TRAILING ZEROES IN A FACTORIAL

1. What is Factorial?

Definition 1.1. The ***factorial*** of a natural number n is the product of all positive integers less than or equal to itself. It is denoted by $n!$.

Example 1.2.

$$3! = 1 \times 2 \times 3 = 6.$$

$$4! = 1 \times 2 \times 3 \times 4 = 24.$$

$$5! = 1 \times 2 \times 3 \times 4 \times 5 = 120.$$

$$6! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720.$$

$$7! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 = 5040.$$

$$10! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 = ?$$

Problem 1.3. *Can you write* $7!$ *in terms of* $4!$*? If yes, then what is it?*

Problem 1.4. *Can you write* $7!$ *in terms of* $6!$*? If yes, then what is it?*

2. Trailing zeros in a factorial

Definition 2.1. Trailing zeros in a factorial is the total number of zeros at the end of the multiplied-out factorial.

Example 2.2.

(1) The number of trailing zeros in the expansion of 6! is one.

(2) The number of trailing zeros in the expansion of 7! is one.

(3) The number of trailing zeros in the expansion of 10! is two.

3. First Problem

Problem 3.1. ***Find the number of trailing zeroes in the expansion of 16!.***

Remark 3.2. If I try to plug this into my calculator, I'll get something in scientific notation, because the answer is too big for the calculator to display. In practical terms, the calculator will show me the beginning of the number, and I'm only caring about the end of the number (the "trailing" part). So the calculator won't help.

Problem 3.3. ***Find the number of trailing zeroes in the expansion of 100!.***

Problem 3.4. ***Find the number of trailing zeroes in the expansion of 1000!.***

4. Some More Problems

Problem 4.1. *What is the maximum power of* 7 *in* 101!*?*

Problem 4.2. *What is the maximum power of* 7 *in* 1000!*?*

Remark 4.3. The algorithm to solve such problems is as follows;
Take the number that you've been given the factorial of.
Divide by 5; if you get a decimal, truncate to a whole number.
Divide by $5^2 = 25$; if you get a decimal, truncate to a whole number.
Divide by $5^3 = 125$; if you get a decimal, truncate to a whole number.
Continue with ever-higher powers of 5, until your division results in a number less than 1. Once the division is less than 1, stop. Sum all the whole numbers you got in your divisions. This is the number of trailing zeroes.