#5 The Birthday Paradox

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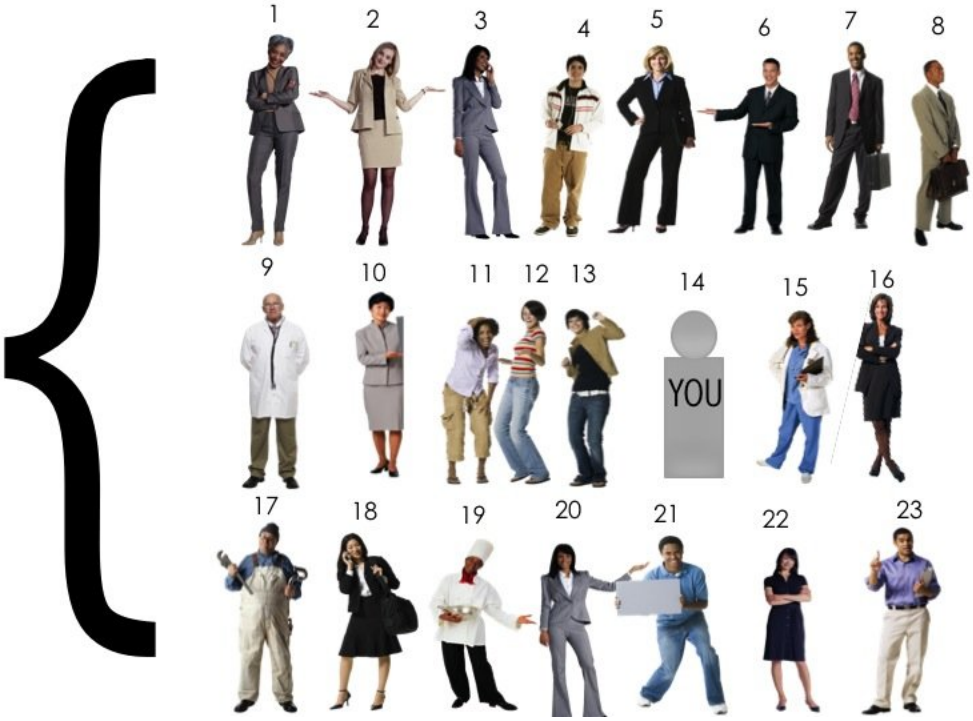
Let's say you work in an office of 23 people.

What is the probability that two people in your office have the same birthday?

(We'll ignore February 29th for the purposes of the problem)

Your 23-person office



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ANSWER: There's a 50% chance two people share a birthday

- Once a population hits 366 people, it is statistically guaranteed that two people have the same birthday since (for the purposes of our problem) there are only 365 possible birthdays
- However, assuming that all birthdays are equally likely, once you have 57 people grouped together there is a 99% chance that two of them have the same birthday.

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How do we figure this out?

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- Let's look back at our 23-person office to understand how this is possible.
- We're going to calculate the **converse probability** — that no two people in the group share the same birthday — to figure out what we want.
- Figuring out the probability that at least two people in the office have the same birthday is difficult if you attack it head on.
- ***Figuring out the probability that nobody in a group of people has the same birthday is much, much easier.***

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The probability that **two** people don't have the same birthday is this:

$$\frac{365}{365} \times \frac{364}{365} = 99.72\%$$

1

2

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The probability that **three** people don't have the same birthday is this:

$$\frac{365}{365} \times \frac{364}{365} \times \frac{363}{365} = 99.17\%$$

1

2

3

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The probability that **four** people don't have the same birthday is this:

$$\frac{365}{365} \times \frac{364}{365} \times \frac{363}{365} \times \frac{362}{365} = 98.36\%$$

1

2

3

4

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See where we're going with this? So, the probability that 23 people don't have the same birthday is:

$$\frac{365}{365} \times \frac{364}{365} \times \frac{363}{365} \times \cdots \times \frac{344}{365} \times \frac{343}{365} = 49.27\%$$

1 2 3 4 – 21 22 23

This means that since there's a 49.3% chance that *nobody* has the same birthday, **there's a 50.7% chance that *at least two people have the same birthday*.**

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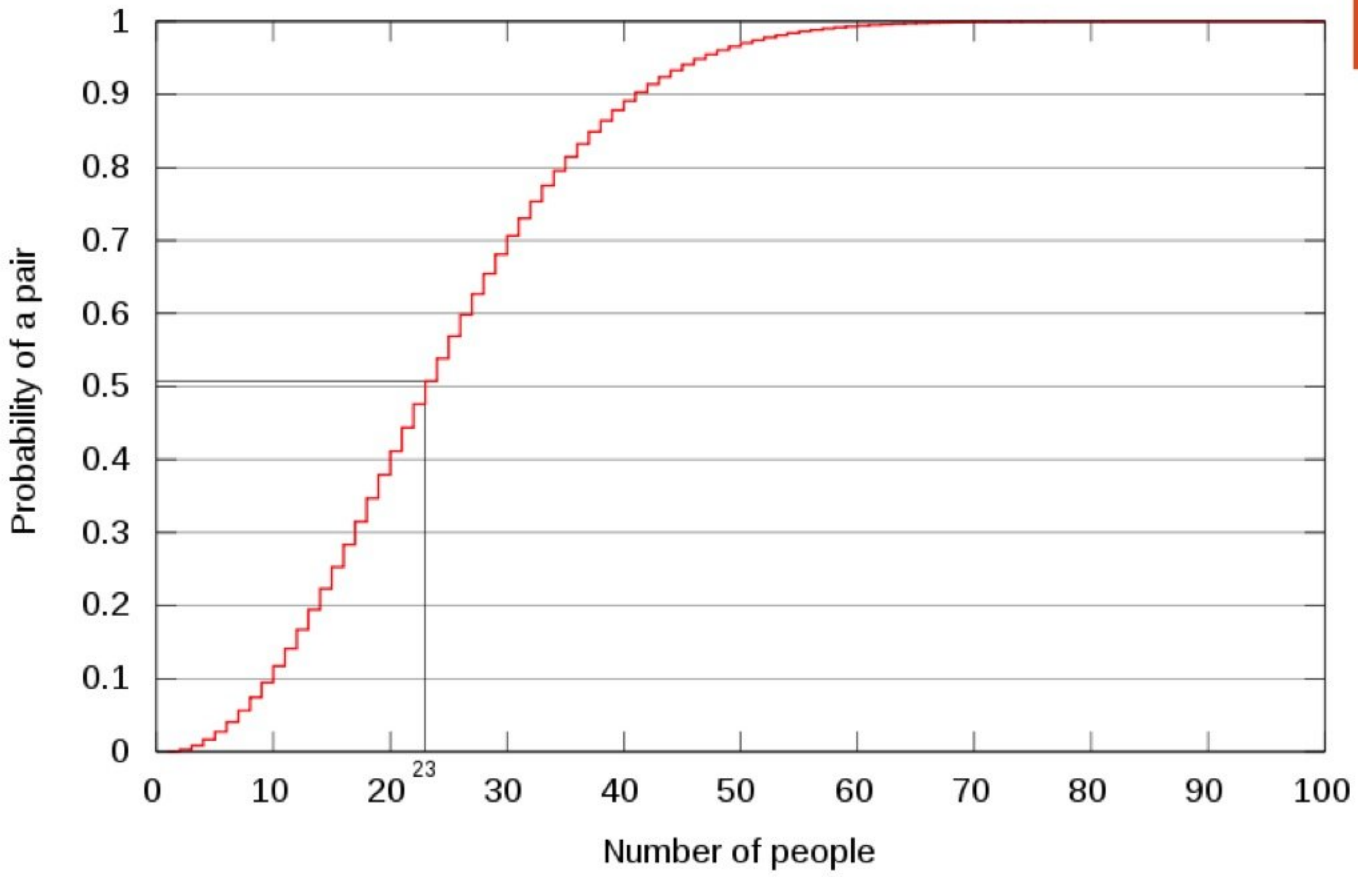
Here's what the probability curve looks like:



Source: Rajkiran g, Wikime

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