CHAMP Activity — September 30, 2013

Exponential Growth and Decay

1. What is exponential growth?

   Describes a process that grows at a rate proportional to its current size.

2. What are some examples from the real world of exponential growth?

   Animal reproduction: The more mother rabbits there are, the more babies a rabbit population can have.

   Yeast is a fungus that splits in half to reproduce; bakers use it in bread and donut dough.

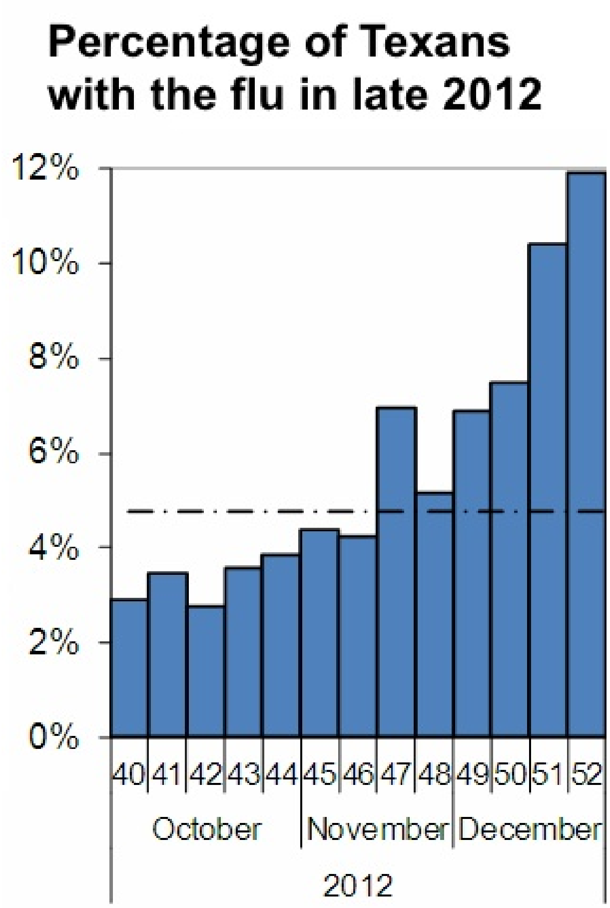
   Money gaining interest in a bank.

   Debt gaining interest.

   Chain reactions in nuclear weapons.

   Lightning.

3. Look at this graph from the State of Texas website on flu cases in 2012. Describe what's happening. Can you draw a **straight** line through the tops of all the bars?



4. We'll now do an activity to understand why the flu spreads in this way.

   Everybody stand up.

   One person will be selected at random to have the flu.

   One person will be selected to be the public health official.

   After one day, person with the flu infects another person. Public health official records this.

   After two days, each of these people infects another person. Public official records.

   Eventually, all are infected.

   This can be visualized as a "branching process."

5. What happened each time to the number of people infected?

   Number of infected people doubled. After k days, there were 2^k infected people.

6. How is an exponential function defined? How does it behave?

   Examples:

   $2^1 = 2$; $2^2 = 2 \cdot 2 = 4$; $2^3 = 2 \cdot 2 \cdot 2 = 8$; $2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$

   $5^5 = 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 3125$

   $9^8 = 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 = 43046721$

   a^b means to multiply 1 by a a total of b times

   This definition must be changed a bit when b is not a whole number, or it is negative. We'll discuss later.

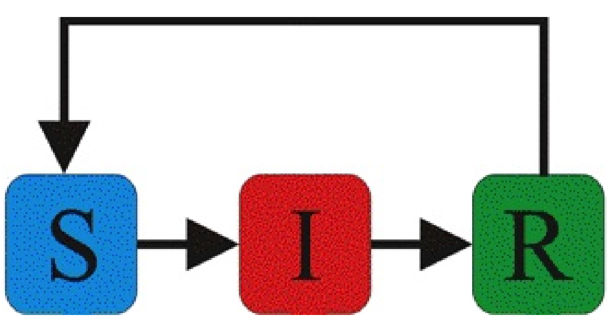
7. Is the model we ran through a reasonable model of flu infection?

   Not really. People don't interact that cleanly. You would expect there to be more "noise" in the data. Also, people can vaccinate against the flu. People can also heal over time.

8. How could it be improved?

   We could start by thinking about including recovery into the model.

   To do this: maybe have 1 person heal each day; maybe have half of all people heal each day; maybe have all people heal at some set time (disease cure).



9. Group exercise. Compare this choice of salaries for a job you work for a month:

   (a) You get paid a hundred dollars a day for a month.

   (b) You get paid a penny on the first day, but your salary doubles every day for a month.

   Which salary would you rather have? You don't have to calculate the exact amount of money you would get at the end of the month, just figure out which salary is better. Work this out in groups of two by calculating your salary over the first week, two weeks,...

   (a) $30 \times \$100 = \$3,000$

   (b) $0.01 + 0.02 + 0.04 + 0.08 + 0.16 + 0.32 + 0.64 + 1.28 + 2.56 + 5.12 + 10.24 + 20.48 + 40.96 + 81.92 + 163.84 + \cdots = (0.01) \times (2^{30} - 1) = \$10,737,418.23$

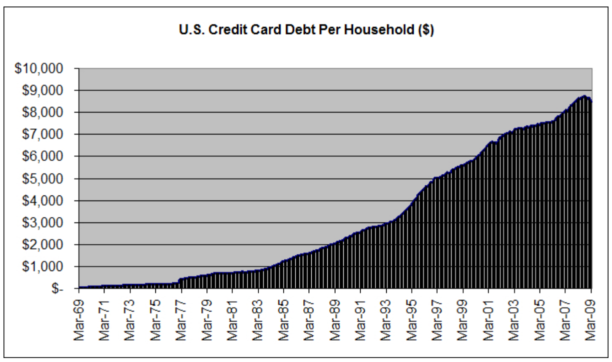
   (b) is better

A dangerous side of exponential growth in money: credit cards.

10. Does anyone have a credit card?

11. Have you ever not paid it off at the end of the month? What happens?

    The money you spent accumulates interest.



12. Most credit cards use **annual percentage rate.**

    Some student cards. VISA Citi (24%); Capitol One (21%); First Progress Mastercard (20%)

    If you don't pay off your balance at the end of the month, the credit card company charges you **more money** - interest.

    Interest is compounded at regular intervals (days or months), this gives the **effective annual rate**, which is actually how much interest you'll be charged if you don't pay a purchase off in a year.

$$EAR = \left(1 + \frac{APR}{n}\right)^n - 1$$

    where n is the number of **compounding periods** in a year. If days, $n = 365$, but if months, $n = 12$. This means the **effective annual rate** is higher than the **annual percentage rate**.

    Example: The effective annual rate when 25% APR is compounded every day for a year is

$$EAR = \left(1 + \frac{0.25}{365}\right)^{365} - 1 \approx 0.2839 = 28.39\%$$

    so the effective annual rate (or how much interest you end up **really** paying) is always a bit bigger than the **annual percentage rate** credit card companies tell you.

13. Say you buy clothes and shoes that cost \$500 using a credit card with 20% APR, and you do not pay it off for a year. If the interest is compounded every day, compute how much will you owe after a year?

$$\$500 \times \left(1 + \frac{0.20}{365}\right)^{365} = \$610.67$$

14. In Brazil, credit card interest rates are much higher, up to 200%. Say you bought \$100 of groceries with a credit card with 200%, but you do not pay this off for a year. If the interest is compounded every day, compute how much you'll owe in a year.

$$\$100 \times \left(1 + \frac{2}{365}\right)^{365} = \$734.88$$

Exponential decay

Exponential decay acts in a related way to **exponential growth**, except the amount of *something* shrinks in proportion to its present amount.

Examples:

15. *Chemical reactions.* Reactants (ingredients) in a chemical reaction get used up at a rate proportional to their present amount.

    For example, rust is formed (on metal roofs, metal chains, railroad tracks, metal pipes, ...) with the overall chemical reaction

    $$[\text{Iron}] + [\text{Water}] + [\text{Oxygen}] \quad \rightarrow \quad [\text{Rust}]$$

    So it will take the same amount of time for the first **half** of the iron to rust as the next **quarter** of iron to rust, and so on.

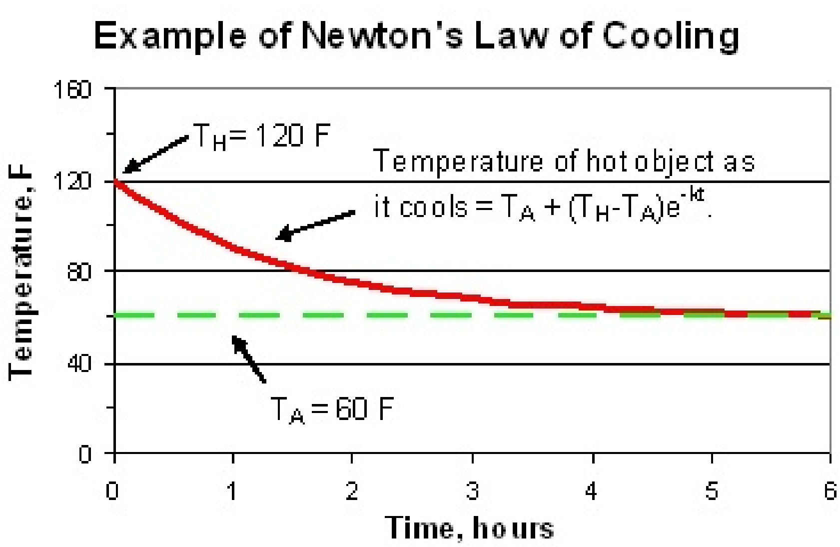
16. *Heat transfer.* The temperature $T(t)$ of an object moves to the temperature of it's surroundings.

    What are some examples of heat transfer in your everyday life?

    Cooking food (especially boiling food). Putting food in the refrigerator. Sun warms earth. Outside air warms us.

    Isaac Newton uncovered a relationship between the temperature of an object T_{object}, the temperature of its surroundings $T_{surroundings}$, and time t:

    $$T_{object}(t) = T_{surroundings} + (T_{starting} - T_{surroundings}) \left(\frac{1}{2}\right)^{at}$$



Having knowledge of Newton's law of cooling can help a detective identify the time at which a dead body was left outside.

17. **Group problem:** With help from the facilitators, and using Newton's Law of Cooling, figure out the time of death for the following murder case. A body is discovered in the woods of Sam Houston State Park forest at 2:00am, and the outside temperature most of the night has been 60°. The coroner measures the body's temperature to be 70°. If we assume the person's body was about 100° (close to 98.6°) at the time of death, how long has the body been in the woods?