# PATTERNS AND GAMES

## "Last One Standing"

Four people are sitting on chairs in a line all facing the same direction, so each person can see all the people in front of them and none of the people in back of them. We start with everyone sitting, and each turn we can make a move and have one person change position (either stand up or sit down) subject to the following rules:

**Rule 1:** The front person is allowed to change position (stand or sit) on any turn.

**Rule 2:** All other people can change position (stand or sit) only if the person in front of them is standing and all other people in front of them are sitting.

Our goal is to have the back person standing and all the other people sitting. We call this a *winning position*.

QUESTION 1: What is the fewest number of moves we need to put 4 people who are all sitting into a winning position?

QUESTION 2: How many moves would it take if we started with 10 people sitting in a line? How many moves would it take if we started with 15 people sitting in a line? How many moves would it take if we started with 20 people sitting in a line?

QUESTION 3: If we did one move per second (so one person sits or one person stands each second), how long would it take to move to a winning position if we start with 4 people? If we start with 10 people? If we start with 15 people? If we start with 20 people?

# PATTERNS AND GAMES

## "Nim"

Two players play a game. There are two piles that each have 5 stones, and the players take turns.

**Rule 1:** On each person's turn they must take away 1 or more stones. However, all stones taken must be from the same pile.

**Rule 2:** The person who takes the last stone wins.

QUESTION 1: Break into groups of two and play this game a few times. Can you figure out a strategy to win? Would you rather go first or go second in this game?

QUESTION 2: What happens if we change the number of stones so there are 6 in each pile? So there are 7 in each pile?

QUESTION 3: What if we changed the game so we had two piles that have an unequal number of stones? Would this change your strategy? Would you want to go first or second in this game?

# PATTERNS AND GAMES

## "Twenty-One"

Two players play a game. We start by writing the number 0 on a piece of paper.

**Rule 1:** On each person's turn, they may add 1, 2, or 3 to the existing number. A player is not allowed to pass or to add zero.

**Rule 2:** The first person to reach 21 (or more) wins.

QUESTION 1: Break into groups of two and play this game a few times. Can you figure out a strategy to win? Would you rather go first or go second in this game?

QUESTION 2: What happens if we change the winning number to 22? Would this change your strategy? Would you want to go first or second in this game?

QUESTION 3: What if we changed the game so you are allowed to add 1, 2, 3, or 4 on any turn? Would this change your strategy? Would you want to go first or second in this game?