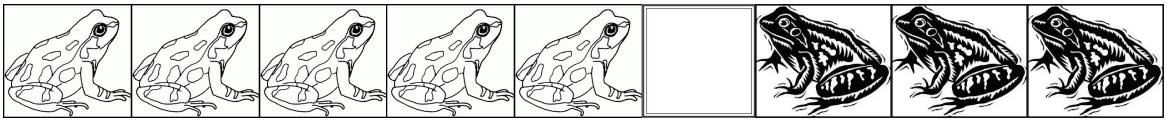
SOLUTION– A Formula for the Number of Moves

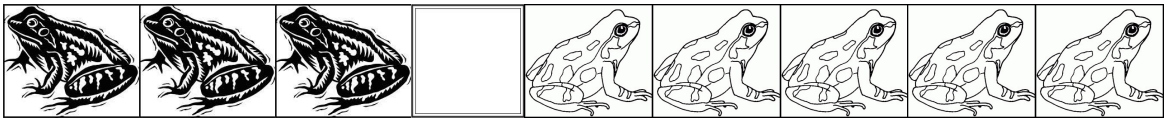
The Formula

Suppose $a =$ the number of white frogs and $b =$ the number of black frogs.



Notice that there are $a + b + 1$ squares.

The frogs move only forward (backward is not allowed) to a final position like this:



We see that each white frog has moved forward $b + 1$ spaces. Since there are a white frogs, the white frogs are moved a total of $a(b + 1)$ spaces.

Likewise, each black frog has moved forward $a + 1$ spaces. Since there are b black frogs, the black frogs are moved a total of $b(a + 1)$ spaces.

This means the total number of spaces moved is: $a(b + 1) + b(a + 1)$.

However, this is not the total number of moves, since when a jump occurs the frog moves two spaces instead of one. Thus $a(b + 1) + b(a + 1)$ overcounts the number of moves by the number of jumps.

How many jumps are there? Each time a white frog and black frog meet, one must jump the other. We don't know which jumps, but we know a jump occurs for each meeting. Since there are a white frogs and b black frogs, there are ab jumps all together.

We can subtract the number of jumps from the number of spaces moved to get the total number of moves. Thus the total number of moves is

$$a(b + 1) + b(a + 1) - ab.$$

We can also simplify this formula to obtain

$$a(b + 1) + b(a + 1) - ab = ab + a + ba + b - ab = ab + a + b.$$

Therefore, the total number of moves needed for a white frogs and b black frogs is

$$ab + a + b.$$