

Basic Probability

A **probability** is a number between 0 and 1 that describes the chances of some outcome happening.

The closer to 0, the less likely the outcome will occur. The closer to 1, the more likely the outcome will occur.

An outcome with a probability of 0 will not occur. An outcome with a probability of 1 must occur.

The probability of some outcome, E happening is given by

$$\frac{\# \text{ of ways } E \text{ can happen}}{\# \text{ of possible outcomes}}$$

Example 1

The basketball team sold 155 raffle tickets for a grand prize Playstation 4. Suppose you bought 4 raffle tickets, what is the probability that you will win the grand prize?

Example 2

There are 4 blue balls and 7 red balls in a bucket. You choose 1 at random. What is the probability that you choose a red ball?

Probability and Counting

Example 3

Let us properly shuffle a deck of 52 cards, is this arrangement of cards unique?

To make things easier let's first figure out the probability that this arrangement is **not** unique.

To simplify let us assume that every properly shuffled deck of cards until now has been unique. Then the probability that this deck of cards not being unique is given by

$$\frac{\# \text{ of card decks that have ever been properly shuffled}}{\# \text{ of ways to arrange a deck of 52 cards}}$$

This requires us to count the number of ways to arrange 52 cards.

How many ways can you arrange 2 cards?

How many ways can you arrange 3 cards?

How many ways can you arrange 4 cards?

Can we conjecture a rule to count the number of ways to arrange n cards?

Using this rule, how many ways are there to arrange 52 cards?

A Surprising Result

The number of ways to arrange a deck of 52 cards is:

$$52! \approx 8.07 \times 10^{67}$$

Now let us grossly overestimate the number of decks that have ever been properly shuffled. There are roughly 7×10^9 people on earth. Now suppose there was a time machine and everyone on earth was able to travel back in time to the beginning of the universe, roughly 4.4×10^{17} seconds ago. Then all 7×10^9 people were able to shuffle a million, 1×10^6 , decks each every second from the beginning of time until now.

How many decks will have been properly shuffled?

Assuming, every deck shuffled until now has been unique (again an overestimation) then the probability of the arrangement of this deck **not** being unique is given by

$$\frac{\# \text{ of decks that have ever been properly shuffled}}{\# \text{ of ways to arrange a deck of 52 cards}} \approx$$

Therefore, is the arrangement of this deck unique?