CHAMP 3/3/2014 – Facilitator Reference

(see also the group worksheets that will be given to the students)

Recap the end of last time and the homework puzzle:

Closing Puzzle from last time: "Suppose we all have a drink and give a toast, and everyone clinks their glass with everyone else once. How many clinks are there?"

Answer from last time: if there are n people, then there are $1 + 2 + 3 + \cdots + (n-1)$ clinks.

Homework Challenge: Can you come up with a formula for this number, so that we can just plug in the number of people n?

Opportunity for students to share answer (if anyone worked on it).

Presentation of answer:

If there are n people, there will be $\frac{(n-1)n}{2}$ clinks.

If we do this in class, we'll show that the sum of the first n integers is $\frac{n(n+1)}{2}$

We'll get a volunteer to walk to a wall using only the following rules:

- The person may only use the given move, and only once per second:
- The move is: "Walk half-way to the wall."
- The person has an unlimited number of moves.

"Can the person reach the wall using only this move?"

<u>Group work 1</u> –Decide yes or no. If yes, how many moves? If no, why not?

Suppose the person starts 1 yard from the wall.

Number of moves	Distance travelled	Distance remaining
0		
1		
2		
3		
4		

And so on…see worksheet.

We'll see where each group stands. Then, we'll show that the answer is "no."

Number of moves	Distance travelled	Distance remaining
1	½	½
2	$\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$	$\frac{1}{4}$
3	$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}=\frac{7}{8}$	$\frac{1}{8}$
4	$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}=\frac{15}{16}$	$\frac{1}{16}$
5	$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}=\frac{31}{32}$	$\frac{1}{32}$
n	$\frac{2^n-1}{2^n}$	$\frac{1}{2^n}$

Why is the answer no? Because it would take infinitely many moves, which would take infinitely many seconds!

This is one of Zeno's paradoxes, examining some of the philosophies at the time.

(Depending on discussion or questions) let's analyze this further (or) let's find the solution or a way around the paradox.

What if we try to add up the distance covered by each move?

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\cdots$$

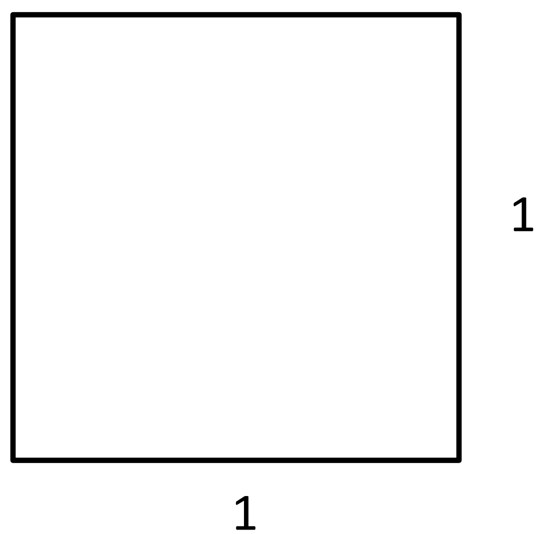
It's impossible to add up infinitely many numbers, right?

<u>Group Work 2</u> – Figure out the value of

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots$$

with the following exercise:

Start with a square that's 1 yard by 1 yard



Write down the area of the whole square:

1) Lightly shade half of the square. Label the area of shaded portion:

2) Shade half of what's not shaded. Label the area of what you just shaded:

3) Shade half of what's not shaded. Label the area of what you just shaded:

4) Repeat this process until you see what the value of $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots$ is, and try to explain why.

When talking about adding up infinitely many numbers, mathematicians call it a series instead of a sum to distinguish it. With a sum you can just figure it out, but a series takes a lot of work or a special trick.

What we just looked at is a great example of what's called a "geometric series" because a geometric argument can be used to figure it out.

Series are pretty hard to deal with. An easier way to start thinking about infinity is with an infinitely long list of numbers. These are called "sequences."

I could just list the counting numbers:

$$1,2,3,4,5,\ldots$$

The numbers in this list just keep getting bigger.

But what happens to this list:

$$1,\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{1}{5},\ldots$$

Do you see the pattern?

What happens if we keep going? Do we get close to anything?

<u>Group Work 3</u> – Try to figure out what the list of numbers "approaches." Use a calculator if you need to.

$$2,\frac{3}{2},\frac{4}{3},\frac{5}{4},\frac{6}{5},\frac{7}{6},\ldots$$

Answer: 1

$$\frac{1}{3},\frac{1}{9},\frac{1}{27},\frac{1}{81},\ldots$$

Answer: 0

$$3,\frac{5}{2},\frac{7}{3},\frac{9}{4},\frac{11}{5},\ldots$$

Answer: 2

$$0.3, 0.33, 0.333, 0.3333, 0.33333, \ldots$$

Answer: 1/3

$$3, 3.1, 3.14, 3.141, 3.1415, 3.14159, \ldots$$

Answer: π