Worksheet #2 An Elevating Experience

The elevator in my apartment building (66 floors) has only two buttons, U and D. When you push the U-button the elevator goes up 8 floors (or not at all if there aren't 8 floors available above); when you push the D-button it goes down 11 floors (or not at all if there aren't 11 floors available below). Is it possible to take the elevator from any given floor to any other given floor?

Exercise: Let us show together that we can get from floor 66 to floor 1:

$$66 \xrightarrow{D} 55 \xrightarrow{D} 44 \xrightarrow{D} 33 \xrightarrow{D} 22 \xrightarrow{D} 11 \xrightarrow{U} 19 \xrightarrow{U} 27 \xrightarrow{U} 35 \xrightarrow{U} 43 \xrightarrow{U} 51$$
$$\xrightarrow{U} 59 \xrightarrow{D} 48 \xrightarrow{U} 56 \xrightarrow{D} 45 \xrightarrow{D} 34 \xrightarrow{D} 23 \xrightarrow{D} 12 \xrightarrow{D} 1$$

Question: We just showed we can get from floor 66 to floor 1. If it is also possible for us to get from every other floor to the one directly above it (i.e., from floor 1 to floor 2, from floor 2 to floor 3, etc.) then it is possible to get from any floor to any other. Why is this?

Exercise: Show that you can get from floor 1 to floor 2 using the elevator. Use the notation we used above.

Exercise: Show that for each floor numbered 1 to 65, you can get from that floor to the one directly above it. Use the notation we used above. Do you see any patterns that allow you to use previous solutions?