CHAMP: Reasoning Challenges

Exercise 1. Card game!

A casino offers a card game using a normal deck of 52 cards. The rule is that you turn over two cards each time. For each pair, if both are black, they go to the dealer's pile; if both are red, they go to your pile; if one is black and one is red, they are discarded. The process repeats until you go through all 52 cards. If you have more cards in your pile, you win the not-yet-released Beyoncé album; otherwise (including ties) you get nothing.

What are the chances that you win the Beyoncé album?

Facilitators. The key to this problem is to think about the final results of the game rather than the probabilities associated with each individual round. When the game is complete, the number of discarded cards is equal to $2k$ for some nonnegative integer k. Exactly k of the discarded cards are red and exactly k of them are black. It follows that both the player and the dealer have exactly $26 - k$ cards, meaning that the game is always a tie!

Exercise 2. Birthdays!

What are the chances that two of the people in our classroom share the same birthday?

Facilitators. The key here is to compute the probability that everyone has a different birthday and then notice that this probability decreases surprisingly quickly as a function of the number of people in the room.

Exercise 3. Tigers and a gazelle!

One hundred tigers and one gazelle are put on a grass-covered magic island. The animals on the island follow these rules.

(a) Tigers can eat grass, but they would rather eat gazelles.
(b) At any given time, only one tiger can eat one gazelle. If a tiger eats a gazelle, that tiger then *turns into a gazelle*.
(c) All tigers are smart, perfectly rational, and want to live.

Will the gazelle be eaten?

Facilitators. Let's have the students approach this problem inductively. If we start with only one tiger, the tiger will eat the gazelle and survive (as a gazelle). If we start with two tigers, the gazelle will not be eaten. This is so because if a tiger eats the gazelle, that tiger, now a gazelle, will be eaten by the other tiger. If we start with three tigers, the gazelle will be eaten. This is so because once the gazelle is eaten, the island will contain two tigers and one gazelle; we know that everyone survives in this scenario. In general, the gazelle will be eaten if we start with an **odd** number of tigers and the gazelle will not be eaten if we start with an **even** number of tigers.