

Is There Just One Geometry?

Warm up Activity

What are some of the major differences between circles and lines when they are on a plane and when they are on a sphere? List as many differences as you can:

Use the globe (or picture of the globe) to draw the shortest path between Los Angeles and Dubai (or Houston and New Delhi) on the flat map below. What are some issues with trying to make maps of the earth? Also, which do you think is bigger: Greenland or Africa?



Activity 1

Directions: Each table is given a different postulate. You will work together with your table to answer the following questions about the postulate you were given.

Does the statement hold in planar geometry?

Does the statement hold on the surface of a sphere?

If the statement doesn't hold on the sphere, what goes wrong? Can the statement be reworded easily so that it will hold? If the statement does hold on the sphere, explain why.

Activity 2

What is the sum of the interior angles of a triangle in the plane?

Is it possible to make a triangle with that angle sum on a sphere?

Construct a triangle with three right angles on the sphere. What happens to the area when you change one of the angles? (See the activity on the next page)

Create triangles of the given size by adjusting *one of the angles of your 90-90-90 triangle*, and compute their angle sum. **Note:** The first row in the table corresponds to your original triangle.

| Fraction of Total Area | Angle Sum |
|--|-----------|
| $\frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$ | |
| $\frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12}$ | |
| $\frac{1}{4} \cdot \frac{2}{3} = \frac{1}{6}$ | |
| $\frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$ | |
| $\frac{1}{4} \cdot \frac{3}{4} = \frac{3}{16}$ | |
| $\frac{1}{4} \cdot \frac{1}{6} = \frac{1}{24}$ | |
| $\frac{1}{4} \cdot \frac{5}{6} = \frac{5}{24}$ | |

Use this table to verify a few cases of the formula:

$$\text{Angle Sum} = 180^\circ(1 + 4f),$$

where f is the fraction of the total surface area of the sphere occupied by the triangle.