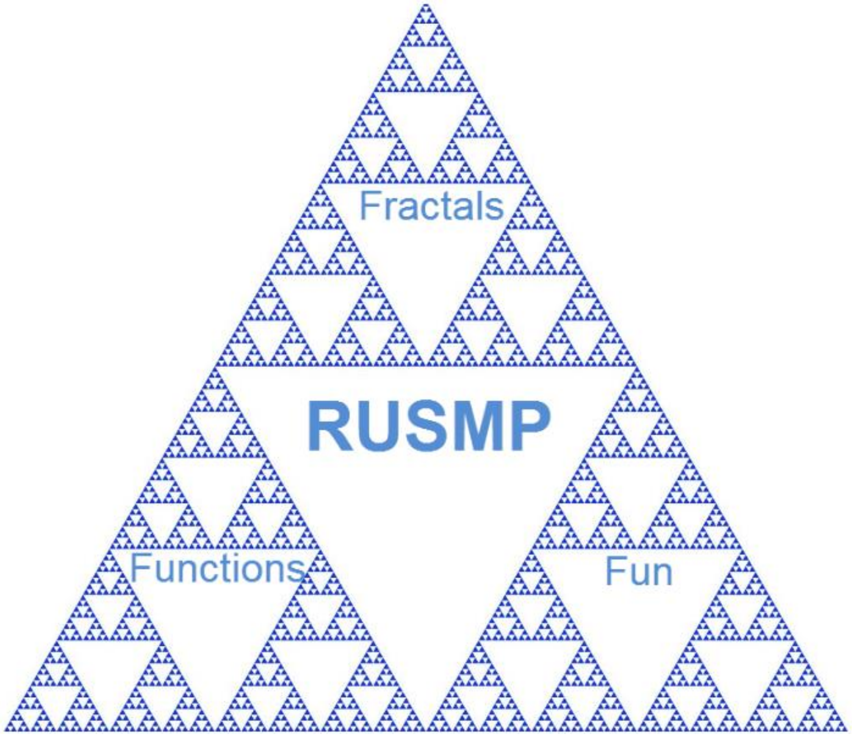
RICE UNIVERSITY
SCHOOL MATHEMATICS PROJECT

Fun with Fractals and Functions



CHAMP at University of Houston

March 2, 2015

Houston, Texas

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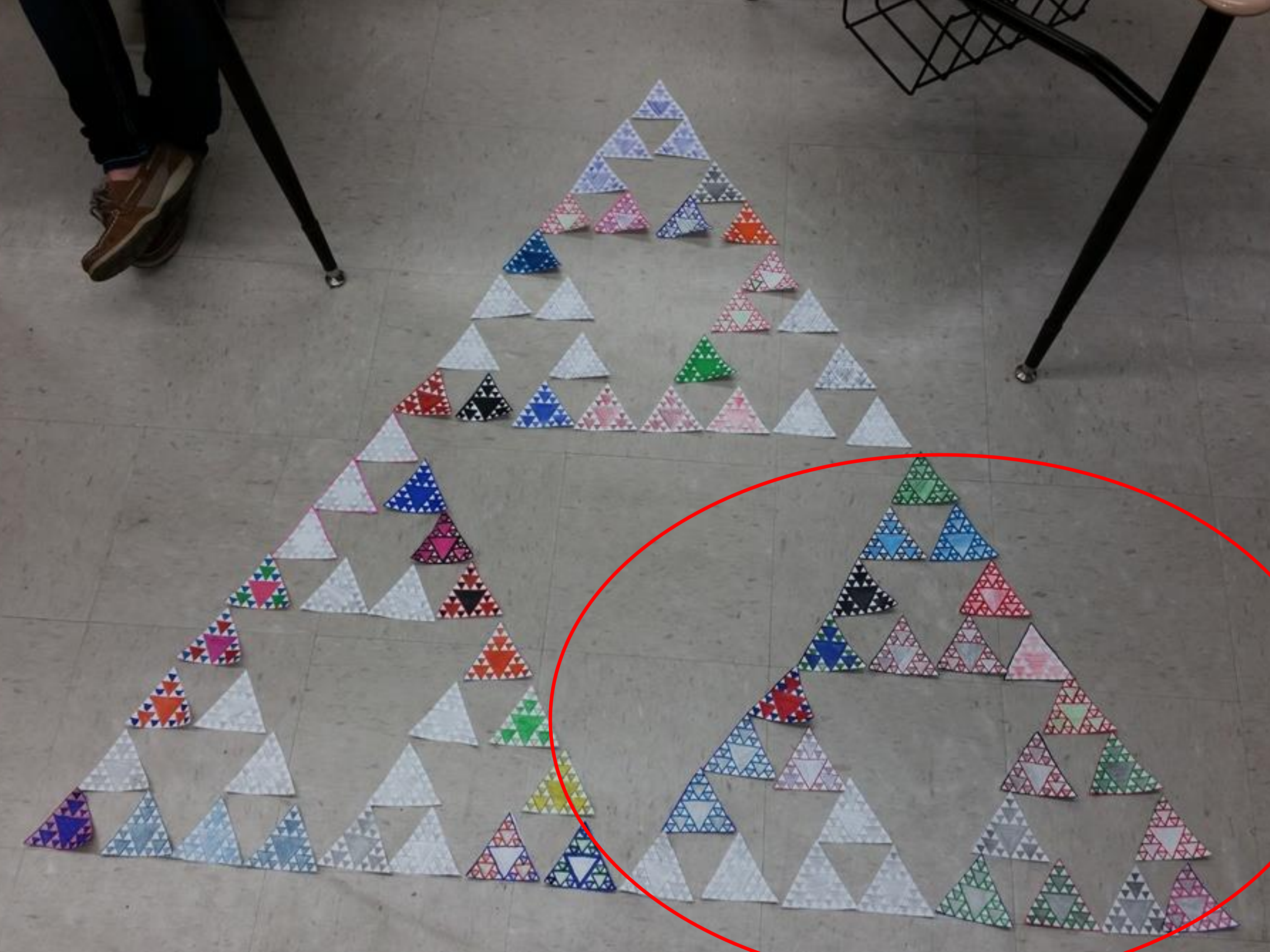
What is a fractal?

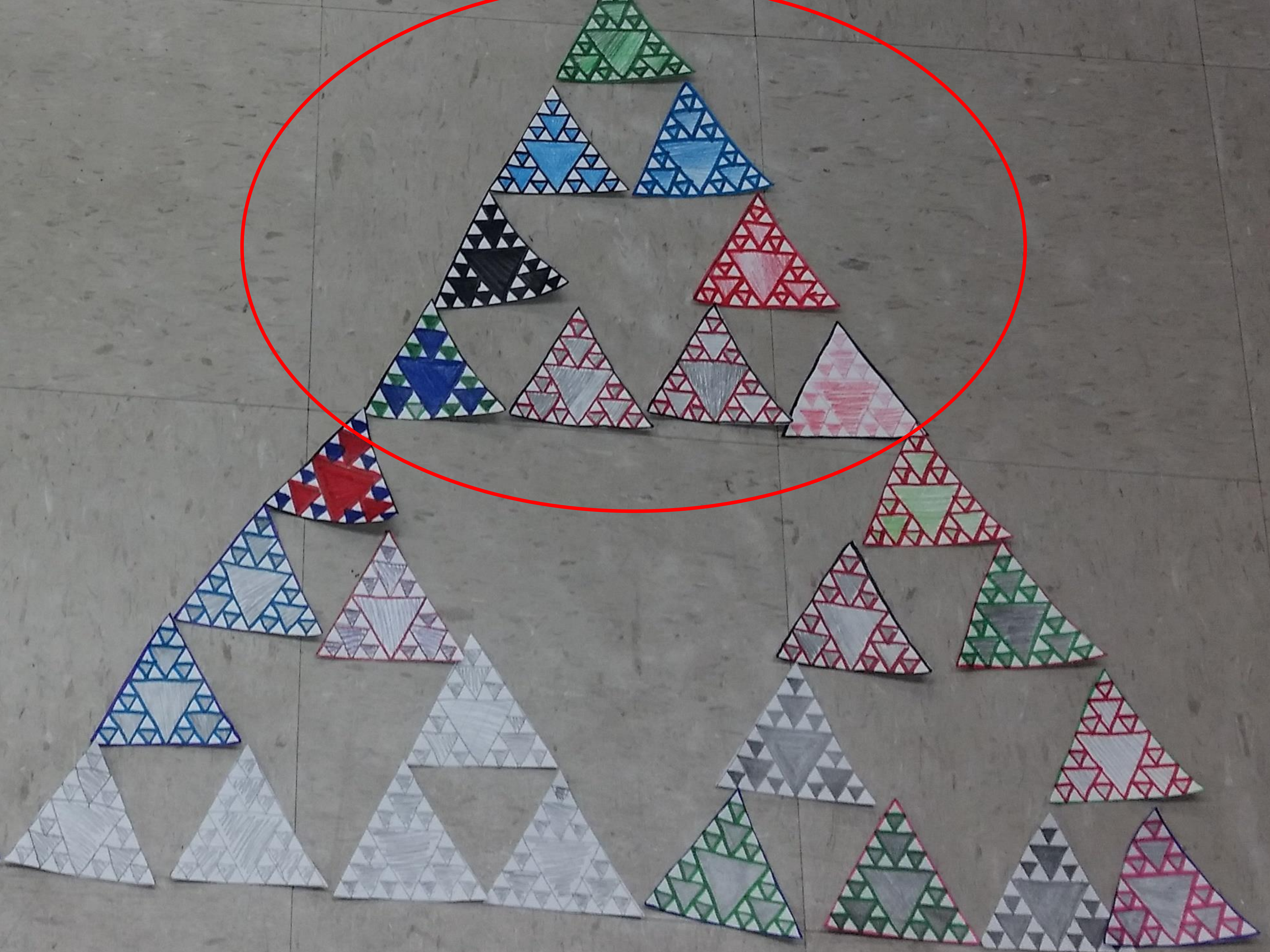
Fractal comes from the Latin word "fractus" for *broken* or *fractured* and the word was created by the mathematician Benoit Mandelbrot in 1975.

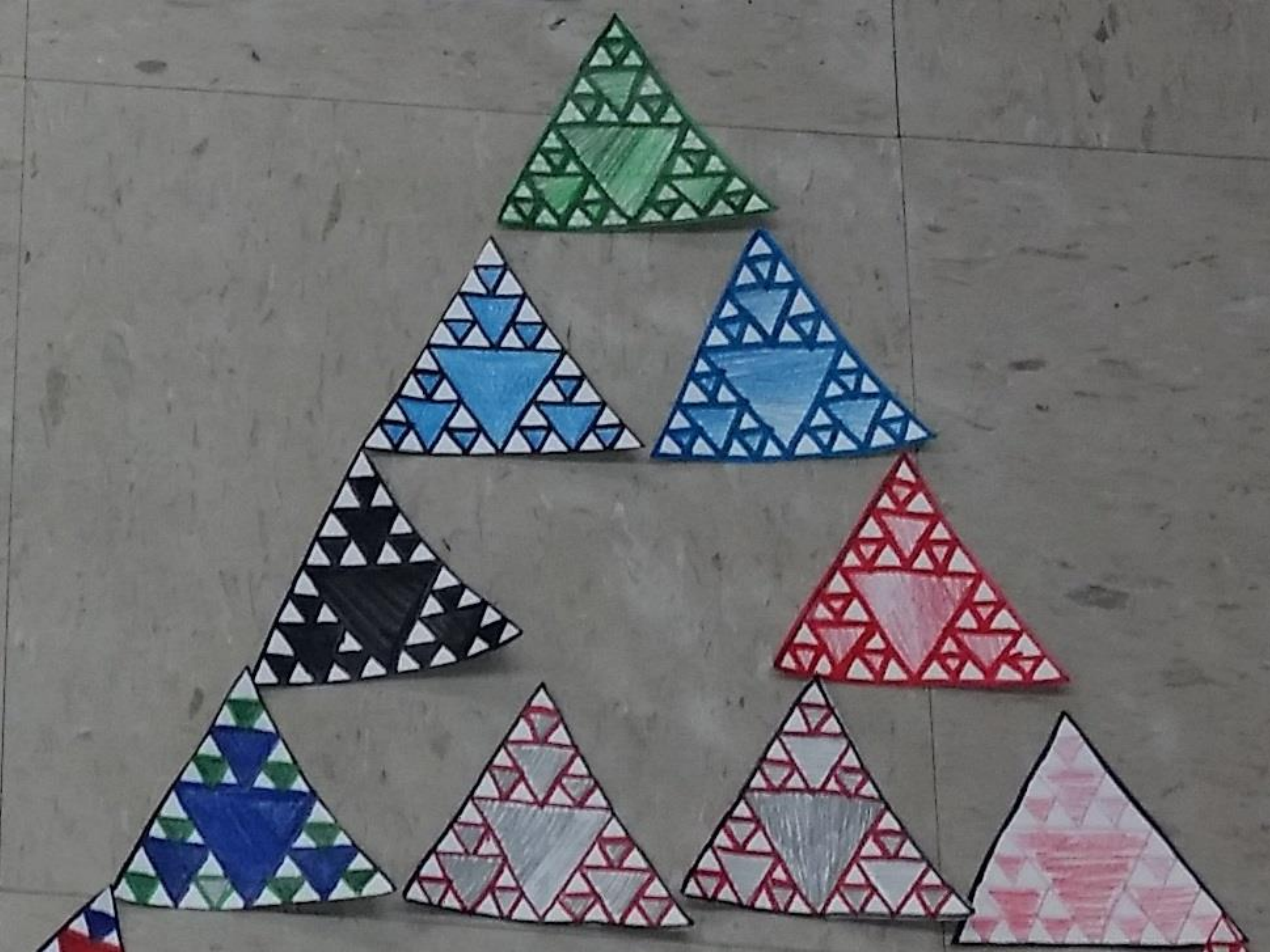
Mandelbrot showed how visual complexity can be created from simple rules.

What is a fractal?

- When viewed closely, fractals are **self-similar**. Thus, if we zoom in on a fractal, we essentially see the same shape, but at a different scale.
- Thus, smaller and smaller copies of a pattern are nested inside each other, so that the same shapes appear no matter how much you zoom in to the whole.



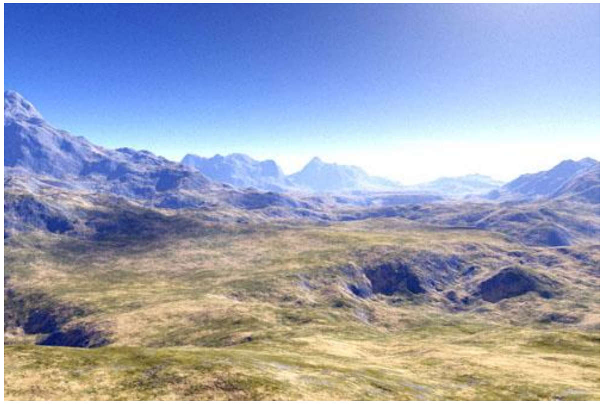


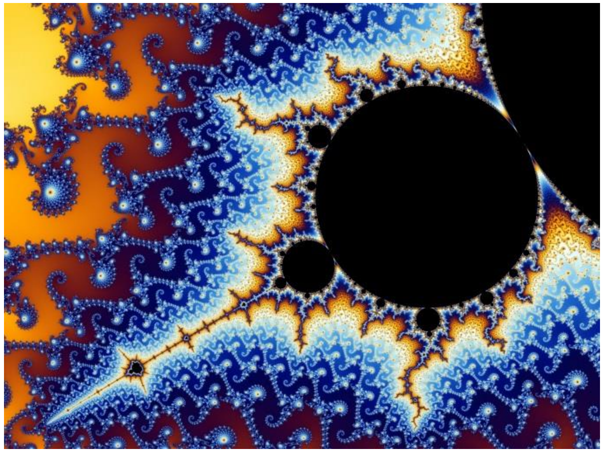


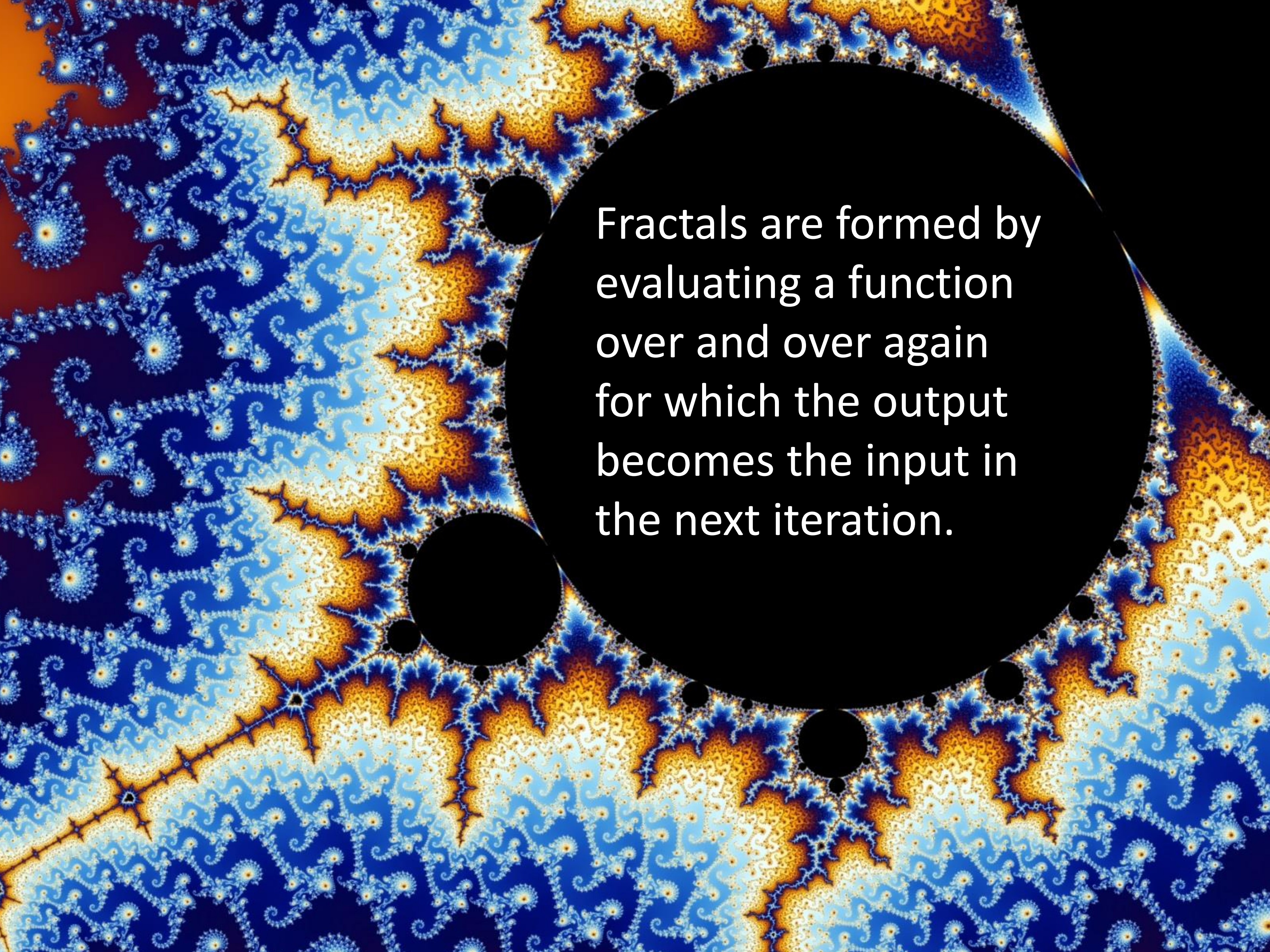
Where do we find fractals?

We find fractals in nature! Examples include **branching patterns** such as trees, river networks, blood vessels, mountains (below left) and **spiral patterns** such as seashells, hurricanes and galaxies (below right).

Fractals Created by Software






Fractals are formed by evaluating a function over and over again for which the output becomes the input in the next iteration.

What is a function?

Fractals are formed by evaluating a function over and over again for which the output becomes the input in the next iteration.

This process is called **recursion.**

Xaos Software Demo

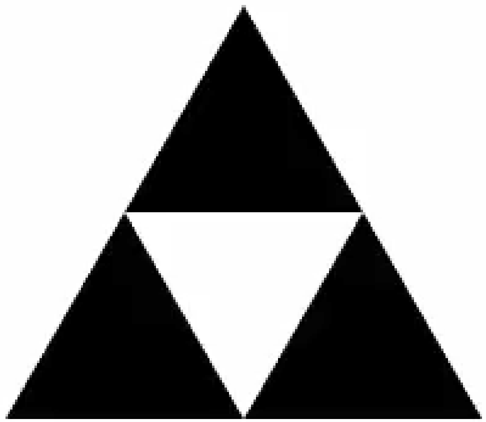
Let's zoom in!

Why learn about fractals?

- Fractals are relevant! They are found almost everywhere in nature.
- There is a lot of mathematics underlying fractals that is accessible to high school students.
- Fractals combine mathematics and art, and their beauty generated from simplicity fascinates a diverse audience.

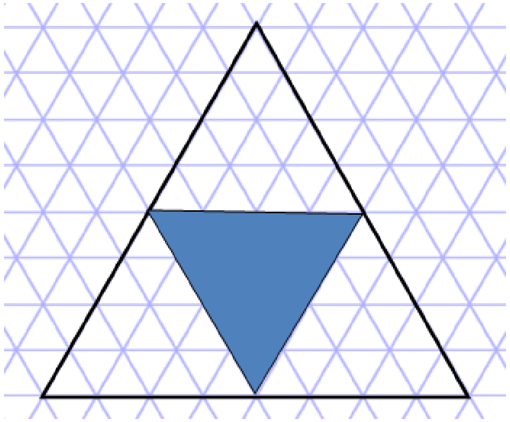
The Sierpinski Triangle

The **Sierpinski triangle** is a fractal named after the Polish mathematician, Wacław Sierpiński, who described it in 1915.

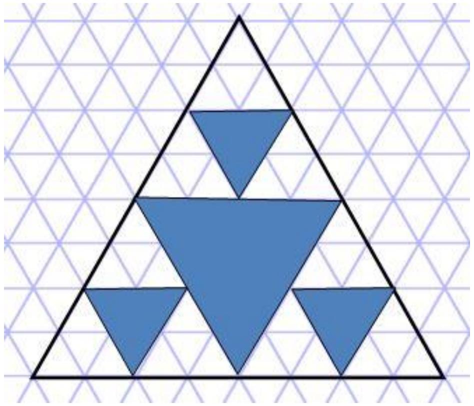


Let's create the first three iterations of the Sierpinski triangle:

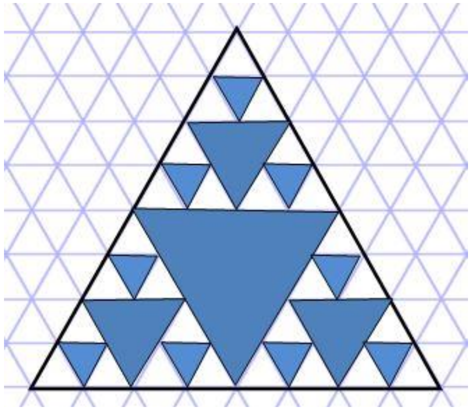
<u>Iteration 1</u>: Draw an equilateral triangle with side length of 8 units on triangular grid paper. Mark the midpoints of the three sides. Then connect the three midpoints and shade in the triangle that is pointing downward.



Iteration 2: Repeat the first iteration with a new triangle. Now mark the midpoints of the three sides of each of the three unshaded triangles. Connect the midpoints and shade the three triangles that are pointing downward.



Iteration 3: Repeat the first and second iterations with a new triangle. Now mark the midpoints of the three sides of each of the nine unshaded triangles. Connect the midpoints and shade/color the nine triangles that are pointing downward. Be creative when you shade in your triangles…use colors! Cut out the three triangles.



What do you notice?

What do you wonder?

Share with your neighbor.

We will examine **a function** derived from looking at **successive iterations** of the Sierpinski triangle:

1. The number of unshaded triangles within the Sierpinski triangle
2. The fraction of the triangle that is unshaded within the Sierpinski triangle

Fractal Trianglethon

During a fractal trianglethon, a giant Sierpinski triangle made of thousands of individual Sierpinski triangles was made by students from all over the world. The Sierpinski triangle on the next slide was composed of 6561 triangles!

Watch how this giant fractal was made…

Fractal Trianglethon 2011

Albuquerque, New Mexico

Introducing the Chaos Game

In 1989, the term “Chaos Game” was coined by Michael Barnsley who developed this technique.

How do you play the Chaos Game?

1. Use the triangle handout and work in pairs.
2. Pick a point inside the triangle to begin the game. This point is called the seed.
3. Each vertex of the triangle has been labeled with two numbers as follows:

   Top vertex: 1, 2
   Bottom left vertex: 3, 4
   Bottom right vertex: 5, 6.

How do you play the Chaos Game?

4. One person rolls the die and the other person plots the next point half the distance from the seed to the vertex that corresponds to the result of the rolled die.
5. Roll the die again, and then plot the next point half the distance from the last plotted point and the vertex that corresponds to the result of the rolled die.
6. Plot 5-10 points on the triangle.

What do you predict you will see after
100 iterations?
1,000 iterations?

Why?

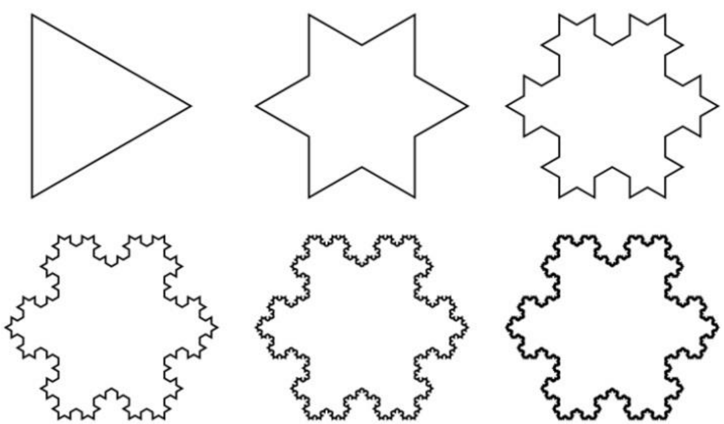
Let's try it!

The Chaos Game on the TI-Nspire

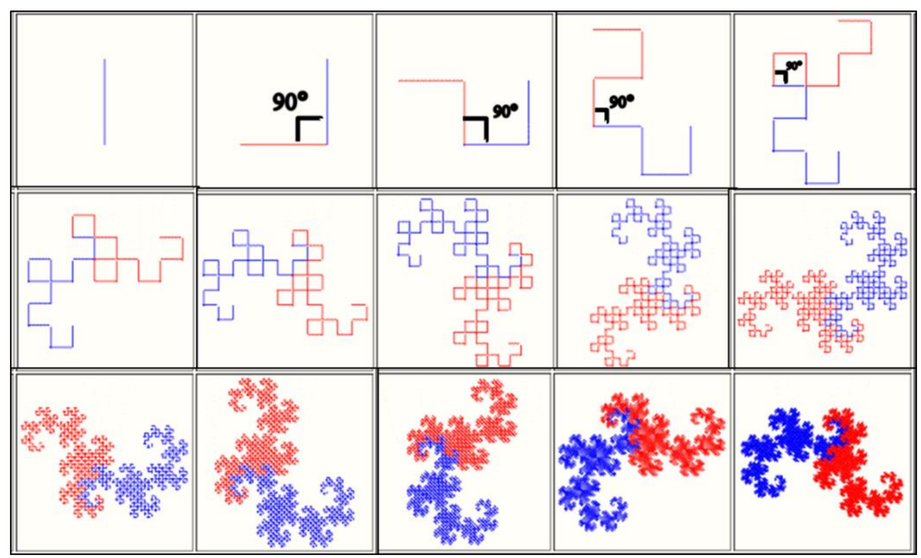
From *www.johnhanna.us/TI-nspire.htm*

Other Fun Fractals!

Koch snowflake



Jurassic Park fractal



Finding the Sierpinski triangle in surprising places!
(iOS app: Geom-e-Tree)