KNIGHTS AND KNAVES — SOLUTIONS

On a certain island there are only two types of people: Knights and Knaves. Every person on the island is either is a Knight or a Knave, an no one is both. When the people on the island speak, the following rules hold:

- Knights always tell the truth.
- Knaves always lie.

Problem 1: There are two native islanders, named Alice and Bob, standing next to each other. You do not know what type either of them is. Suddenly, Alice says "At least one of us is a Knave."

What are Alice and Bob?

Solution: Consider four possible cases, as shown in the four rows of this table.

Alice	Bob
Knight	Knight
Knight	Knave
Knave	Knight
Knave	Knave

Considering each of the four cases, we see that Alice's statement is False in Row 1 and True in Rows 2, 3, and 4. If Alice is a Knight, what she says must be True, which means Row 1 cannot occur, but Row 2 can. If Alice is a Knave, what she says must be False, meaning neither Row 3 nor Row 4 can occur. Thus the only possibility is Row 2, and hence Alice is a Knight and Bob is a Knave.

Alternate Solution: Since Alice says "At least one of us are Knaves", her statement is true if she is a Knave. However, if she is a Knave, she must be lying. Thus Alice cannot be a Knave, and hence Alice is a Knight. Since Alice is a Knight, her statement is true, and since Alice is not a Knave, we must have that Bob is a Knave.

Problem 2: Again, there are two native islanders standing next to each other. One is named Claire and the other is named Desmond. You do not know what type either of them are. Claire suddenly says, "We are both Knights". After this, Desmond says "Either Claire is a Knight or I am a Knight, but we are not both Knights."

What is Claire?

Is it possible to determine what Desmond is?

Solution: Again, consider four cases.

Claire	Desmond
Knight	Knight
Knight	Knave
Knave	Knight
Knave	Knave

Claire's statement is True in Row 1, and False in Rows 2, 3, and 4. If Claire is a Knight, what she say must be true, which means that Row 2 cannot occur. Hence we must be in the situation of Row 1 or Row 3 or Row 4. In Row 1, Desmond is a Knight and must tell the truth; but since Desmond's statement is False in Row 1. Thus Row 1 cannot occur, and we are either in the situation of Row 3 or Row 4. In Row 3, Desmond is a Knight and his statement is true; so there is no problem and Row 3 is possible. In Row 4, Desmond is a Knave and his statement is false; so there is no problem and Row 4 is possible. Thus either Row 3 or Row 4 occurs (but we don't know which). This means Claire must be a Knave. However, we have no idea what Desmond is, and he could be either a Knight or a Knave.

Alternate Solution: Claire cannot be a Knight, because if she were her statement would be true, implying Desmond is a Knight and his statement (that they are not both Knights) is False, which is a contradiction since Knights cannot lie. Thus Claire must be a Knave. Once can then check, Desmond's statement is True if he is a Knight and false if he is a Knave; so Desmond could be either.

Problem 3: There are three native islanders, named Elena, Fernando, and Gary, standing together. You ask Elena, "How many knights are among you?", and Elena answered but you couldn't quite hear what she said. You then ask Fernando, "What did Elena say?", and Fernando replies, "Elena said there is one knight among us." At this point, Gary says "Don't believe Fernando; he is lying."

Is it possible to determine what Elena is?

Is it possible to determine what Fernando is?

Is it possible to determine what Gary is?

Solution: In this case, we need to consider 2 possibilities for each person, giving us 8 total cases to consider.

Elena	Fernando	Gary
Knight	Knight	Knight
Knight	Knight	Knave
Knight	Knave	Knight
Knight	Knave	Knave
Knave	Knight	Knight
Knave	Knight	Knave
Knave	Knave	Knight
Knave	Knave	Knave

If Elena is a Knight, she must have answered with the correct number of Knights (e.g., in Row 1 she would say "Three", and in Row 2 she would say "Two"). If Elena is a Knave she must have answered with an incorrect number of Knights (e.g., in Row 5, she must have said a number other than "Two").

If Fernando is a Knight, then we must be in Row 1, 2, 5, or 6. Furthermore, if Fernando is a Knight, he is telling the truth, and Elena must have indeed said there is One Knight. Row 1 cannot happen, because then Elena is a Knight who is lying. Similarly, Row 2 cannot happen, because then Elena is again a Knight telling a lie. In Row 5, Elena would be a Knave and lying (which is fine), but Gary would be a Knight and he is saying that Fernando (who is also a Knight) is lying, which is impossible; thus Row 5 cannot occur. In addition, Row 6 cannot happen since then Elena would be a Knave who is telling the truth. We have just argued that Rows 1, 2, 5, and 6 are all impossible. Thus we must be in Rows 3, 4, 7, or 8, and Fernando must be a Knave.

Consider Rows 3, 4, 7, and 8. In all these rows, Fernando is lying, which means Elena must have said that number of Knights is something other than One. In Row 3, Elena is a Knight and thus must have said there are Two

Knights, Fernando is lying (since he is a Knave), and Gary is truthful when he says Fernando is lying (which is consistent with Gary being a Knight). Thus Row 3 is possible.

In Row 4, Elena is a Knight, which means she truthfully said there is One Knight, and that means Fernando is telling the truth, which contradicts that he is a Knave. Thus Row 4 cannot occur.

In Row 7, Elena is a Knave, which means she lied about the number of Knights and said a number other than One. Fernando is a Knave, and his statement is a lie (which is consistent). In addition, Gary is a Knight, and his statement that Fernando is lying is true (which is also consistent). Thus Row 7 is possible.

In Row 8, Fernando is a Knave, so his statement must be a lie. In addition, Gary is a Knave, but his statement that Fernando is lying is true. Thus Row 8 cannot occur.

We see from above that either Row 3 or Row 7 is possible, and all other rows are impossible. In Rows 3 and 7 we have that Fernando is a Knave and Gary is a Knight. However, we cannot determine what Elena is.

Alternate Solution: There are two possibilities for Fernando: either he is a Knight or he is a Knave. If Fernando is a Knight, then he is telling the truth and Gary is lying, so Gary is a Knave. However, since Fernando is a Knight, that means Elena did indeed say there is One Knight among them, which is false is Elena is a Knight (contradiction) and True if Elena is a Knave (contradiction). Thus we cannot have Fernando be a Knight.

Therefore Fernando is a Knave, which means Gary is telling the truth, and hence Gary is a Knight. One can then check its consistent for Elena to be either a Knight or Knave.

Problem 4: You travel along a road that comes to a fork producing a path to the right and a path to the left. You know that one path leads to Death and the other path leads to Freedom, but you have no idea which is which. You can only choose one path to follow. Fortunately, at the fork in the road are two native islanders, named Horace and Ingrid. You know that one of Horace and Ingrid is a Knight and the other is a Knave, but you don't know which is which. You are allowed to ask only one question. You can ask either Horace or Ingrid (but not both), and the person you ask will answer you.

How can you ask a question that will allow you to determine the path to Freedom?

Note to Facilitators: Give the students a chance to think about this for a while, and poke holes in their initial attempts. If the students try for a while and seem to be stuck or frustrated, try giving them this hint.

Hint: Think of a question that each person will answer the same regardless of whether they are a Knight (telling the truth) or a Knave (telling a lie).

Solution: Pick either person (it doesn't matter which) and ask them

"Which direction would the other person tell me to go to reach Freedom?"

If the person you ask is a Knight, then the other person is a Knave. The Knave would lie and tell you the opposite direction of Freedom, and the Knight will then tell you what the Knave would say, thus giving you the opposite direction of Freedom.

If the person you ask is a Knave, then the other person is a Knight. The Knight would tell you the correct direction to Freedom, but then the Knave you are asking will lie and tell you the opposite direction of Freedom.

In either case, the answer you get is the opposite direction of Freedom. So, you should ask one of the people this question, and then go in the direction opposite to the answer they give you.