CHAMP: Mathematical Reasoning

1. Games

The game of three doors. You are a contestant on a game show. The host has placed a prize behind one of three doors (labeled $D1, D2, D3$). The game is played in two steps. First, you guess which door leads to the prize. Second, the host selects a door that you did not guess and that does not contain the prize. Once the host selects his door, he opens it and shows you that it does not contain the prize. The host then gives you a choice: you may either keep your original guess or switch your guess to the remaining closed door. Once you have made your choice, the host opens your door and shows you if you have won the prize.

For example, suppose you initially guess $D3$. Suppose further that the host then opens $D1$ and shows you that the prize is not behind $D1$. You now have a choice: keep your original $D3$ guess, or switch your guess to $D2$.

(1) Is it better to (a) keep your original guess or (b) switch your guess, or (c) are both strategies equally good? Why do you think so?

(2) Play this game many times with your group. For each game, record two letters. First, record S if the player switches her guess or N if she does not switch her guess. Second, record W if the player wins the prize or L is she does not win the prize. For example, a recording of NW for a given game means that the player does not switch her guess and wins the prize.

(3) Take a look at the results from the games. Now what do you think: Is it better to (a) keep your original guess or (b) switch your guess, or (c) are both strategies equally good? Why do you think so?

(4) What conclusions may be drawn from a mathematical analysis of the game? Do these conclusions surprise you?

The game of two doors. You enter a room with two unlabeled, identical doors. One of these doors leads to a fortune; the other leads to a ravenous horde of zombies. You must choose a door and then walk through it (you certainly want to avoid zombies). Two identical twins are with you in the room to help you make a decision. One of these twins always tells the truth and the other always lies; you do not know who is honest. Before you select a door, you may ask exactly one of the twins exactly one question.

(1) If you were required to select a door without any assistance from the twins, what is the probability that you select the door that leads to a fortune?

(2) Does this game have a winning strategy? Discuss with your group.

Birthday magic. Here is an interesting algorithm you may use to guess a friend's birthday.

- (a) Start with the number of the month in which your friend was born.
- (b) Multiply by 5.
- (c) Add 6.
- (d) Multiply by 4.
- (e) Add 9.
- (f) Multiply by 5 again.
- (g) Add the day of the month on which your friend was born.
- (h) Subtract 165.

After completing this process, you will be able to read off your friend's birthday.

(1) Try this algorithm using your own birthday. Does it work?

(2) Discuss why this algorithm works with your group.