

1. A few (literally) words about paradoxes [1 min]
 - a. A paradox is a seemingly contradictory conclusion of a logical argument. In mathematics, it's useful to look at paradoxes so we can understand counter-intuitive concepts and (sometimes) discover flaws in the foundation of our reasoning.
 - b. Let's start with a game called Prisoner's Dilemma.
2. Prisoner's Dilemma [30-40 minutes]
 - a. Draw the payoff matrix
 - b. Describe the situation
 - c. Pair students up, distribute a red and a black card to each, have them play 6 rounds of PD and record their results on the board
 - d. Discuss the results and the strategies students used
 - e. Discussion question: what is paradoxical about PD?
 - f. [Optional activity, if time allows: mix pairs, and have students play PD but without a fixed number of rounds... play until I say stop. Put results on the board. What is different about this activity from the previous one where the number of rounds was fixed?]
 - g. Nash equilibrium
 - h. A short plug for game theory
3. Cardinality and Hilbert's Hotel [50-80 minutes]
 - a. How do we know that 4 is bigger than 3?
 - i. It comes after 3 – ordinals. Interesting topic, but we won't discuss it much... instead:
 - ii. 1-1 matching and left overs – cardinals.
 - b. Is there a largest natural number?
 - c. Hilbert's hotel [perhaps put students in small groups and have them discuss each question internally before discussing it altogether?]
 - i. Empty
 - ii. Taxi with numbers 0, 1, 2 arrives – where do we

put them up?

- iii. Stay one night, then leave
 - iv. Bus with all natural numbers arrives – where do we put them up?
 - v. 0 returns – where do we put it up?
 - vi. All the negative integers arrive – what do we do?
 - vii. Pi arrives – where to?
 - viii. All leave.
 - ix. [Optional, if time permits: Infinite number of busses arrive, labeled 1, 2, 3, ... Each is carrying all natural numbers. What do we do?]
 - x. [Optional, if time permits: All positive fractions arrive. What do we do?]
 - xi. Real numbers between 0 and 1 arrive. What do we do?
- d. Cantor's diagonalization
- i. Refresher on real number, terminating vs. non-terminating decimal expansions
 - ii. Diagonalization argument
 - iii. Discuss: is the conclusion of the diagonalization argument paradoxical?
4. [If time remains: Sets and Russell's paradox, 20 minutes]
- a. What is a set?
 - b. What sets have we discussed so far?
 - c. Assignment: come up with outlandish sets.
 - d. Russell's paradox
 - e. Paradox as one of the propelling forces in the development of our mathematical understanding