CHAMP presentation #1
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Why do we have all these numbers anyhow?

Well the short answer is to solve problems.
The longer answer is because mathematicians find them fun!

Let’s look at numbers.

The first collection you learn about is the Natural numbers and you learn these early because they correspond to body parts – two eyes, 10 toes, 1 belly button.

Some of these get really big like how many dollars does Bill Gates have?
(47×10^7)

Notice that you needed a teacher to learn to write that number efficiently!

Then, you get Whole numbers…like how many chips are there when your brother is done with the bag? zero.

$\{0,1,2,3,...\}$

In diagrams:

Then, with lemonade stands, come negative numbers and fractions.

If you spend $7 on ingredients, how much money have you made when you open the stand?

Integers: diagrams and number “line”

If your first customer pays 50 cents where are you…and what is that 50 cents anyhow?

Rational numbers…numberline “dust”

Diagrams again:

Then after you’ve been in school for a while: irrational numbers….

How long is that diagonal on a square with 1 foot sides?

What is the ratio of circumference to diameter for any circle?

At last, the real numberline!

And the set diagram?

Let’s list some irrational numbers:

Here's my favorite irrational number

.01001000100001000001…

What is that definition again? Nonrepeating and nonterminating!

Now let's look at what mathematicians do to these sets!

First off, let's go back to natural numbers: {1, 2, 3, 4, 5, …}

Let's look at a couple of subsets of the natural numbers:

Composites and Primes

Definitions:

Primes:

Composites:

And, of course

One:

Picture:

Now, just naturally mathematicians got into playing with natural numbers. Early in the 1800's someone started playing around with number strings of one.

You know them,

1, 11, 111, 1111, 11111, …

There are an infinite number of these. How do you know that?

By 1966, there were enough people playing around with these, that Dr. Albert Beiler gave them a name:

Repunit	repeated unit

SOME repunits are prime, while others are composite, and there's that 1, too:

Diagram:

How can you tell which is which? Well, there's a way to tell for SURE which are composite.

Not so long ago, rather than writing down all the one's, someone came up with subscripts:

$R_1 = 1$
$R_2 = 11$
$R_3 = 111$
$R_4 = 1111$

$R_{\textit{your age}}$

R_{57}

R_{2013}

R_n

And then some other people noticed a pattern with primes and composites…let's look at the next two pages and see if we can see the pattern.

$R_1 = 1$ *that one*

$R_2 = 11$ *prime*

$R_3 = 111$ *prime*

$R_4 = 1111 = 11(101)$ *composite*

$R_5 = 11111$ *prime*

$R_6 = 111111 = 3(7)(11)(13)(37)$

$R_7 = 239(4649)$

$R_8 = 11(73)(101)(137)$

$R_9 = 3^3(37)(333667)$

$R_{10} = 11(41)(271)(9091)$

$R_{11} = 21649(513239)$

$R_{12} = 3(7)(11)(13)(37)(101)(9901)$

$R_{13} = 53(79)(265371653)$

$R_{14} = \textit{composite}$

$R_{15} = \textit{composite}$

$R_{16} = \textit{composite}$

$R_{17} = \textit{composite}$

$R_{18} = \textit{composite}$

$R_{19} = \textit{prime}$

$R_{20} = \textit{composite}$
...
$R_{23} = \textit{prime}$
...
$R_{317} = \textit{prime}$
...
$R_{1031} = \textit{prime}$
...
$R_{49081} = \textit{probably prime}$
...

What's the pattern? Let's take some time and see if we can see it!

The study of repunits bloomed with the advances in computers! Factoring primes is still hard work…someone will find a "probably prime" repunit and it takes a couple of YEARS to factor it! For example, R_{317} was called "probably prime" in about 1966; it took until 1977 to PROVE it prime!

Now all of this was about base 10 primes. But there are other bases for numbers, right? …how many of you are familiar with binary? Hex? Base 7?

"There are 11 kinds of people in the world…those who know binary and those who don't."

Computer science tee shirt.

We can review binary: digits {0, 1}

$1_2 = 1$

$10_2 = 2 + 0 = 2$

$11_2 = 2^1 + 2^0 = 3$

$100_2 = 2^2 + 0 + 0 = 4$

Let's make a number line with base 10 on top and base 2 on the bottom

Let’s do some adding base 2.

Ok, now let’s get back to Primes, repunits and base 2 repunits.

Base 2 repunit primes are called Mersenne Primes. Let's unpack that by talking about Mersenne Primes first – these are natural base 10 numbers. Then we'll tie in base 2 repunits.

Many early (like 16th century) mathematicians felt that all the numbers of the form 2^n-1 were prime for *all* primes *n*, (NOT all *n*, just prime *n*'s)

$n = 2,3,5...$

$2^n - 1$

but in 1536 Hudalricus Regius showed that 2^{11}-1 = 2047 was not prime (it is $23 \cdot 89$). By 1603 Pietro Cataldi had correctly verified that 2^{17}-1 and 2^{19}-1 were both prime, but then incorrectly stated 2^n-1 was also prime for 23, 29, 31 and 37. In 1640 Fermat showed Cataldi was wrong about 23 and 37; then Euler in 1738 showed Cataldi was also wrong about 29. Sometime later Euler showed Cataldi's assertion about 31 was correct.

Enter French monk Marin Mersenne (1588-1648). Mersenne stated in the preface to his *Cogitata Physica-Mathematica* (1644) that the numbers 2^n-1 were prime for

n = 2, 3, 5, 7, 13, 17, 19, **31, 67, 127 and 257**

and were composite for all other positive integers $n < 257$ From 2 to 257 is called Mersenne's range. Mersenne's (incorrect) conjecture fared only slightly better than Regius', but still got his name attached to these numbers.

Definition: When 2^n-1 is prime it is said to be a **Mersenne prime**.

It was obvious to Mersenne's peers that he could not have tested all of these numbers (in fact he admitted as much), but they could not test them either. It was not until over 100 years later, in 1750, that Euler verified the next number on Mersenne's and Regius' lists, 2^{31}-1, was prime. After another century, in 1876, Lucas verified 2^{127}-1 was also prime. Seven years later Pervouchine showed 2^{61}-1 was prime, so Mersenne had missed this one. In the early 1900's Powers showed that Mersenne had also missed the primes 2^{89}-1 and 2^{107}-1.

Finally, by 1947 Mersenne's range, $n \le 258$, had been completely checked and it was determined that the correct list is:

$n = 2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 107$ and 127. Now these are the exponents, not the primes…Let’s look at a list of the Mersenne Primes:

The table below lists some Mersenne primes

#	***p***	M_p	M_p **digits**	**Discovered**	**Discoverer**	**Method used**
1	2	3	1	c. 430 BC	Ancient Greek mathematicians[15]	
2	3	7	1	c. 430 BC	Ancient Greek mathematicians[15]	
3	5	31	2	c. 300 BC	Ancient Greek mathematicians[16]	
4	7	127	3	c. 300 BC	Ancient Greek mathematicians[16]	
5	13	8191	4	1456	Anonymous[17][18]	Trial division
6	17	131071	6	1588[19]	Pietro Cataldi	Trial division[20]
7	19	524287	6	1588	Pietro Cataldi	Trial division[21]
8	31	2147483647	10	1772	Leonhard Euler[22][23]	Enhanced trial division[24]
9	61	2305843009213693951	19	1883 November[25]	I. M. Pervushin	Lucas sequences
10	89	618970019…449562111	27	1911 June[26]	R. E. Powers	Lucas sequences
11	107	162259276…010288127	33	1914 June 1[27][28][29]	R. E. Powers[30]	Lucas sequences
12	127	170141183…884105727	39	1876 January 10[31]	Édouard Lucas	Lucas sequences
13	521	686479766…115057151	157	1952 January 30[32]	Raphael M. Robinson	LLT / SWAC
14	607	531137992…031728127	183	1952 January 30[32]	Raphael M. Robinson	LLT / SWAC
15	1,279	104079321…168729087	386	1952 June 25[33]	Raphael M. Robinson	LLT / SWAC
16	2,203	147597991…697771007	664	1952 October 7[34]	Raphael M. Robinson	LLT / SWAC
17	2,281	446087557…132836351	687	1952 October	Raphael M. Robinson	LLT / SWAC

				9[34]		
18	3,217	259117086…909315071	969	1957 September 8[35]	Hans Riesel	LLT / BESK
19	4,253	190797007…350484991	1,281	1961 November 3[36][37]	Alexander Hurwitz	LLT / IBM 7090
20	4,423	285542542…608580607	1,332	1961 November 3[36][37]	Alexander Hurwitz	LLT / IBM 7090
21	9,689	478220278…225754111	2,917	1963 May 11[38]	Donald B. Gillies	LLT / ILLIAC II
22	9,941	346088282…789463551	2,993	1963 May 16[38]	Donald B. Gillies	LLT / ILLIAC II

This is from Wikipedia…

Now let’s look at some base 2 numbers and expand them and see this connection!

$1_2 = 1$

$11_2 = 2 + 1 = 3 = 2^2 - 1$

$111_2 = 4 + 2 + 1 = 7 = 2^3 - 1$

$1111_2 = 8 + 4 + 2 + 1 = 15 \quad \textit{nope}$

$11111_2 = 16 + 15 = 31 = 2^5 - 1$

what's the next one and the one after that?

Note that Mersenne Primes are a proper subset of the Primes. Let's draw the set diagram.

How will we work in the base 2 repunits?

Let's review the connections.

Somebody started looking a repunits and then named them. Then noticed an interesting connection between the subscripts and the type of number it was, prime or composite.

Then somebody with an interest in numbers other than base 10 noticed repunits in those bases and found yet another, more complicated connection right back to primes again, but only a certain proper subset of the primes, the Mersenne primes.

And notice how things have gone throughout time…MUCH older work is being woven into newer work with living mathematicians.

Now let's talk about the number 3.

What are all the things we know about 3?

With respect to repunits?

With respect to Mersenne Primes?

Now let's look at some special websites. One of my favorites is Number Gossip.

http://www.numbergossip.com/list

Number Gossip

(Enter a number and I'll tell you everything you wanted to know about it but were afraid to ask.)

Unique Properties of 3

3 is the greatest number of consecutive integers that can be pairwise relatively prime

3 is the only prime sandwiched between a prime and a composite number

3 is the only prime followed by a square

Every positive integer is the sum of at most 3 triangular numbers

If the n-th Fibonacci number is prime, then n must itself be prime, with the exception of 3, which is the 4th Fibonacci number

3 is the only number which is equal to the sum of all the natural numbers less than it

3 is the only triangular number that is also prime

Rare Properties of 3

- Fibonacci
- Mersenne
- Mersenne prime
- Narcissistic
- Palindromic prime

Common Properties of 3

- Deficient
- Evil
- Lucky
- Odd
- Palindrome
- Prime
- Square-free
- Triangular
- Twin
- Ulam

Let’s explore some of these properties, using the definitions from Number gossip:

Common Properties of 3

Deficient

The number n is *deficient* if the sum of all its positive divisors except itself is less than n.

Evil

The number n is *evil* if it has an even number of 1's in its binary expansion.

Guess what odious numbers are.

- **3**,
- 5,
- 6,
- 9,
- ...

Lucky

Odd

Palindrome

Prime

Square-free

Triangular

Twin Prime

Ulam

Fibonnaci

Mersenne

Mersenne Prime

Palindrome Prime