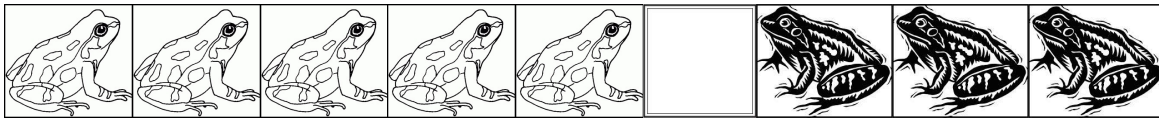


Leap Frog – A Lesson in Strategy and Counting

The Setup

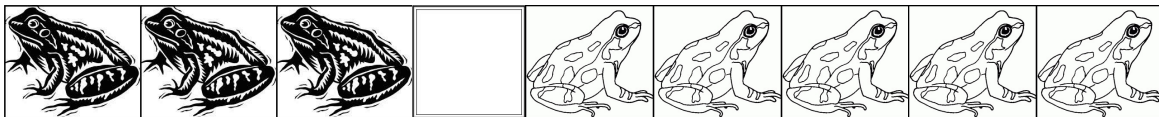
You have a row of boxes with some number of white frogs in the left boxes facing right, and some number of black frogs in the right boxes facing left. There is one box in between the frogs.

Here is an example with 5 white frogs and 3 black frogs:



Let a equal the number of white frogs and b equal the number of black frogs. Notice that there will be $a + b + 1$ total boxes. You can check that in the example above $a = 5$ and $b = 3$, and there are 9 boxes total.

The goal is to move the frogs so that all the black frogs are on the right and all white frogs are on the left. For the example above, we want to move the frogs to the following position:



Rules for Moving Frogs

On each turn you can move one frog and the color doesn't matter.

- (1) If the space in front of a frog is empty, the frog can move into that space.
- (2) If one frog is facing a second frog of another color, and the space behind that second frog is open, the first frog can jump the second frog.

Doing either (1) or (2) above constitutes one move. When you do a move of type (1) the frog moved progresses one space. When you do a move of type (2) the frog moved progresses two spaces due to the jump.

Be Careful! Frogs cannot move backward. Frogs can only jump ONE other frog at a time, and the space behind the frog that is jumped needs to be open for the jump to be allowed.

QUESTIONS

Question 1: Is it always possible to move the frogs from starting position to the ending position? Does it depend on the number of white frogs and black frogs for it to be possible?

Question 2: If you have a white frogs and b black frogs, how many moves (in terms of a and b) does it take to move from a starting position to an ending position?

Use the frogs and boxes on the next two pages to work out examples. Fill out the following chart with the number of moves needed to go from a starting position to an ending position.

Number of Moves Needed From Start to Finish

$a = \text{number of white frogs}$

		1	2	3	4	5
b = n u m b e r o f b l a c k f r o g s	1					
	2					
	3					
	4					
	5					

As you fill out the table, try to determine a formula for the number of moves needed in terms of a and b . (Hint: First fill out the diagonal of the chart, and look for a formula when $a = b$.)

FROG TOKENS TO CUT OUT

Use your scissors to cut out the frog tokens on this page. You can then use the rows of squares on the following page to try combinations of moves for various values of a and b .

