PATTERNS AND GAMES

"Last One Standing"

Four people are sitting on chairs in a line all facing the same direction, so each person can see all the people in front of them and none of the people in back of them. We start with everyone sitting, and each turn we can make a move and have one person change position (either stand up or sit down) subject to the following rules:

Rule 1: The front person is allowed to change position (stand or sit) on any turn.

Rule 2: All other people can change position (stand or sit) only if the person in front of them is standing and all other people in front of them are sitting.

Our goal is to have the back person standing and all the other people sitting. We call this a *winning position*.

QUESTION 1: What is the fewest number of moves we need to put 4 people who are all sitting into a winning position? (We'll do this problem together. Let's establish some notation and write 1 for a person standing and 0 for a person sitting.)

QUESTION 2: Get into groups and make a chart for how many moves are needed when we have n people sitting, for $n = 1, 2, 3, 4, 5$. Try to find a formula for the number of moves in terms of n.

QUESTION 3: Based on your formula from question 2, how many moves would it take if we started with 10 people sitting in a line? How many moves if we started with 15 people? How many moves if we started with 20 people?

QUESTION 4: If we did one move per second (so one person sits or one person stands each second), how long would it take to move to a winning position if we start with 4 people? If we start with 10 people? If we start with 15 people? If we start with 20 people?