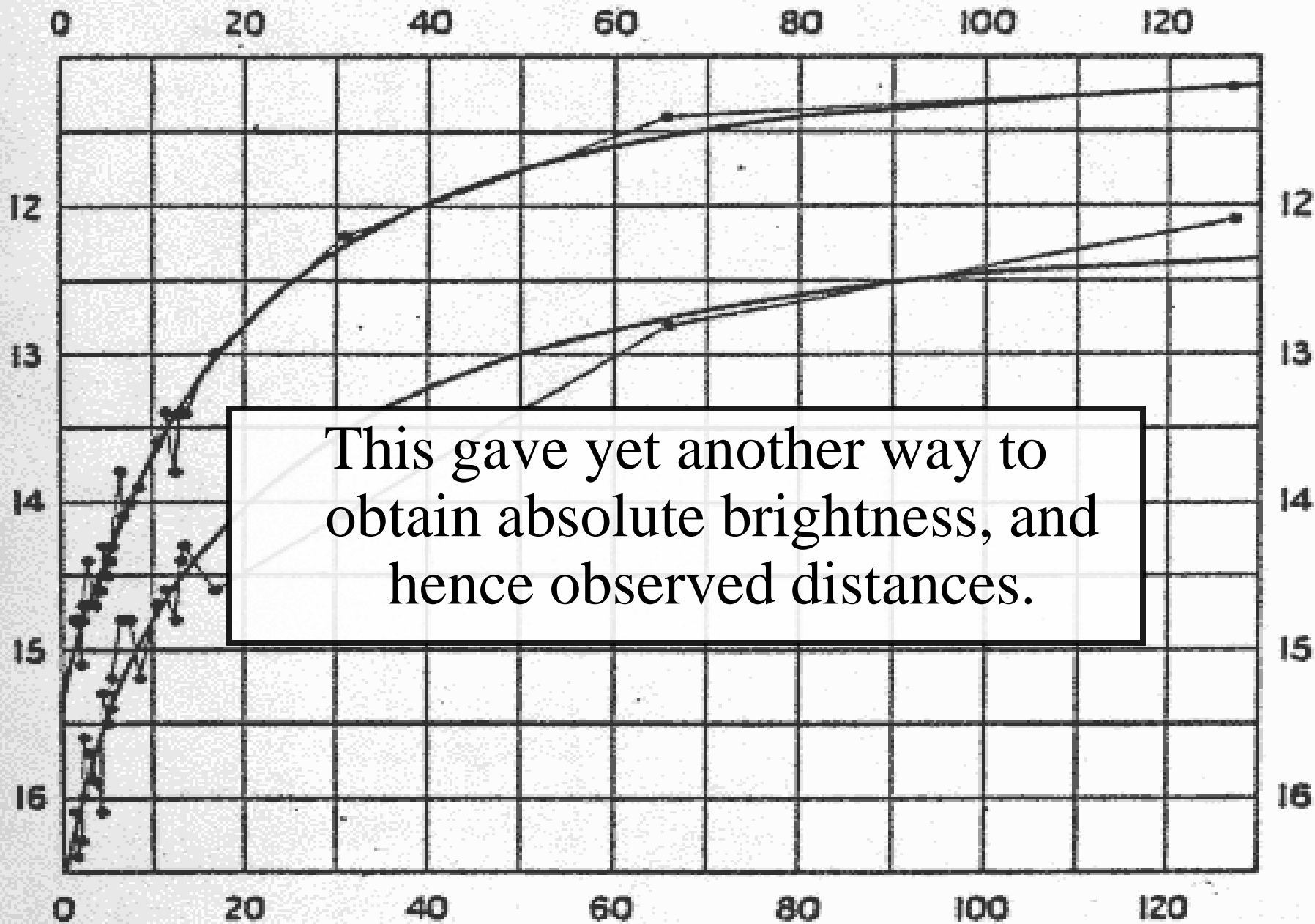
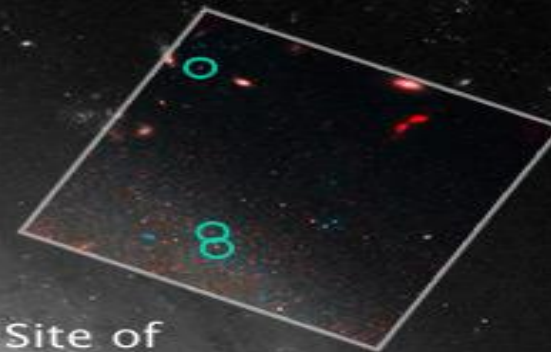


FIG. 1.

Henrietta Swan Leavitt, 1912



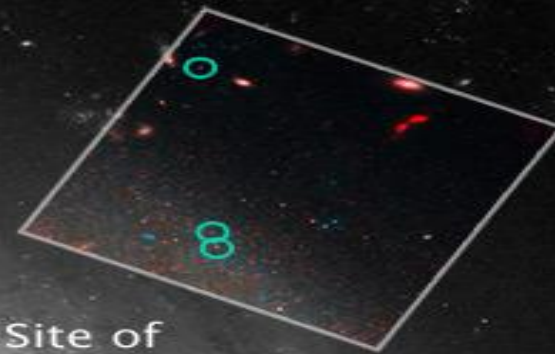
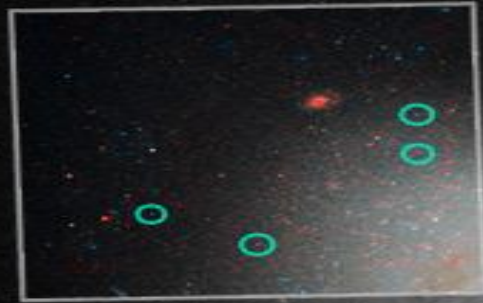
Site of
SN 1995al

Because Cepheids are so bright,
this method works up to
100,000,000 light years!

Diameter of Milky Way = 100,000 light years

Most distant Cepheid detected (Hubble Space Telescope) = 108,000,000 light years

Diameter of universe > 76,000,000,000 light years



Site of
SN 1995al

Most galaxies are fortunate to have at least one Cepheid in them, so we know the distances to all galaxies out to a reasonably large distance.

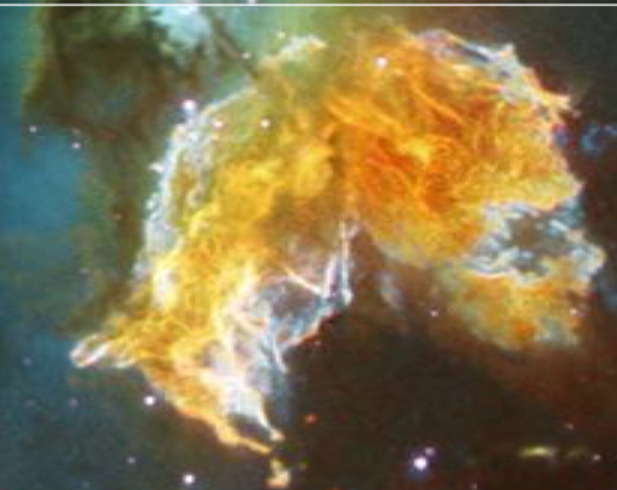


Diameter of Milky Way = 100,000 light years

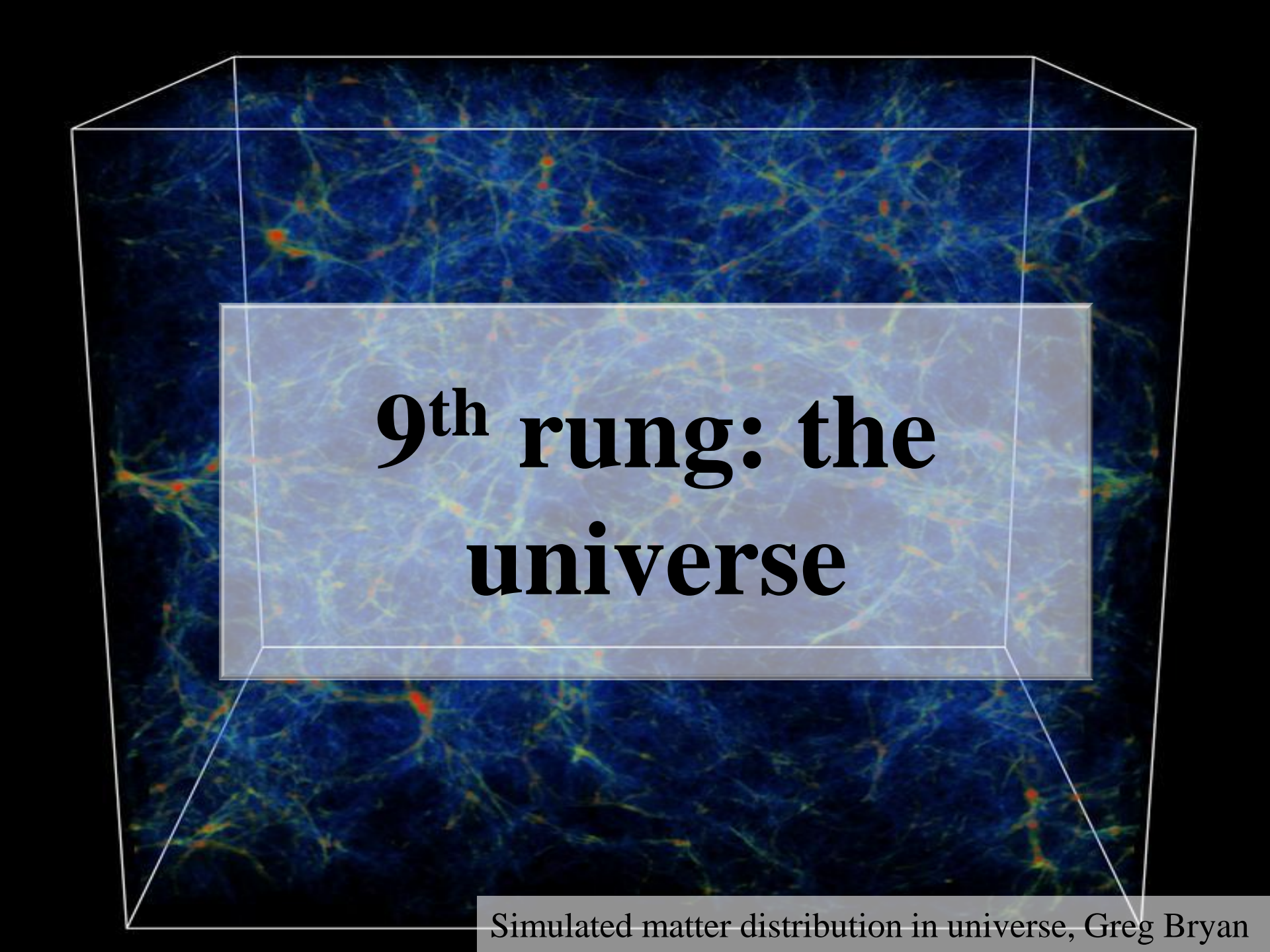
Most distant Cepheid detected (Hubble Space Telescope) = 108,000,000 light years

Most distant Type 1a supernova detected (1997ff) = 11,000,000,000 light years

Diameter of universe > 76,000,000,000 light years



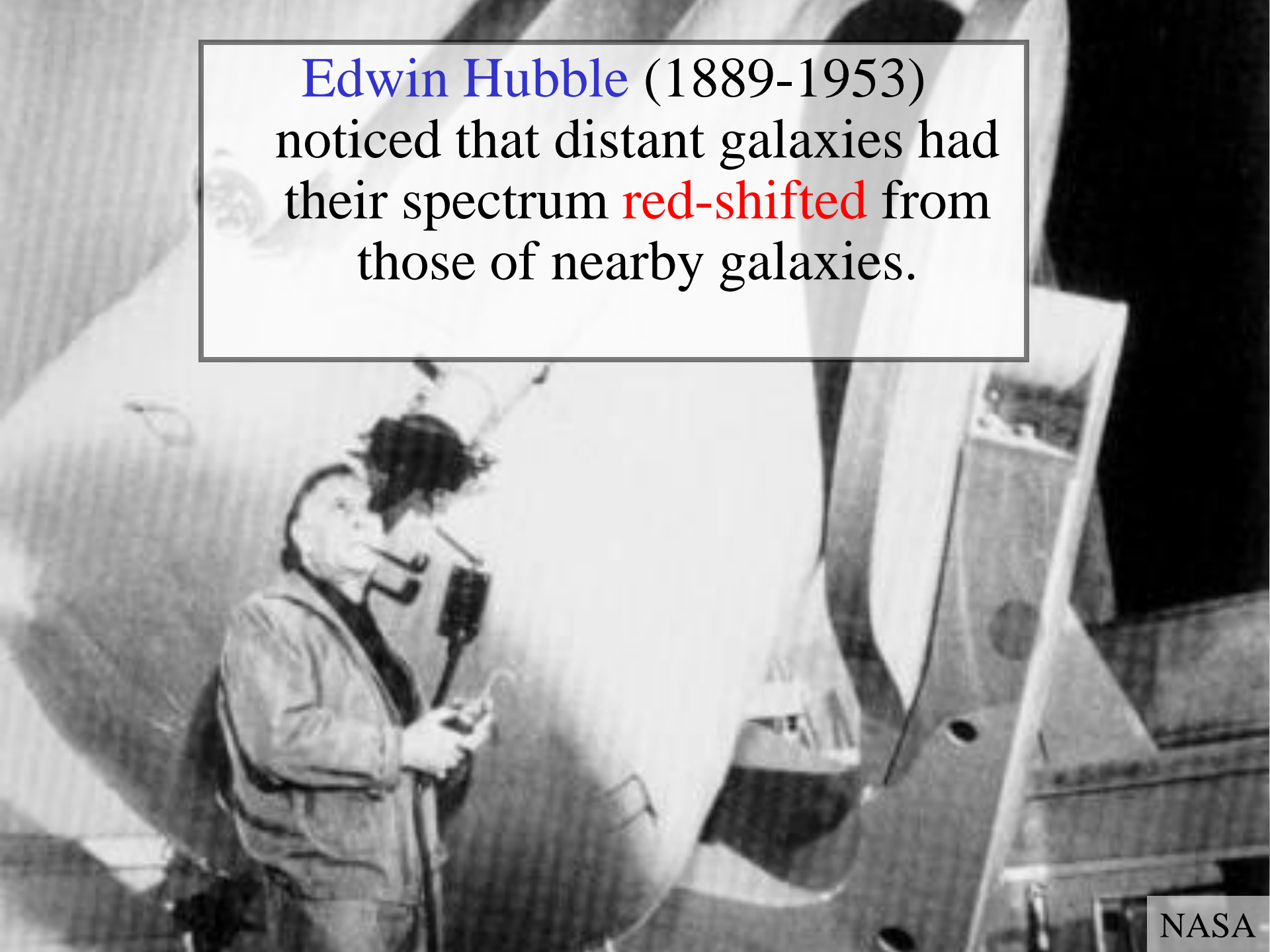
Similar methods, using supernovae instead of Cepheids, can sometimes work to even larger scales than these, and can also be used to independently confirm the Cepheid-based distance measurements.

A 3D wireframe box containing a simulated matter distribution in the universe. The background is a dark blue field filled with a complex network of thin, glowing lines in shades of blue, green, and yellow, representing the cosmic web. Scattered throughout are small, bright orange and red spots, likely representing galaxy clusters or individual galaxies. The text is centered within a semi-transparent rectangular box in the middle of the scene.

9th rung: the universe

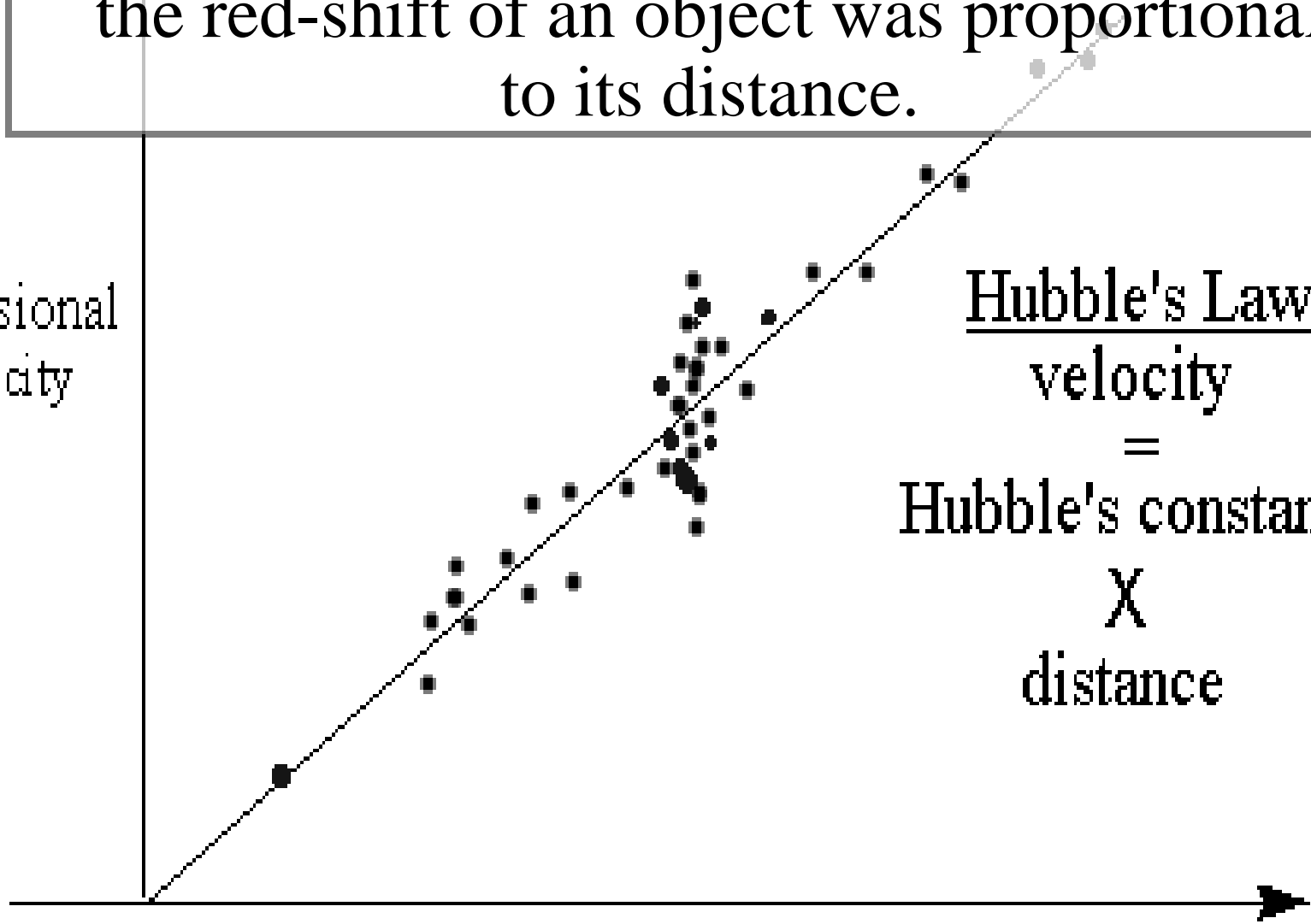
Simulated matter distribution in universe, Greg Bryan

Edwin Hubble (1889-1953)
noticed that distant galaxies had
their spectrum **red-shifted** from
those of nearby galaxies.



With this data, he formulated **Hubble's law**:
the red-shift of an object was proportional
to its distance.

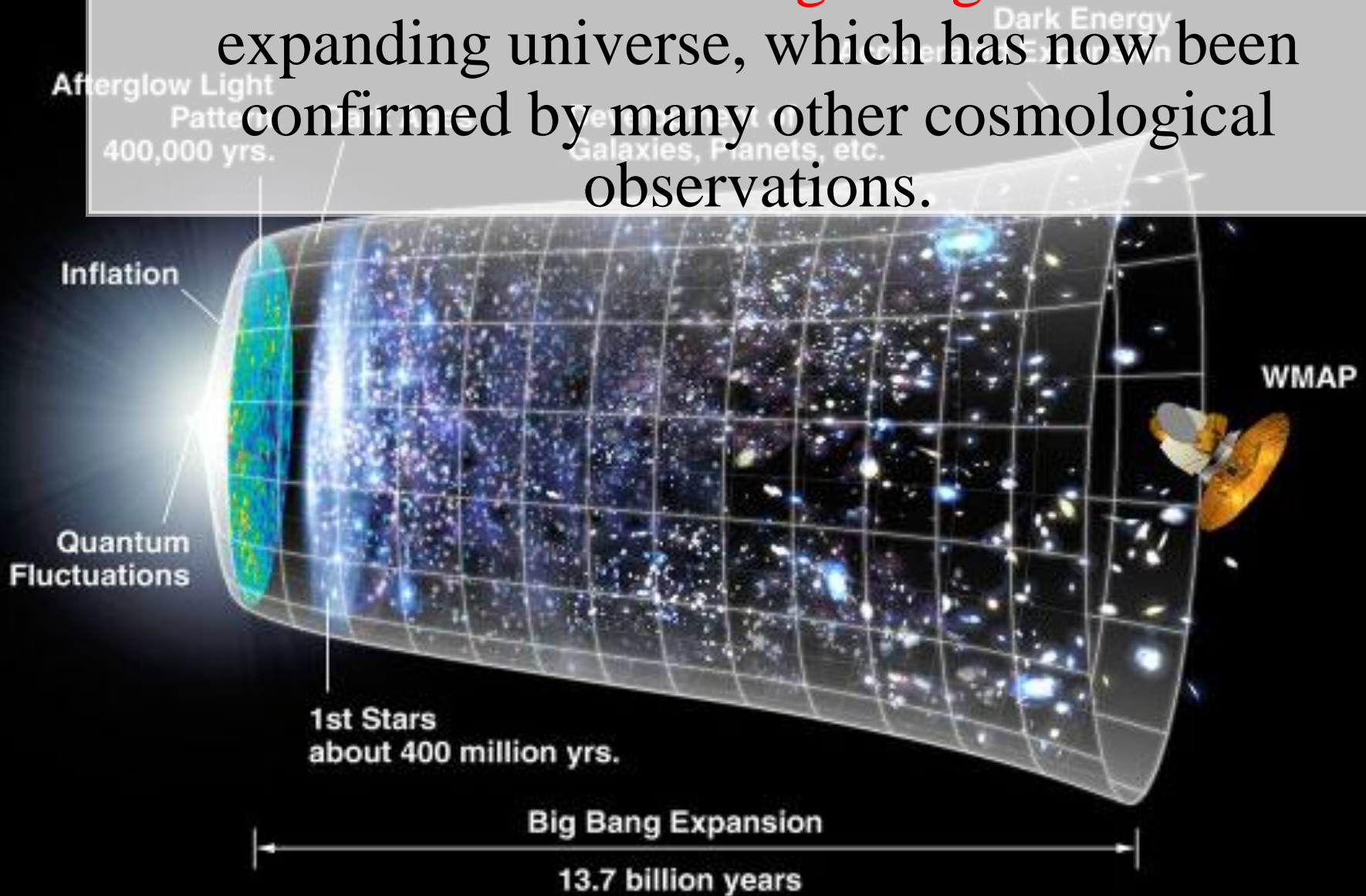
recessional
velocity

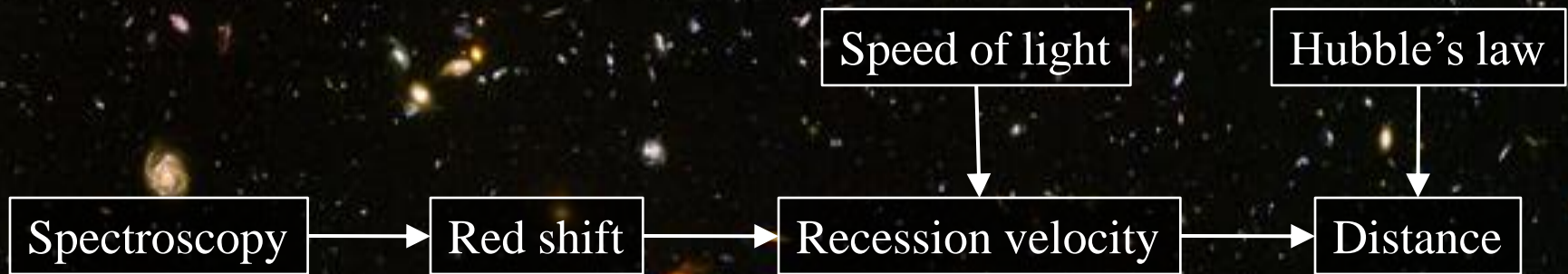


Hubble's Law
velocity
=
Hubble's constant
 \times
distance

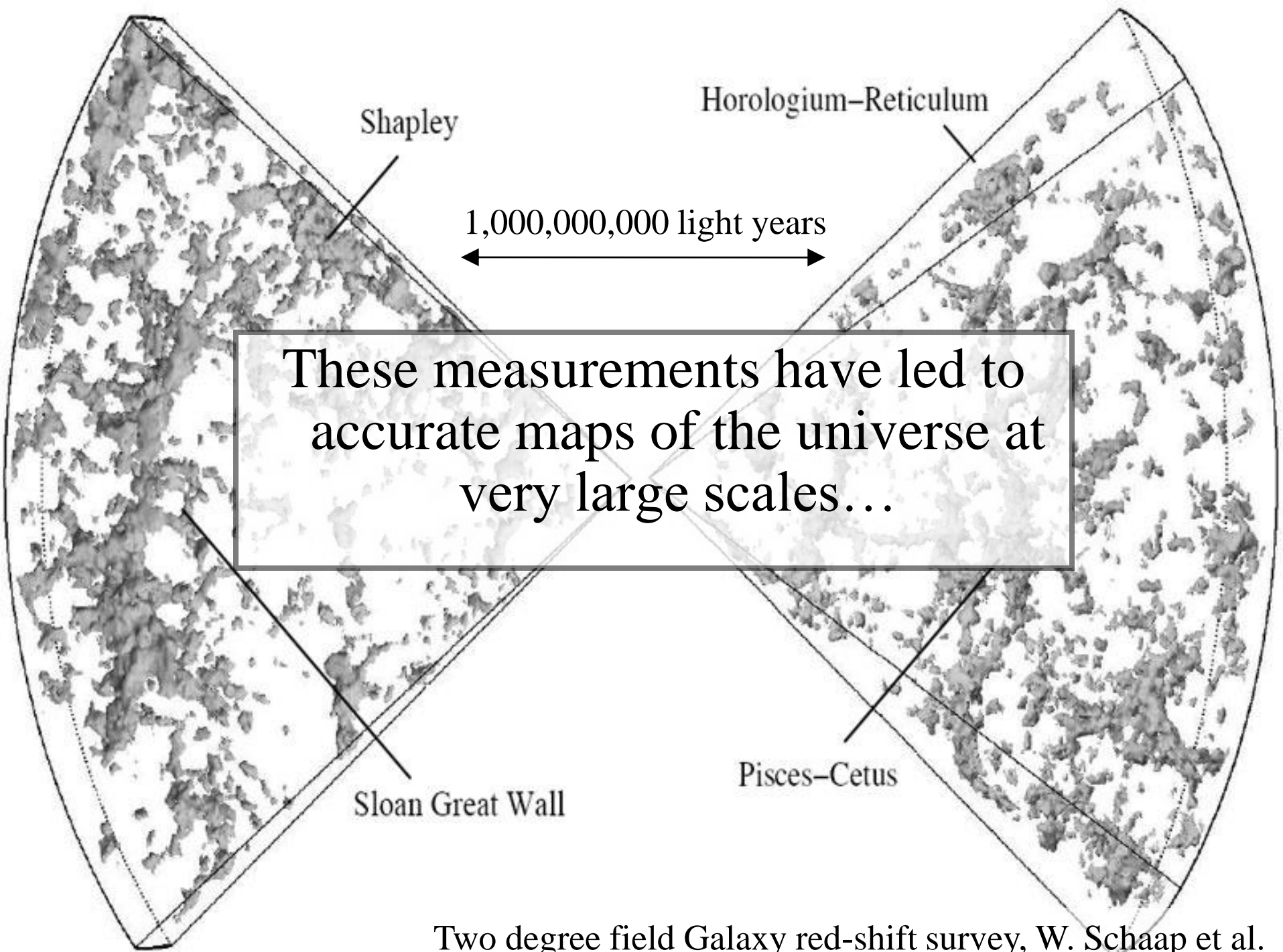
distance

This led to the famous **Big Bang** model of the expanding universe, which has now been confirmed by many other cosmological observations.

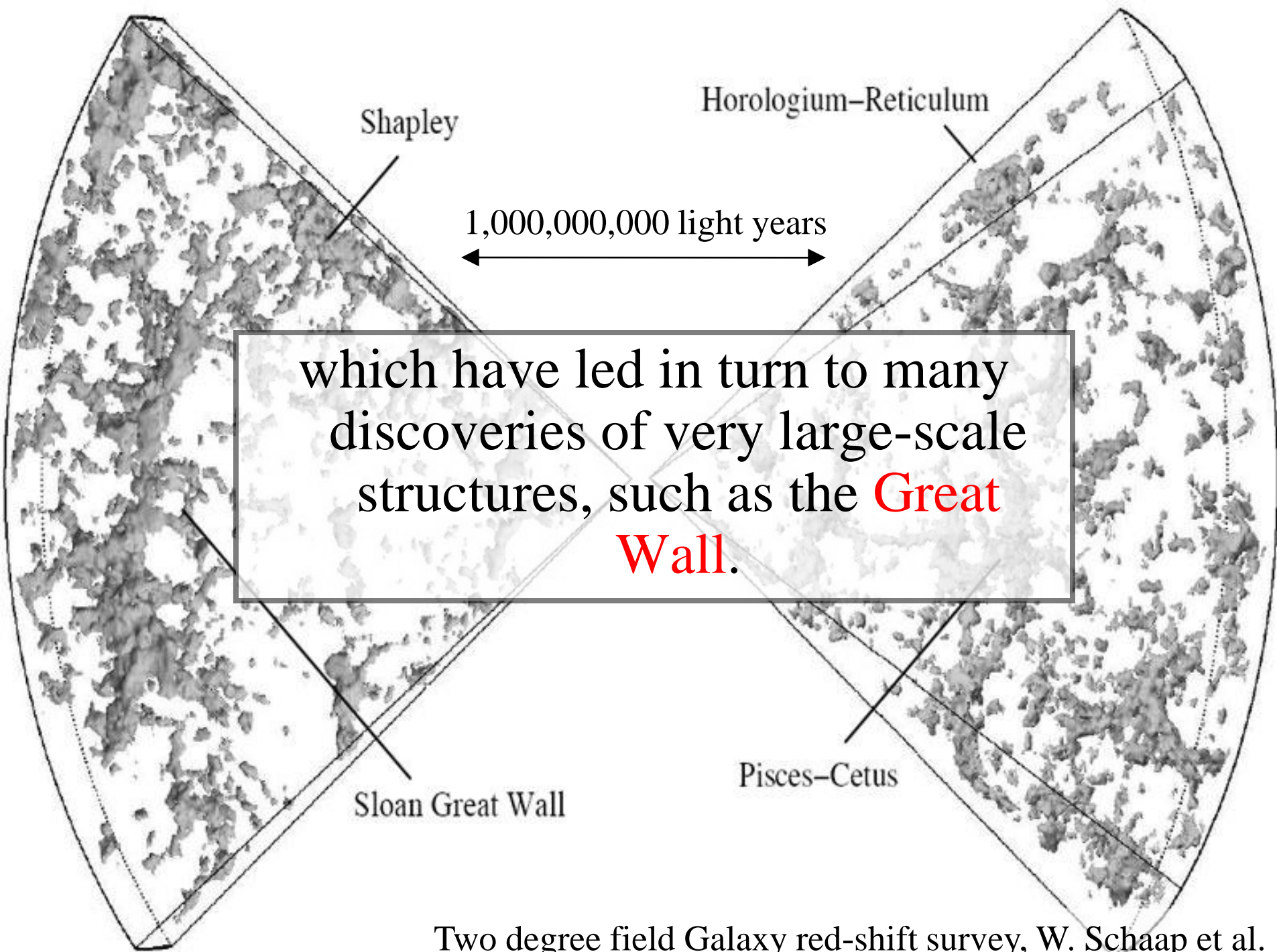




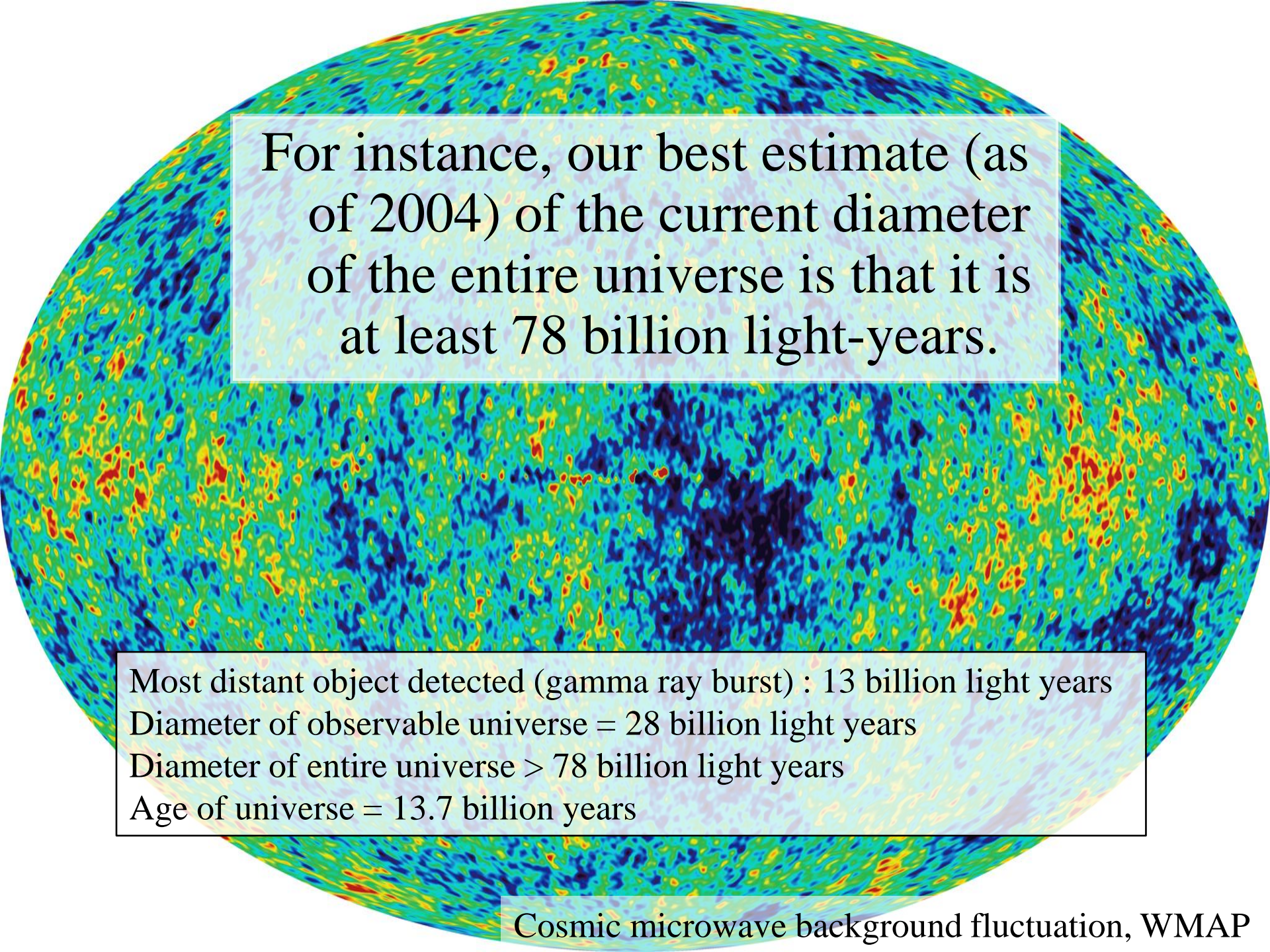
But it also gave a way to measure distances even at extremely large scales... by first measuring the red-shift and then applying Hubble's law.



Two degree field Galaxy red-shift survey, W. Schaap et al.



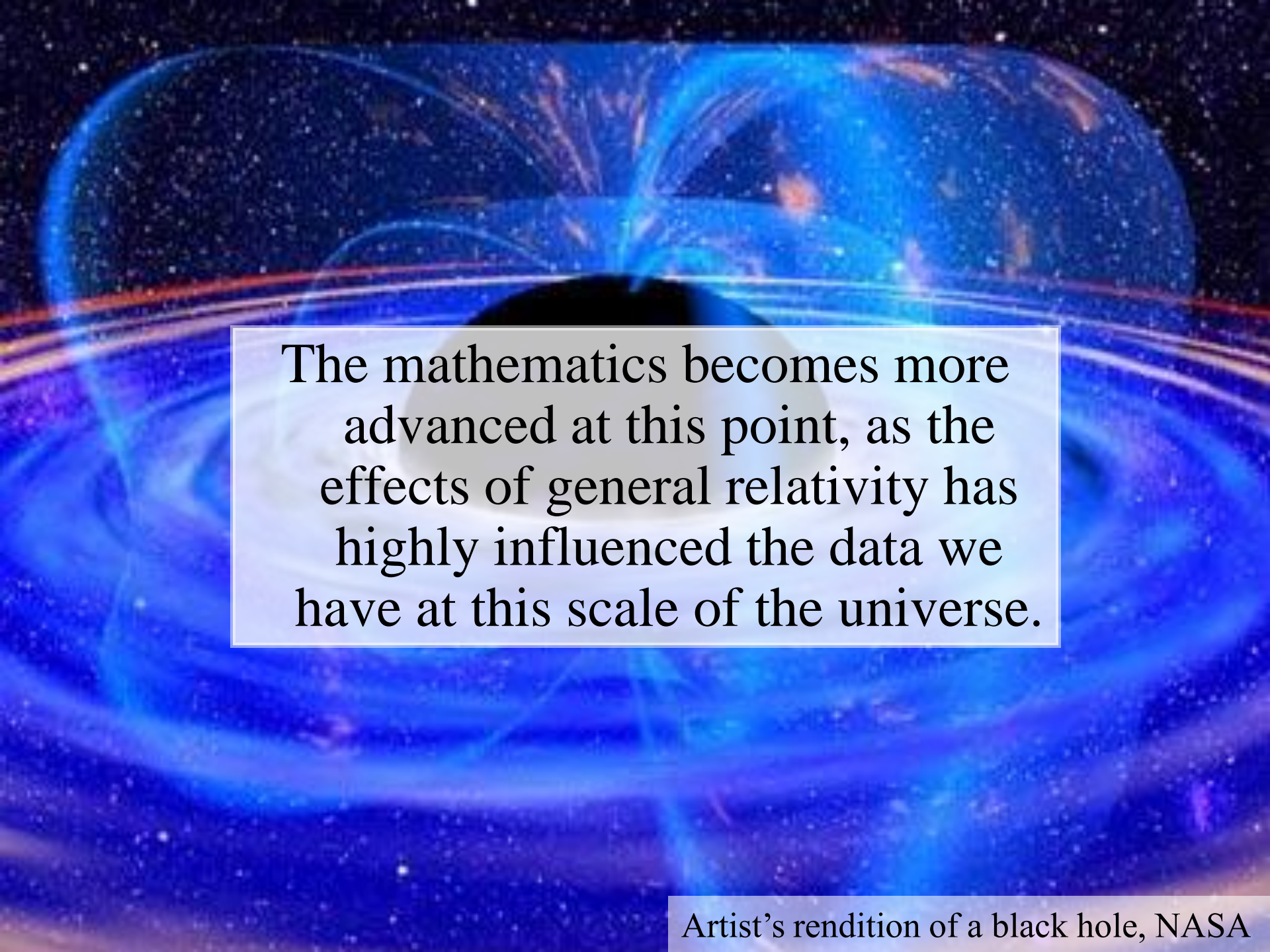
Two degree field Galaxy red-shift survey, W. Schaap et al.

A full-sky map of the Cosmic Microwave Background (CMB) temperature fluctuations, showing a complex pattern of hot (red/yellow) and cold (blue) spots. The map is presented in a circular projection, with the center of the universe at the top.

For instance, our best estimate (as of 2004) of the current diameter of the entire universe is that it is at least 78 billion light-years.

Most distant object detected (gamma ray burst) = 13 billion light years
Diameter of observable universe = 28 billion light years
Diameter of entire universe > 78 billion light years
Age of universe = 13.7 billion years

Cosmic microwave background fluctuation, WMAP

An artist's rendering of a black hole, showing a dark central region surrounded by a glowing accretion disk. The disk is composed of blue and purple gas, with bright spots and filaments. The background is a starry space. The text is overlaid on a semi-transparent white box in the center.

The mathematics becomes more advanced at this point, as the effects of general relativity has highly influenced the data we have at this scale of the universe.

Artist's rendition of a black hole, NASA

Cutting-edge technology (such as the [Hubble space telescope](#) (1990-) and [WMAP](#) (2001-2010)) has also been vital to this effort.



Hubble telescope, NASA

Climbing this rung of the ladder (i.e. mapping the universe at its very large scales) is still a very active area in astronomy today!



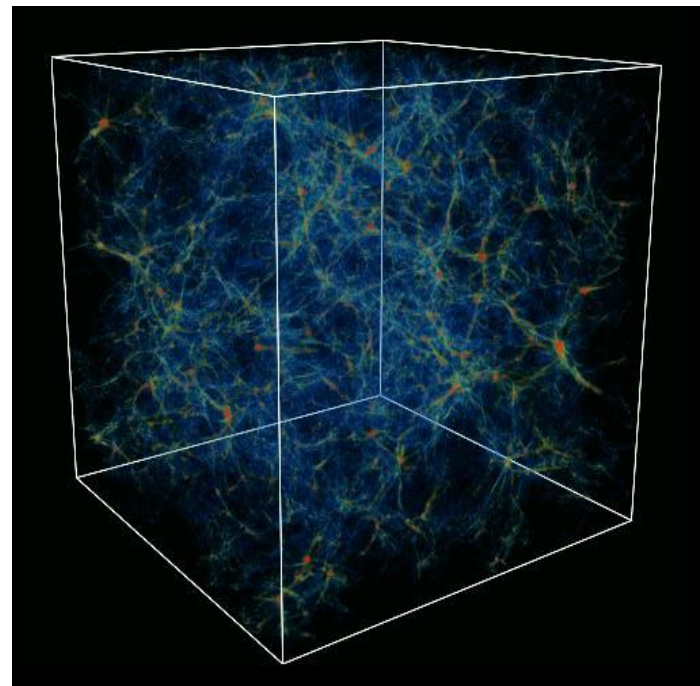
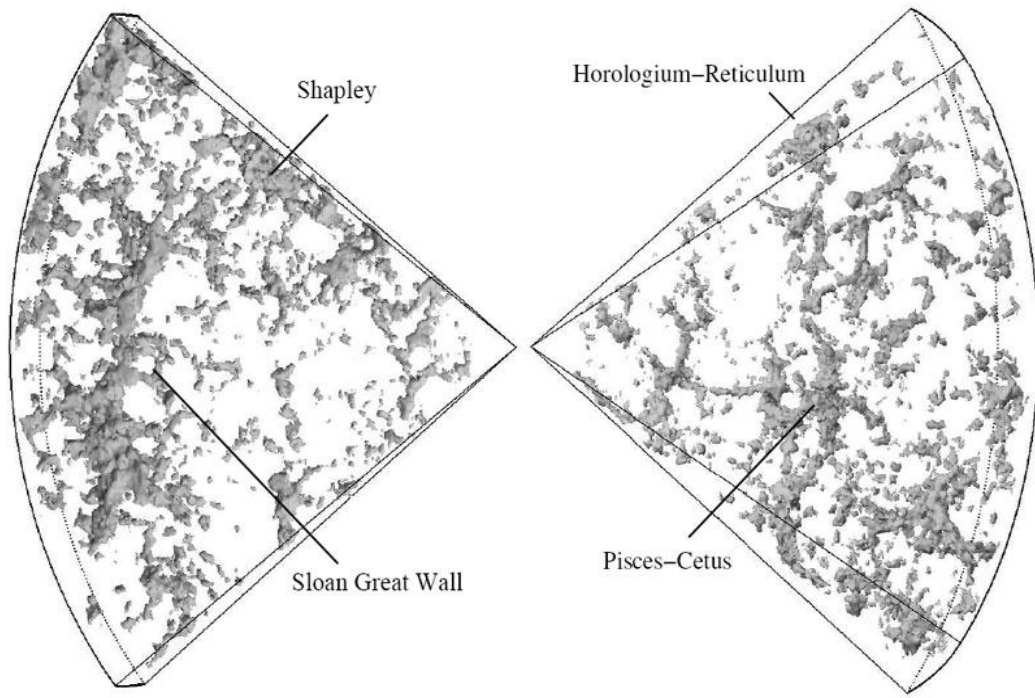
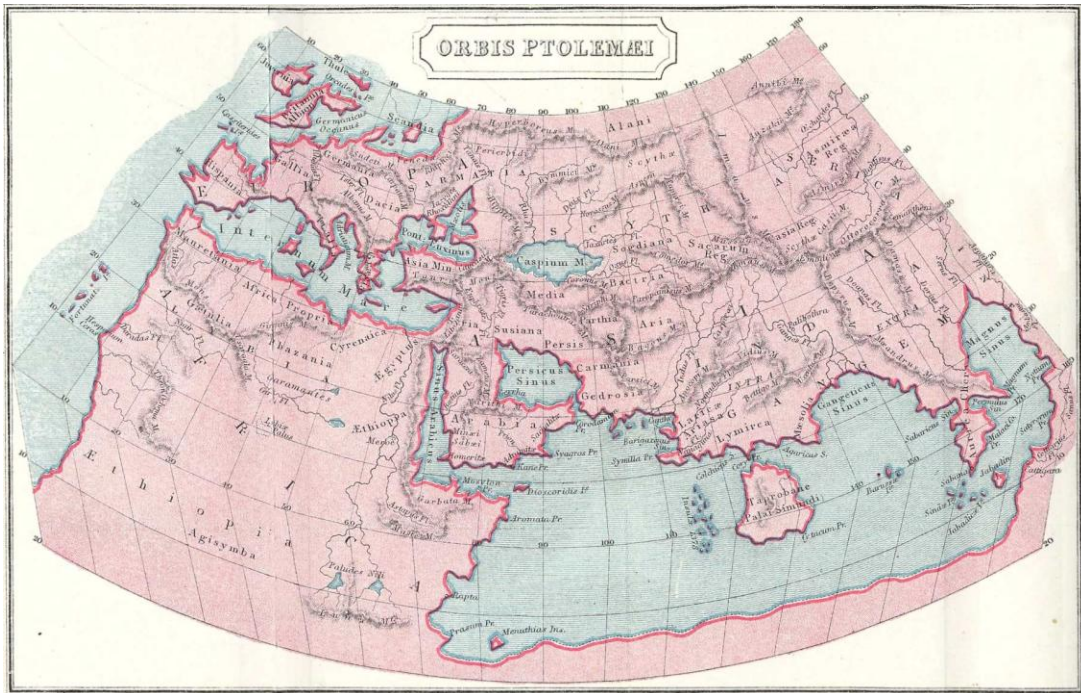


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